統計學（一）

第六章 抽樣分佈
(Distributions of Sample Statistics)

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本課程內容參考書目

- **教科書**
  - P. Newbold, W. L. Carlson and B. Thorne(2007). *Statistics for Business and the Economics*, 7^{th} Edition, Pearson.
- **參考書目**
  - Berenson, M. L., Levine, D. M., and Krehbiel, T. C. (2009). *Basic business statistics: Concepts and applications*, 11^{th} EditionPrentice Hall.
  - Larson, H. J. (1982). *Introduction to probability theory and statistical inference*, 3^{rd} Edition, New York: Wiley.
  - Miller, I., Freund, J. E., and Johnson, R. A. (2000). *Miller and Freund's Probability and statistics for engineers*, 6^{th} Edition, Prentice Hall.
  - Montgomery, D. C., and Runger, G. C. (2011). *Applied statistics and probability for engineers*, 5^{th} Edition, Wiley.
  - Watson, C. J. (1997). *Statistics for management and economics*, 5th Edition. Prentice Hall.
  - 唐麗英、王春和（2013），「從範例學MINITAB統計分析與應用」，博碩文化公司。
  - 唐麗英、王春和（2008），「SPSS 統計分析 」，儒林圖書公司。
  - 唐麗英、王春和（2007），「Excel 統計分析」，第二版，儒林圖書公司。
  - 唐麗英、王春和（2005），「STATISTICA與基礎統計分析」，儒林圖書公司。

抽樣分布
(Sampling Distributions)

Sampling Distributions (抽樣分佈)

Recall: Parameter and statistic (參數與統計量)

- A **quantity** computed from the observations in a **population** is called a **parameter**.
- A **quantity** computed from the observations in a **sample** is called a **statistic.**

Example: 1) μ, σ and P are parameters.

2) $\bar{\mathbf{X}}$, S and $\hat{\mathbf{p}}$ are statistics.

Sampling Distributions (抽樣分佈)

- **Sampling Distribution 抽樣分佈**
  - The **probability distribution** of a **statistic** that results when random sample of size **n** are repeatedly drawn from a given population is called the **sampling distribution** of the **statistic**.
  - **統計量之機率分佈**稱為「**抽樣分佈**」。

- **Example：**
  - The distribution of **sample mean** $\overline{X}$ is one type of **sampling distribution.** The distribution of **sample proportion** $\hat{p}$ is another type of **sampling distribution.**

樣本平均數的抽樣分布
(The Sampling Distributions of Means)

1) What is the Sampling Distribution of the Sample Mean, $\overline{X}$ (When σ is known) ？

i.e., How the statistic $\overline{X}$ behaves in the repeated sampling ?

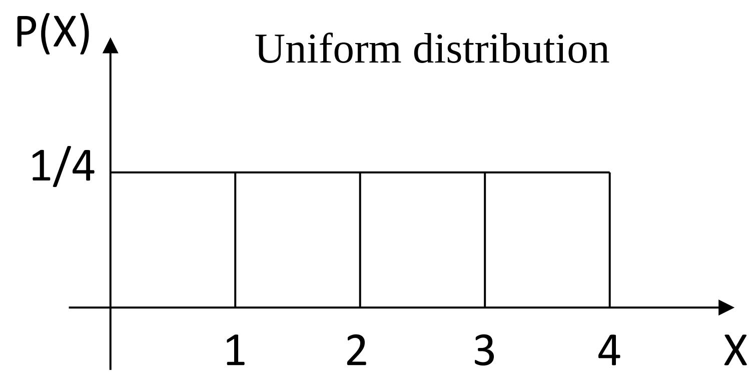
Sampling Distribution of the Sample Mean

- **例 1：**

Suppose a population consists of four numbers (N=4): 1,2,3,4

Since the four values are distinct, the population probability distribution assigns an equal probability of **1/4** to each value of x in the population.

x	P(x)	xP(x)	$x^2P(x)$
1	1/4	1/4	1/4
2	1/4	2/4	4/4
3	1/4	3/4	9/4
4	1/4	4/4	16/4



The **population mean** and **variance** are

$$\mu = \sum_{i=1}^{n} xP(x_i) = 2.5, \qquad \sigma^2 = \sum_{i=1}^{N} x^2P(x_i) - \mu^2 = 7.5 - 2.5^2 = 1.25$$

Sampling Distribution of the Sample Mean

- **Step 1 :** Take a **random sample** of size **2 with replacement** from the population. How many possible samples are there?

