Fonction

In[ ]:=
```
Quit
```

In[ ]:=
```
Poisson[lambda_, i_] := Exp[(-lambda)] * (lambda ^ i / (i!));
HyperGeo[n_, Nn_, m_, i_] := Binomial[m, i] Binomial[Nn - m, n - i] / Binomial[Nn, n];
Geo[n_, p_] := (1 - p) ^ (n - 1) p;
Bin[n_, p_, x_] := Binomial[n, x] * p ^ x * (1 - p) ^ (n - x);
BinNeg[n_, p_, r_] := Binomial[n - 1, r - 1] p ^ r (1 - p) ^ (n - r);
Centre[x_, mu_, sigma_] := (x - mu) / sigma;
Expo[lambda_, x_] := lambda Exp[-lambda x];
Pnorm[z_] := N[CDF[NormalDistribution[0, 1], z]];
Qnorm[p_] := N[InverseCDF[NormalDistribution[0, 1], p]];
(*≈*)
```

```
(*Il peut y avoir des conflits avec les fonctions de même symbole e.g. f[x_]=
 x et f[x_]=x^2. Faire Clear["Global`*"] ou Quit*)
```

Probabilité

Chapitre 2

Problème

Exercice 44 (p.66)

In[ ]:=
```
A = (2 × 3 × 3!) / 5!
B = (2 Binomial[3, 2] 2 × 2) / 5!
Cc = 2 × 3! / 5!
```

Out[ ]= $\frac{3}{10}$

Out[ ]= $\frac{1}{5}$

Out[ ]= $\frac{1}{10}$

Auto-Évaluation

Exercice 2 (p.71)

In[ ]:=
```
A = (Binomial[13, 4] (Binomial[4, 1])^4) / Binomial[52, 4]
N@%
13 × 12 × 11 × 10 × 4^4 / (52 × 51 × 50 × 49)
B = Binomial[13, 1]^4 / Binomial[52, 4]
N@%
```

Out[ ]= $\frac{2816}{4165}$

Out[ ]= 0.67611

Out[ ]= $\frac{2816}{4165}$

Out[ ]= $\frac{2197}{20\,825}$

Out[ ]= 0.105498

Exercice 11 (p.72)

In[ ]:=
```
Binomial[13, 1]^4 * 48 / Binomial[52, 5]
N@%
% / 2
1 - ((4 Binomial[39, 5] - 6 Binomial[26, 5] + 4 Binomial[13, 5]) / Binomial[52, 5])
N@%
```

Out[ ]= $\frac{2197}{4165}$

Out[ ]= 0.527491

Out[ ]= 0.263745

Out[ ]= $\frac{2197}{8330}$

Out[ ]= 0.263745

Chapitre 3

Auto-Évaluation

Exercice 4 (p.143)

In[ ]:=
```
Bb = (2 / 7) (2 / 3) + (1 / 7) (1 / 3)
(2 / 7) (2 / 3) / Bb
```

Out[ ]= $\frac{5}{21}$

Out[ ]= $\frac{4}{5}$

Exercice 10 (p.144)

In[ ]:=
```
r = 8;
v = 10;
b = 12;
n = 6;
cardS = Binomial[r + b + v, n];
(*A*)
A = 1 - Binomial[b + v, n] / cardS
N@%
(*B*)
Binomial[v, 2] Binomial[b, 4] / Binomial[b + v, n]
N@%
```

Out[ ]= $\frac{24722}{28275}$

Out[ ]= 0.874341

Out[ ]= $\frac{675}{2261}$

Out[ ]= 0.29854

Exercice 13 (p.144)

a)

In[ ]:=
```
r = 20;
b = 10;
(*Même probabilité que de pigée une boule bleue en premiers*)
A = Binomial[b, 1] / Binomial[r + b, 1]
```

Out[ ]= $\frac{1}{3}$

b)

In[ ]:=
```
r = 20;
b = 10;
v = 8;
(*Même probabilité que de pigée une boule bleue en premiers*)
B = 10 / 30
```

Out[ ]= $\frac{1}{3}$

Exercice 28 (p.144)

a)

In[ ]:=
```
pc = 7 / 10;
pd = 4 / 10;
c = 8;
d = 6;
(*Par la formule des probabilités totales*)
F = pc * (c / (c + d)) + pd * (d / (c + d))
N@%
```

Out[ ]= $\frac{4}{7}$

Out[ ]= 0.571429

b)

