Assignment 9

Kummitha Jhanavi (CS21BTECH11032)

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Outline

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Question

(a). Reasoning as in (6.31), Show that if the random variables x,y, and z are independent and their joint density has spherical symmetry:
f(x,y,z) = f($\sqrt{x^2+y^2+z^2}$)
then they are normal with zero mean and equal variance.
(b). The components v_x,v_y,v_z of the velocity v = $\sqrt{v_x+v_y+v_z}$ of a particle are independent random variables with zero mean and variance kT/m. Furthermore, their joint density has spherical symmetry .Show that v has a Maxwell density and

E(v) + $2\sqrt{\frac{2kT}{m\pi}}$

E(v^2) = $\frac{3kT}{m}$

E(v^4) = $\frac{15k^2T^2}{m^2}$

Description

x and y are independent Rayleigh random variables with common parameter σ^2. then the density of $\frac{x}{y}$ is

$f_{XY}(x,y) = \frac{xy}{\sigma^4} e^{-\frac{(x^2+y^2)}{2\sigma^2}}, x, y \geq 0$

$$Z = \frac{X}{Y}$$

$F_Z(z) = P(Z \leq z) = P(\frac{X}{Y} \leq z) = \int_0^\infty \int_0^{xy} f_{XY}(x,y)dxdy.(1)$
This gives the density function of z to be

$$f_Z(z) = \int_0^\infty y f_{XY}(zy, y)dy = \int_0^\infty \frac{zy^3}{\sigma^4} e^{-(\frac{z^2y^2+y^2}{2\sigma^2})} dy \tag{2}$$

$= \frac{z}{\sigma^4} \int_0^\infty y^3 e^{-y^2(z^2+1)/2\sigma^2} dy$
let , t $= \frac{y^2(z^2+1)}{2\sigma^2}$
$= \frac{2z}{(z^2+1)^2} \int_0^\infty te^{-t} dt = \frac{2z}{(z^2+1)^2}$

Solution

(a) the joint density f(x,y) has circular symmetry because

$$f(x,y) = \int_{\infty}^{-\infty} f(\sqrt{x^2+y^2+z^2})dx \tag{3}$$

depends only on (x^2+y^2). The same holds for f(x,z) and f(y,z). And since the RVs x,y and z are independent, they must be normal

(b)
From (a) it follows that the RVs $\mathrm{v}_x, v_y, v_z areN(0,\sqrt{kT/m}).With\sigma^2$ $=\frac{kT}{m}$and n $=$3 it follows
$\mathrm{f}_v(v) = \sqrt{\frac{2m^3}{\pi k^3 T^3}} v^2 e^{-mv^2/2kT} U(v)$
E(v)$= 2\sqrt{\frac{2kT}{\pi m}} e(v^2 n) = 1*3*........2n+1)(\frac{kT}{m})^n$

Reason

(a) If the random variables x and y are circularly symmetrical and they are normal with zero meam and equal variance

$$g(\sqrt{x^2+y^2}) = f_x(x)f_y(y) \tag{4}$$

(b)

$$f_Z(Z) = \frac{1}{\pi^\pi |C_{ZZ}|} = e^{-ZC_{ZZ}^{-1}Z^+} \tag{5}$$

$$\phi_Z(\Omega) = e^{-\frac{1}{4}\Omega C_{ZZ}\Omega^+} \tag{6}$$

If x has a Rayleigh density
$f(x) = \frac{x}{\sigma^4} e^{-x^2/2\sigma^2} U(x)$
then , $E(x^n) = \frac{1}{\sigma^2}\int_0^\infty x^{n+1} e^{-x^2/2\sigma^2} dx = \frac{1}{2\sigma^2}\int_{-\infty}^\infty |x|^{n+1} e^{-x^2/2\sigma^2} dx$