  4 * 4 = 16 possible samples.

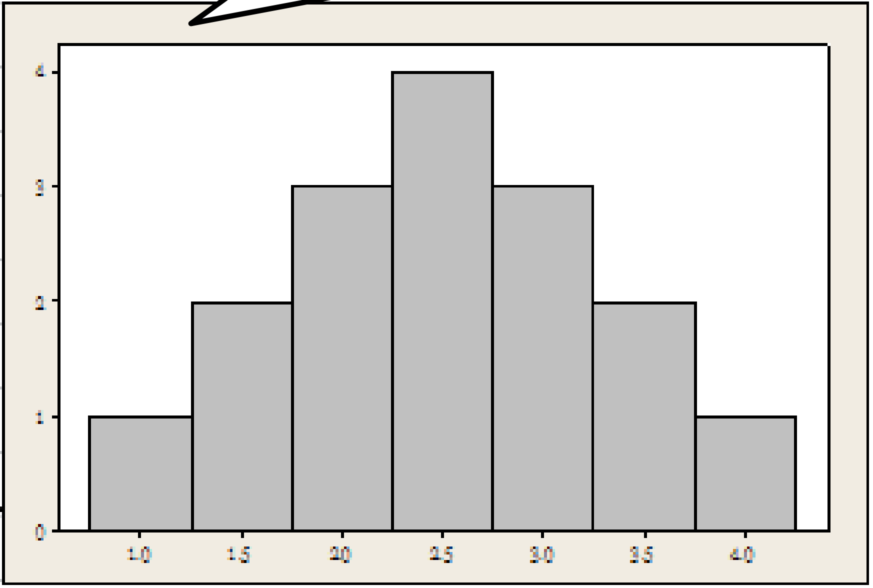
Sample	Sample mean $\bar{X}$	Sample	Sample mean $\bar{X}$
(1 , 1)	1	(3 , 1)	2
(1 , 2)	1.5	(3 , 2)	2.5
(1 , 3)	2	(3 , 3)	3
(1 , 4)	2.5	(3 , 4)	3.5
(2 , 1)	1.5	(4 , 1)	2.5
(2 , 2)	2	(4 , 2)	3
(2 , 3)	2.5	(4 , 3)	3.5
(2 , 4)	3	(4 , 4)	4

Sampling Distribution of the Sample Mean

- **Step 2:** Construct the **probability distribution** for the sample mean $\bar{X}$.

Close to Normal Distribution

$\bar{X}$	P($\bar{X}$)	$\bar{X}$ *P($\bar{X}$)	$\bar{X}^2$ *P($\bar{X}$)
1	1/16	1/16	1/16
1.5	2/16	3/16	4.5/16
2	3/16	6/16	12/16
2.5	4/16	10/16	25/16
3	3/16	9/16	27/16
3.5	2/16	7/16	24.5/16
4	1/16	4/16	16/16
Total	1	40/16	110/16



For this probability distribution of $\bar{\mathbf{X}}$, the **mean** of the **sample mean $\bar{\mathbf{X}}$** and the **variance** of the sample mean $\bar{\mathbf{X}}$ are :

