

Assignment 7

Kummitha Jhanavi (CS21BTECH11032)

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Question

The random variables x and y are $N(0,4)$ and independent. Find $f_z(z)$ and $F_z(z)$ if (a) $z=2x + 3y$, and (b) $z= \frac{x}{y}$

Definition

For any two random variables x and y , let

$$\sigma_x^2 = \text{Var}x, \sigma_y^2 = \text{Var}y \text{ and } \sigma_{x+y}^2 = \text{Var}(x+y)$$

$$\text{Then, } \frac{\sigma_{x+y}^2}{\sigma_x^2 + \sigma_y^2} \leq 1$$

Solution

(a) Given $z=2x + 3y$

$$E(z)=0$$

$$\sigma_z^2 = 4\sigma_x^2 + 9\sigma_y^2 = 5^2 \quad (1)$$

$$\sigma_x = \sigma_y = 2$$

$$\sigma_z^2 = 4(4) + 9(4) \quad (2)$$

Hence z is $N(0, \sqrt{52})$

(b) Given $z=\frac{x}{y}$

From definition $\sigma_1 = \sigma_2 = 2, r = 0$

$$F_z(z) = \frac{1}{2} + \frac{1}{\pi} \arctan z \implies f_z(z) = \frac{1}{\pi(1+z^2)} \quad (3)$$