Assignment 8

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Outline

1. Question

2. Solution

Question

If x and y are independent exponential random variable with common parameter λ. Show that $\frac{x}{(x+y)}$ is a uniformly distributed random variable in (0,1).

Solution

X,Y are independent identically distributed exponential random variables

$$Z = \frac{X}{X+Y}$$

F_Z(z) =P($\frac{X}{X+Y}$≤z)=P($\frac{X}{Y}$≤ $\frac{z}{1-z}$)
=P(X $\leq \frac{zY}{1-z}$) $= \int_0^\infty \int_0^{\frac{zy}{1-z}} f_{XY}(x, y)dxdy(1)$

f$_Z$(z)=$\int_0^\infty \frac{y}{(1-z)^2} f_{XY}(\frac{zy}{1-z}, y)dy(2)$

$$= \frac{1}{(1-z)^2} \int_0^\infty y \frac{1}{\lambda^2} e^{-(z/(1-z)+1)(y/\lambda)} dy$$
$$= \int_0^\infty ue^{-u}du = 1, 0 < z < 1$$

$=\frac{X}{X+Y} \sim U(0, 1)$