$\mu_{\bar{X}} =$ $\sigma^2_{\bar{X}} =$

Sampling Distribution of the Sample Mean

- **Conclusions :**

1) $\mu_{\overline{X}} = \mu$

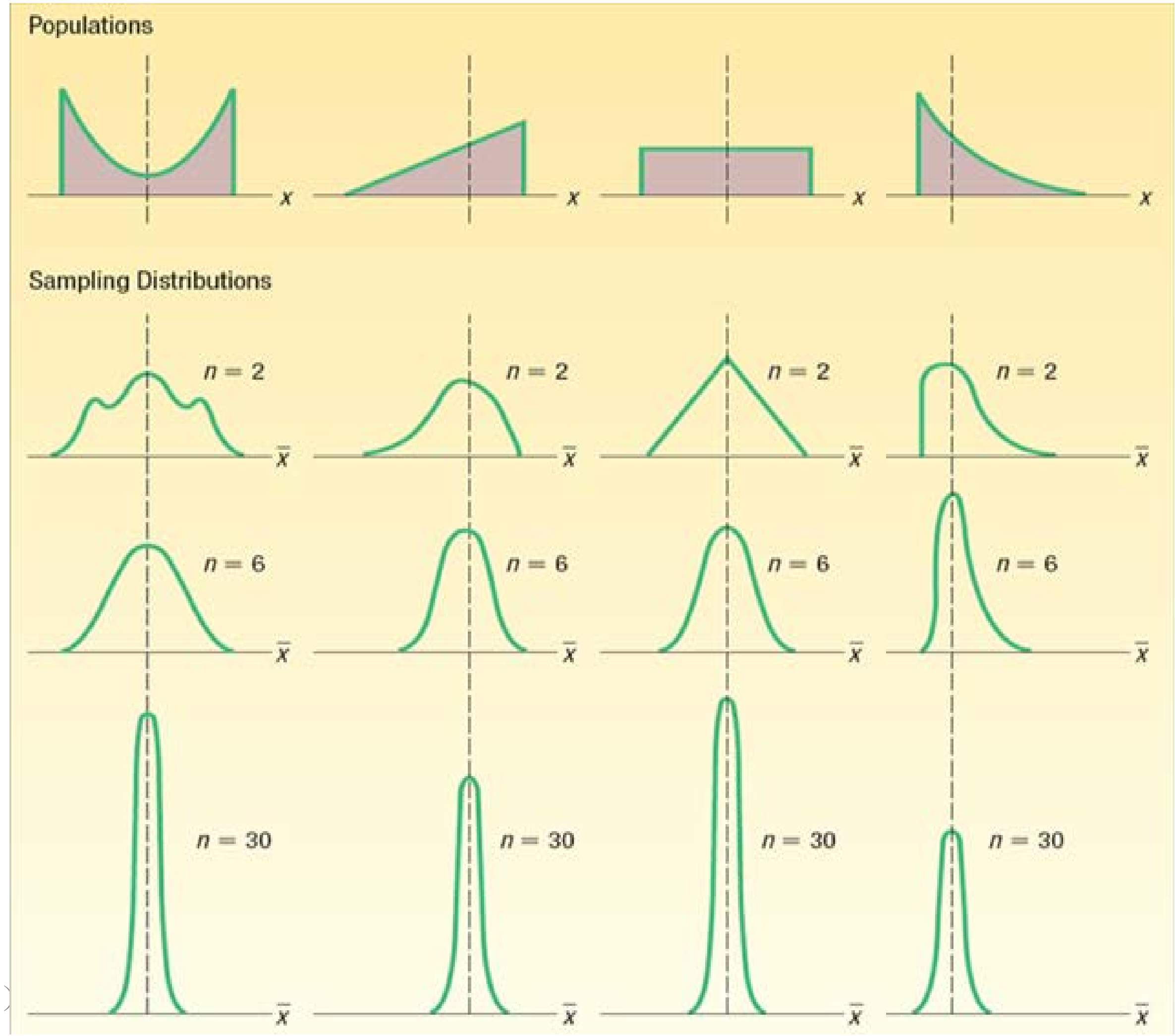
2) $\sigma_{\bar{x}} = \sigma / \sqrt{n}$, where **n** is the sample size.

3) Whether the distribution of the original population is normal or **not**, the distribution of the **sample mean** is close to **Normal**.

Note: When n gets **larger**, the distribution of $\bar{x}$ gets closer to the **Normal distribution**.

Central Limit Theorem 中央極限定理

Following Figures gives the sampling distributions of $\bar{\mathbf{X}}$ for four different population probability distributions with n= 2, 6, 30, respectively.



