$X=\{x_1, x_2, \ldots, x_n\}$，標準化為 $z=\frac{x-\mu}{\sigma}$　$z\sim N(0,1)$.

goal：是否有統計顯著

給定 time t. $\Delta z_t = |z_t - z_{t-1}| \sim$ folded normal.

因 $z_t \sim N(0,1)$，故 $E[\Delta z_t] = 0.798$.（均值為 $\sigma\sqrt{\frac{2}{\pi}}$）.

$Pr\{|z_t - z_{t-1}| > 2\} = 2 \cdot Pr\{\Delta z_t > 2\}$.

$= 2 \cdot Pr\{\hat{z} > \frac{2}{\sqrt{2}}\} = 2 \cdot 0.0787 = 0.1574$.

其中 $\hat{z} \sim N(0,1)$.　0.1574 為 false alarm rate under no change point.

同理 $2 \cdot Pr\{\hat{z} > \frac{3}{\sqrt{2}}\} = 0.0027$.　3 倍 σ 為 0.27%.

在無斷点下 0.27% 為異常

假設斷点大小為 δ.

檢定力. $\Delta z_t = \delta$. $Pr\{\Delta z_t > \theta \mid \Delta z_t = \delta\} = 1$.

False Alarm Rate：$Pr\{\Delta z_t > \theta \mid \text{No change}\}$.

Detection Rate：$Pr\{\Delta z_t > \theta \mid \text{change}\} = \begin{cases} 1 & \text{if } \delta > \theta \\ 0 & \text{o.w.} \end{cases}$

實作优化

1°. 动態調 σ_t s.t. $\theta = 3 \cdot \sigma_t$　$\frac{\alpha}{m}$.

2°. Bonferoni 校正 [整体顯著水準分到 m 次 test 上].

$Pr\{\text{至少 1 次誤報}\} \le m \cdot \frac{\alpha}{m} = \alpha$

∴ 各異常以 $\frac{\alpha}{m}$ 去決定.

threshold 推論

by law of large deviation

表示偏離 range

$\overline{X} = \frac{1}{N}\sum X_i$, for some set A, a mapping I

$$\frac{1}{N}\ln \Pr\{\overline{X}_N \in A\} \to -\inf_{x \in A} I(x) \text{ as } N \to \infty$$

代表N愈大 $\overline{X}$ 偏離 μ 之機率以指數下降

即 $\Pr\{\overline{X}_N \in A\} \approx \exp\{-N \cdot \inf I(x)\}$

by Bernoulli Distribution,

KL Div for p is: $I(x) = x \cdot \ln\frac{x}{p} + (1-x) \cdot \ln\frac{x}{(1-p)}$

s.t. $\Pr\{\overline{X}_N > p\} \approx -\exp\{N \cdot [x \cdot \ln\frac{x}{p} + (1-x)\ln\frac{x}{1-p}]\}$

$\Pr\{\Delta Z_t > \theta\} \approx \exp\{-N \cdot I(\theta)\} = \alpha$ false alarm rate

$-N\,I(\theta) = \ln\alpha \Rightarrow I(\theta) = \frac{-\ln\alpha}{N}$

又 $\Pr\{\Delta Z_t > \theta\} \approx \exp\{-\frac{(\theta-\mu)^2}{2\sigma^2}\} = \alpha$

$\Rightarrow \theta = \mu + \sigma \cdot \sqrt{-2 \cdot \ln\alpha} = \mu + \sigma \cdot \sqrt{2 \cdot \ln\frac{1}{\alpha}}$

設 $\mu = K \cdot \sigma$ s.t. $\theta = K \cdot \sigma + \sqrt{2 \cdot \ln\frac{1}{\alpha}} \cdot \sigma$

在 $\alpha = \frac{\alpha}{m}$ 下, $\theta = (K + \sqrt{2 \cdot \ln\frac{1}{\alpha}}) \cdot \sigma$