Assignment 5

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Outline

Problem Statement

Question

Three switches connected in parallel operate independently. Each switch remains closed with probability p. (a) Find the probability of receiving an input signal at the output.(b) Find the probability that switch S_1 is open given that an input signal is received at the output.

Figure

figure

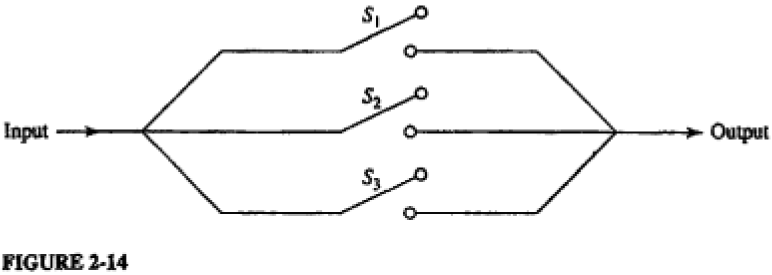

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Figure: Figure 1

Solution (a)

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et A_i = "Switch S_i is closed." then P(A_i) = p, i=1,2,3.
Since switches operate independently, we have

$$\mathrm{P}(A_iA_j) = \mathrm{P}(A_i)\mathrm{P}(A_j)$$

$$\text{Similarly , } \mathrm{P}(A_1A_2A_3) = \mathrm{P}(A_1)\mathrm{P}(A_2)\mathrm{P}(A_3)$$

Let R represents the event "Input signal is received at the output"

R= $A_1 \cup A_2 \cup A_3$

$$\mathrm{P(R)} = \text{1-P(R)=1-P}(\overline{A}_1\overline{A}_2\overline{A}_3)\text{= 1-P}(\overline{A}_1)\mathrm{P}(\overline{A}_2)\mathrm{P}(\overline{A}_3)$$
$$=\text{1- (1-p)}^3 = 3p - 3p^2 + p^3$$

(b). We need P($\frac{\overline{A}_1}{R}$).From Bayes theorem
P($\frac{\overline{A}_1}{R}$)=$\frac{P(R/\overline{A}_1)P(\overline{A}_1)}{P(R)}$=$\frac{(2p-p^2)(1-p)}{3p-3p^2+p^3}$=$\frac{2-3p+p^2}{3-3p+p^2}$.

Conclusion

Because of the symmetry of the switches, we also have
$P(\frac{\overline{A_1}}{R})=P(\frac{\overline{A_2}}{R})=P(\frac{\overline{A_3}}{R})$