- **The Central Limit Theorem (C.L.T.) 中央極限定理**

If random samples of n observations are drawn from a population with mean μ and standard deviation σ，when **n** is **large** ($n \geq 30$), the **sampling distribution** of $\bar{X}$ is approximately normally distributed with $\mu_{\bar{X}} = \mu$, and $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$.

That is, $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$, if $n \geq 30$.

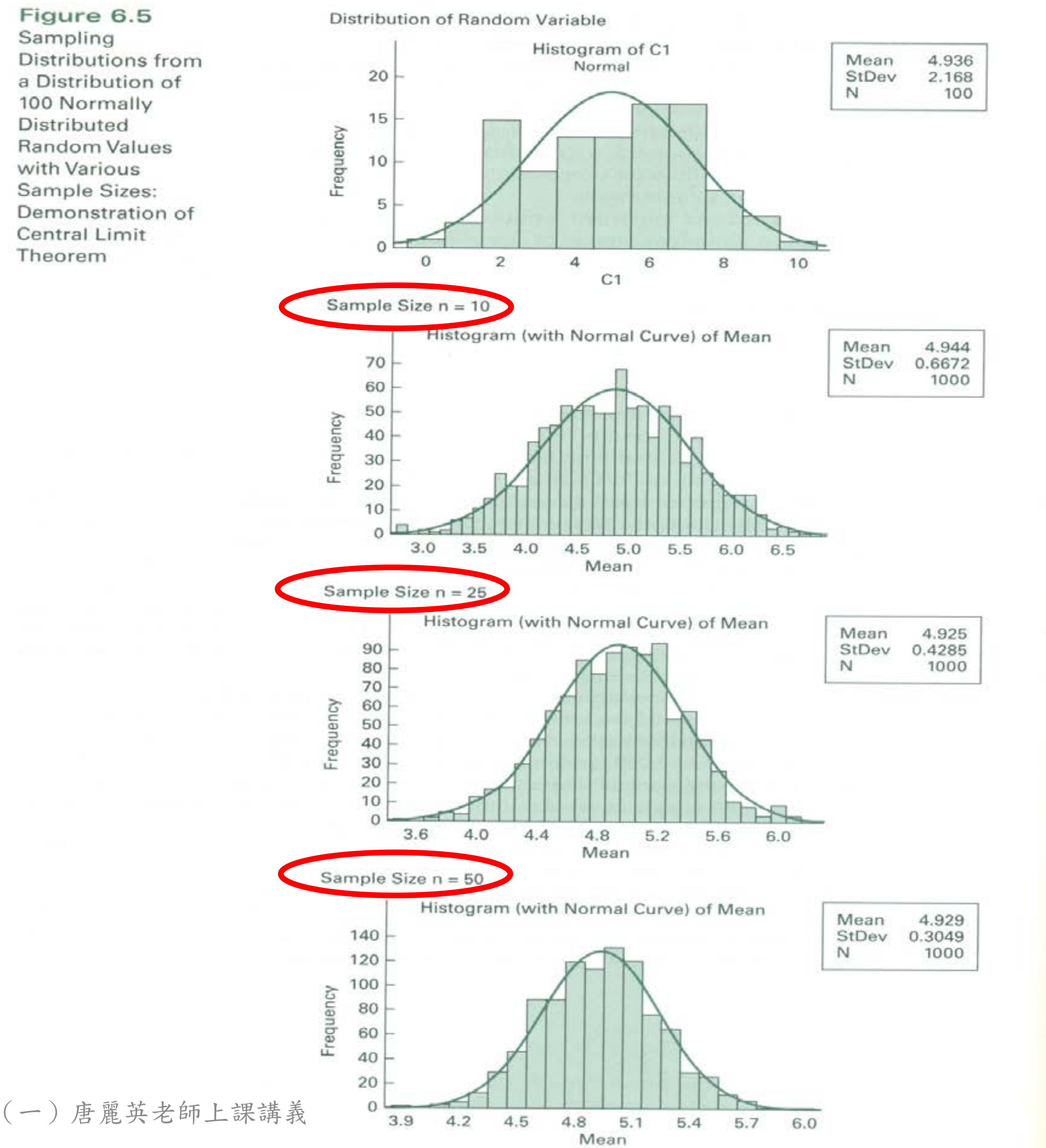
The **approximation** will become more and more **accurate** as **n** becomes large.

