Assignment 13

Kummitha Jhanavi (CS21BTECH11032)

June 19, 2022

Outline

Question

Referring to Example 15.8 , let e_1,e_2,,,,e_N represent the states with two end reflecting barriers and transition matrix as in (15-22).Thus $p_{i,i+1}$=p, $p_{i,i-1}$=q for 2 $\leq$ i $\leq$ N-1,, p_{11}=q,p_{12}=p, and $p_{N,N-1}$=q,p_{NN}=p.

Reference

Example 15.8: Suppose the two boundaries in Example 15-7 reflect the particle to the adjacent state instead of adsorbing it. With e_1,e_2,...,e_N representing the N states , the end reflection probabilities to the right and left are given by

$$p_{1,2}\text{=p and } p_{N,N-1}\text{=q}$$

(15-22) and this gives the N*N transition matrix to be

$$p= \begin{pmatrix} q & p & 0 & 0 & . & . & . & 0 \\ q & 0 & p & 0 & 0 & . & . & . \\ 0 & q & 0 & p & 0 & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & q & 0 & p \\ 0 & 0 & 0 & . & . & 0 & q & p \end{pmatrix}$$

Solution

$\sum_{j=1}^{N} \mathrm{p}_{ij} x_j^{(k)} = \lambda_k x_i^{(k)}$
Substituting in above equation we get

$\mathrm{x}_1 = \mathrm{s}(\mathrm{q}\mathrm{x}_1 + p\mathrm{x}_2)(2)$

$\mathrm{x}_i = \mathrm{s}(\mathrm{q}\mathrm{x}_{i-1} + p\mathrm{x}_{i+1}) i = 2, 3, ...N - 1(3)$

$\mathrm{x}_N = \mathrm{s}(\mathrm{q}\mathrm{x}_{N-1} + p\mathrm{x}_N)(4)$

Where we have used s $= 1/\lambda$. Clearly λ=1 corresponds to the specific solution $x_i = 1$.To find all other solutions , notice that $x_i =$ s(q$x_{i-1} +$ px_{i+1}) satisfies the particular solution
$x_i = \xi^i$
provided ξ is a root of the quadratic equation

ξ=qs+psξ^2(5)
The two roots of the quadratic equation are

$$\xi_1(\mathrm{s}) = \frac{1+\sqrt{1-4pqs^2}}{2ps}$$

$\xi_2(\mathrm{s}) = \frac{1+\sqrt{1-4pqs^2}}{2ps}$ and the general solution to $x_i =$ s(q$x_{i-1} + px_{i+1}) isgivenby$
for i = 2,....,N-1

x$_i$=a(s)$\xi_1{}^i(s) + b(s)\xi_2{}^i(s)(6)$

where a(s) and b(s) are yet to be determined .For equation 2 to satisfy equation 6 , it must have the same form as equation 3 and hence we must have $x_1 = x_0$. Similarly for equation 4 to satisfy equation 6 , on comparing it with equation 3 , we must have $x_N = x_{N+1}$.But

$$\mathrm{x}_1 = x_0 \Rightarrow \mathrm{a(s)[1\text{-}}\xi_1\mathrm{(s)]= \text{-}b(s)[1\text{-}}\xi_2\mathrm{(s)]} \ (7)$$

and

$$\mathrm{x}_N = x_{N+1} \Rightarrow \mathrm{a(s)[1\text{-}}\xi_1\mathrm{(s)]}{\xi_1}^N(s) = -b(s)[1-\xi_2\mathrm{(s)]}{\xi_2}^N(s)(8)$$

a

nd from equation 7 anad 8 we must have

$$\xi_1{}^N(s) = \xi_2{}^N(s) with \xi_1(\mathrm{s}) \neq \xi_2(\mathrm{s})$$

But from equation 5 , we get $\xi_1(\mathrm{s})\ \xi_2(\mathrm{s}) = \mathrm{q/p}$ so that equation $\xi_1{}^N(s) = \xi_{12}{}^N(s) reducesto$
$\lambda_k = \frac{1}{s_k} For \lambda_0 = 1$ we obtain directly $x_i^{(0)} = 1$ we get
$x_i{}^{(k)} = (\frac{q}{p})^{i/2} sin \frac{\pi ki}{N} - (\frac{q}{p})^{(i+1)/2} sin \frac{\pi k(i-1)}{N} wherek = 1, 2, , , N-1$