Central Limit Theorem 中央極限定理

Remark:

1. If the population is normal, then distribution of the sample mean $\bar{X}$ will always be normal, **regardless of the sample size** (n).
2. If $\bar{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right), \sum_{i=1}^{n} x \sim N(n\mu, \sqrt{n}\sigma)$。

Central Limit Theorem 中央極限定理



Resoucre:

“Statistics for Business and the Economics”,

7th Edition, by P. Newbold, W. L. Carlson

and B. Thorne, Pearson, 2007.

Sampling Distribution of the Sample Mean

- **例 1：**

  Suppose that X follows a distribution with mean $\mu=10$ and variance $\sigma^2= 4$. A sample of size 25 are drawn from this population. What is the probability distribution of $\overline{X}$?

Application of the Central Limit Theorem

- **例 2 :**

  The **average** vitamin B-2 content of a certain brand of vitamins is **30** mg with a **standard deviation** of **2** mg. A quality control inspector selects **36** pills for testing. What is the probability that the **average** vitamin B-2 content of these 36 pills is **less than 28 mg** ?

Application of the Central Limit Theorem

- **例 3 :**

  If a 1-gallon can of a certain kind of paint covers on the average 513.3 square feet with a standard deviation of 31.5 square feet, what is the probability that the mean area covered by a sample of 40 of these 1-gallon cans will be anywhere from 510.0 to 520.0 square feet ?

Application of the Central Limit Theorem

Note : When the sample size, n, is not a small fraction of a finite Population with size N, the sampling Distribution of $\bar{\mathbf{X}}$ is Normal with

$$\text{mean } \mu_{\bar{X}} = E(\bar{\mathbf{X}}) = \mu$$

$$\sigma_{\bar{X}}^2 = \mathrm{Var}(\bar{\mathbf{X}}) = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$$

Note : $\frac{N-n}{N-1}$ is called the finite population correction (fpc)factor

(有限母體校正因子)

Sampling Distribution of the Sample Proportion

- **What is the Sampling Distribution of the Sample Proportion, $\hat{p}$**
  - **P**：Population Proportion
  - $\boldsymbol{\hat{p}}$：Sample proportion = x/n = 成功次數/總試驗次數

- **Theorem：Sampling Distribution of $\hat{p}$**

When the sample size **n** is large, the sampling distribution of $\boldsymbol{\hat{p}}$ is approximately normal with **mean** P and **standard deviation** $\sqrt{\frac{pq}{n}}$.

$$\boxed{\hat{p} \sim N(p, \sqrt{\frac{pq}{n}})}$$

- **例 1 :**

  A production line at a manufacturing company produces **10%** defective items. If a sample of n=64 items is taken, what is the probability that the **sample defective rate** is **less** than 8%?

Sampling Distribution of the Sample Proportion

- **例 2：請參考課本289頁 例6.7**

  A random sample of 270 homes was taken from a large population of older homes to estimate the proportion for homes with unsafe wiring. If, in fact, 20% of the homes have unsafe wiring, what is the probability that the sample proportion will be between 16% and 24% of homes with unsafe wiring?

Sampling Distribution of the Sample Proportion

- **[Ans]**

$$P = 0.2 \quad n = 270$$

$$\sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{0.2(1-0.2)}{270}} = 0.024$$

$$\begin{aligned} P(0.16 < \hat{p} < 0.24) &= P\left(\frac{0.16-P}{\sigma_{\hat{p}}} < \frac{\hat{p}-P}{\sigma_{\hat{p}}} < \frac{0.24-P}{\sigma_{\hat{p}}}\right) \\ &= P(-1.67 < \mathrm{Z} < 1.67) \\ &= 0.9050 \end{aligned}$$

Sampling Distribution of the Sample Mean

2) What is the Sampling Distribution of the Sample Mean, $\bar{X}$ (When σ is unknown) ?

Sampling Distribution of the Sample Mean

- **Theorem**
  - If $\overline{X}$ is the mean of a random sample of size **n** taken from a **normal** population having the mean μ and the variance σ^2 , the sample statistic

$$\boxed{t = \frac{\overline{X} - \mu}{s / \sqrt{n}}}$$

has a **t-distribution** with **degrees of freedom**(d.f.)(自由度) ν= n-1.

Note: **t-distribution** is also called “**Student’s t-distribution**”.

Student's t Distribution

• What is "Student's" t-Distribution ?

The probability distribution of **t** statistic was first published in 1908 in a paper by **W. S. Gosset**. At the time, Gosset was employed by an Irish brewery (釀酒場) that disallowed publication of research by members of its staff. To circumvent this restriction, he published his work secretly under the name "**Student**". Consequently, the distribution of **t** is usually called the **Student's t-distribution**, or simply the **t**-distribution.

William Sealy Gosset

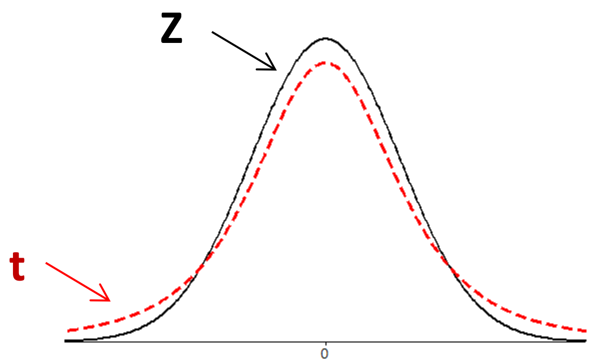
Student in 1908

Born	June 13, 1876 Canterbury, Kent, England
Died	October 16, 1937 (aged 61) Beaconsfield, Buckinghamshire, England

source: http://en.wikipedia.org/wiki/William_Sealy_Gosset

Student's t Distribution

- **Properties of t-Distribution**
  - The **t**-distribution is very much like a **Z**-distribution



- **Comparison of t-distribution and Z-distribution**
  1) Both are **symmetric**, bell-shaped.
  2) Both have a **mean** of 0.
  3) t is **more variable** than Z in repeated sampling. (There is more area in the tails of the t-distribution, and the Z-distribution is higher in the middle).
  4) As the number of **d.f.** increases (i.e., as **n** increases) without limit, the **t**-distribution approaches **Z**-distribution.

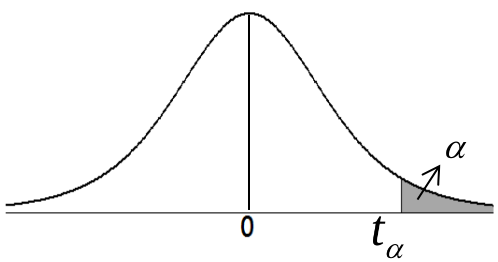
Degree of Freedom

- **What is the Degree of Freedom?**
  - We use the **degrees of freedom** as a measure of **sample information**.
  - For example, we say that the **t** statistic has degrees of freedom ***n*-1**.
- **Why？**
  - There are **n *degrees of freedom*** or **independent pieces** of information in the random sample of size ***n*** from the normal distribution.
  - In calculating $\mathbf{t} = \frac{\bar{X}-\mu}{s/\sqrt{n}}$, we **do not know σ** and need to use the sample data to estimate **σ**. When the data (the values in the sample) are used to compute the mean $\bar{\mathbf{X}}$ for obtaining $S^2 = \sum_{i=1}^{n}(x_i - \bar{X})^2/(n-1)$, there is **1** less degree of freedom in the information used to estimate $\boldsymbol{\sigma}^2$.

Student's t Distribution

- **t-distribution table**
  - **Table 8 in the Appendix Tables** of the textbook **(page 866**) gives the value of t_α which locates an area of α in the **<u>upper</u>** tail of the t-distribution for various values of α and for **d.f.** ranging from **1** to ∞ .

- **Note**
  - When d.f. $\geqq$29 (or **n**$\geqq$30), **Z**-distribution is very close to **t**-distribution.



For selected probabilities, α, the table shows the values $t_{\nu,\alpha}$ such that $P(t_\nu > t_{\nu,\alpha}) = \alpha$, where t_ν is a Student's t random variable with ν degrees of freedom. For example, the probability is .10 that a Student's t random variable with 10 degrees of freedom exceeds 1.372.

ν	α				
	0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
∞	1.282	1.645	1.960	2.326	2.576

Student's t Distribution

- **例 1：**
  - Find t_{α} and $t_{\alpha/2}$ when α=0.05 and n=6.
- **[Ans]**

ν	α				
	0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032

$$t_{.05,\,5} = 2.015 \quad , \qquad t_{.025,\,5} = 2.571 \,.$$

Student's t Distribution

- **例 2：**
  - Find t_α and $t_{\alpha/2}$ when α=0.01 and n=20.
- **[Ans]**

ν	α				
	0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
…			…		
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845

$t_{.01,\,19} = 2.539$, $t_{.005,\,19} = 2.861.$

Student's t Distribution

- **例 3 :**
  - Find t_α and $t_{\alpha/2}$ when α=0.10 and n=42.
- **[Ans]**

ν	α				
	0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
…			…		
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
∞	1.282	1.645	1.960	2.326	2.576

$t_{.10,\infty} = 1.282$, $t_{.05,\infty} = 1.645$.

Sampling Distributions of Sample Variance, S^2

χ^2 -Distribution

- If $\mathbf{s^2}$ is the variance of a random sample of size **n** taken from a ***Normal*** population having the variance $\boldsymbol{\sigma^2}$, then

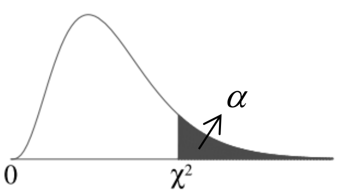
$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has a (Greek letter, **Chi**) distribution with the **d.f.** = ν =n-1.

- **Table 7 on pages 865** of the Appendix Tables gives the value of which locates an area of **α** in the <u>**upper**</u> tail of the χ^2 -distribution for various values of **α** and **d.f**.

χ^2 -Distribution Table

- **例 1：**
  - If n= 20, use Table 7 (p.865) to determine $\chi^2_{0.05}$ = ?



df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997

χ^2 -Distribution

- **例 2:**

  Consider a cannery that produces 8-ounce cans of processed corn. Quality control engineers have determined that process is operating properly when the true variation σ^2 of the fill amount per can is less than 0.0025. A random sample of n=10 cans is selected from a day's production, and the fill amount (in ounces) recorded for each. Of interest is the sample variance, S^2. If, in fact, σ^2 =0.001, find the probability that S^2 exceeds 0.0025. Assume that the fill amounts are normally distributed.

χ^2-Distribution

- **[Ans] (1/2)**

We want to calculate $P(S^2 > 0.0025)$. Assume the sample of 10 fill amount is selected from a normal distribution.

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has a chi-square probability distribution with $\nu=(n-1)$ degrees of freedom. Consequently, the probability we seek can be written

$$P(S^2 > 0.0025) = P\left[\frac{(n-1)S^2}{\sigma^2} > \frac{(n-1)(0.0025)}{\sigma^2}\right] = P\left[\chi^2 > \frac{(n-1)(0.0025)}{\sigma^2}\right]$$

Substituting n =10 and σ^2 =0.001, we have

$$P(S^2 > 0.0025) = P\left[\chi^2 > \frac{9(0.0025)}{0.001}\right] = P(\chi^2 > 22.5).$$

χ^2 -Distribution

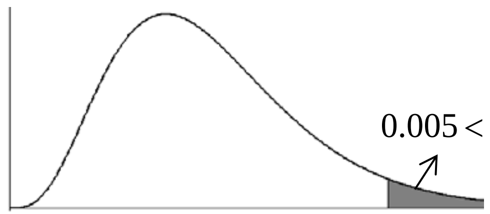
- **[Ans] (2/2)**

We want to find the probability **α** such that $\chi^2_\alpha > 22.5$

for n=10 (ν =9) , we obtain

$$\chi^2_{0.01} = 21.666 \text{ and } \chi^2_{0.005} = 23.589$$

i.e., $0.005 < P(\chi^2 > 22.5) < 0.01$



df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188

Thus, the probability that the variance of the sample fill amounts exceeds 0.0025 is small (between 0.005 and 0.01) when the true population variance σ^2 equals 0.001.

χ^2 -Distribution

- **例 3：請參考課本297頁 例6.10**

  Shirley Mendez is the manager of quality assurance for Green Valley Foods Inc., a packer of frozen vegetable products. Shirley wants to be sure that the variation of package weight is small so that the company does not produce a large proportion of packages that are under the stated package weight. She has asked you to obtain upper and lower limits for the ratio of the sample variance divided by the population variance for a random sample of n = 20 observations. The limits are such that the probability that the ratio is below the lower limits is 0.025 and the probability that the ratio is above the upper limit is 0.025. Thus, 95% of the ratios will be between these limits. The population distribution can be assumed to be normal.

χ^2 -Distribution

- **[Ans] (1/2)**

$$P\left(\frac{s^2}{\sigma^2} < K_L\right) = 0.025 \;\; and \quad P\left(\frac{s^2}{\sigma^2} > K_U\right) = 0.025$$

$$0.025 = P\left[\frac{(n-1)s^2}{\sigma^2} < (n-1)K_L\right] = P[\chi^2_{19} < (n-1)K_L]$$

$$0.025 = P\left[\frac{(n-1)s^2}{\sigma^2} > (n-1)K_U\right] = P[\chi^2_{19} < (n-1)K_U]$$

查表可得 $\chi^2_{19L} = 8.91$，$\chi^2_{19U} = 32.85$

χ^2 -Distribution

- **[Ans] (2/2)**

$$0.025 = P\left[\frac{(n-1)s^2}{\sigma^2} < (n-1)K_L\right] = P[8.91 < (19)K_L]$$

$$0.025 = P\left[\frac{(n-1)s^2}{\sigma^2} > (n-1)K_U\right] = P[32.85 < (19)K_U]$$

K_L=0.469, K_U=1.729

The 95% acceptance interval for the ratio of sample variance divided by population variance is as follows:

$$P\left(0.469 \le \frac{s^2}{\sigma^2} \le 1.729\right) = 0.95$$

Sampling Distributions of Sample Variance, ${s_1^2}/{s_2^2}$

F-Distribution

– Let χ_1^2 and χ_2^2 be two **independent chi-square** random variables with ν_1 and ν_2 degrees of freedom, respectively, then

$$F = \frac{\chi_1^2/\nu_1}{\chi_2^2/\nu_2}$$

has a **F** distribution with ν_1 numerator d.f. (分子自由度) and ν_2 denominator d.f. (分母自由度)

- **Theorem :**

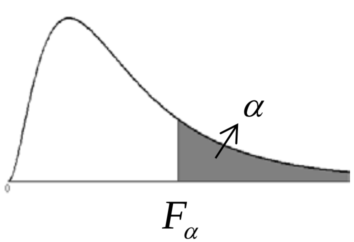
– If s_1^2 and s_2^2 are the variances of a random sample of size n_1 and n_2 taken from two normal population having the same variances, then

$$F = \frac{s_1^2}{s_2^2}$$

has a **F** distribution with **d. f.** $= (\nu_1, \nu_2) = (n_1 - 1, n_2 - 1)$.

F-Distribution Table

- **F-Distribution Table**
  - TABLE 9 on pages 867-869 of the Appendix Tables gives the value of F_α which locates an area of α in the <u>**upper**</u> tail of the F-distribution for various values of **α** and **d.f.**
- **例 4**: If $n_1 = 7$, $n_2 = 13$, use TABLE 9 to determine $F_{.01} = ?$



DENOMINATOR v_2	1	2	3	4	5	6	7	8	9
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	10.04	7.56	6.55	5.99	5.64	5.39	5.30	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03

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