In[ ]:=
```
(*Par Bayes*)
B = (1 - pc) * (c / (c + d)) / (1 - F)
```

Out[ ]= $\frac{2}{5}$

Chapitre 4

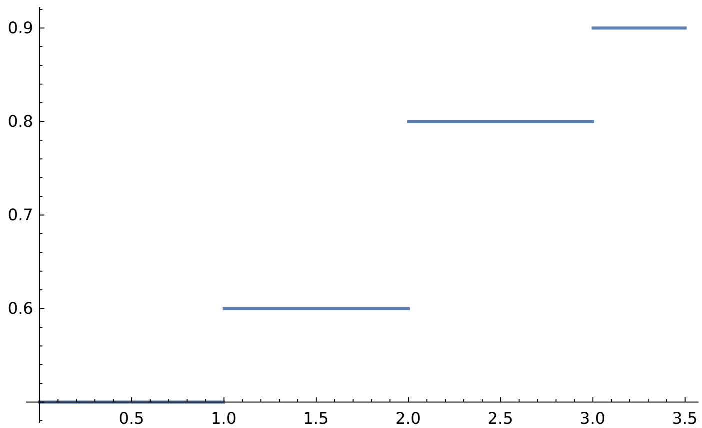
Problème

Exercice 15 (p.209)

Exercice 19 (p.210)

In[ ]:=
```
Clear[b]
Ff[b_] := Piecewise[{
   {0, b < 0},
   {2 / 4, 0 ≤ b < 1},
   {3 / 5, 1 ≤ b < 2},
   {4 / 5, 2 ≤ b < 3},
   {9 / 10, 3 ≤ b < 7 / 2},
   {1, b ≥  7 / 2}}]
Plot[Ff[b], {b, 0, 7 / 2}]
loi = {}; For[i = 0, i < 4, i++, AppendTo[loi, F[i] - F[i - 1]]]
AppendTo[loi, F[3.5] - F[3]]
```

Out[ ]=



Out[ ]= $\left\{-\frac{4}{7}[-1]+\frac{4}{7}[0],\ -\frac{4}{7}[0]+\frac{4}{7}[1],\ -\frac{4}{7}[1]+\frac{4}{7}[2],\ -\frac{4}{7}[2]+\frac{4}{7}[3],\ -\frac{4}{7}[3]+\frac{4}{7}[3.5]\right\}$

Exercice 25 (p.212)

a)

In[ ]:=
```
p1 = 6 / 10;
p2 = 7 / 10;
A = p1 * (1 - p2) + (1 - p1) * p2
```

Out[ ]= $\frac{23}{50}$

b)

In[ ]:=
```
B = 0 + 1 * A + 2 (p1 * p2)
```

Out[ ]= $\frac{13}{10}$

Exercice 28 (p.212)

In[ ]:=
```
n = 3;
Nn = 20;
m = 4;
Sum[i HyperGeo[n, Nn, m, i], {i, 0, 4}]
```

Out[ ]= $\frac{3}{5}$

Exercice 42 (p.214)

In[ ]:=
```
n = 10;
pa = 0.7;
pb = 0.4;
(*Binomial*)
(*A*)
p = pa pb;
esp = n p
(*B*)
var = n p (1 - p)
```

Out[ ]= 2.8

Out[ ]= 2.016

Exercice 49 (p.215)

In[ ]:=
```
(*A*)
n = 10;
p1f = 4 / 10;
p2f = 7 / 10;
A = Bin[10, p1f, 7];
N@%;
B = Bin[10, p2f, 7];
N@%;
Aa = N[(A + B) / 2]
(*p=p1f*1/2+p2f*1/2;
Bin[10,p,7];
N@%*)
(*B*)
Cc = 1 - 0.5 (Bin[10, p1f, 0] + Bin[10, p2f, 0]);
Bb = 0.5 (Bin[9, p1f, 6] + Bin[9, p2f, 6]);
Bb / Cc
```

Out[ ]= 0.154648

Out[ ]= 0.171091

Exercice 53 (p.216)

In[ ]:=
```
n = 80 000;
p = (1 / 365) ^ 2
A = 1 - Bin[n, p, 0];
N@%
(*365 jours possible*)
B = 1 - Poisson[365 n p, 0];
N@%
```

Out[ ]= $\frac{1}{133\,225}$

Out[ ]= 0.451457

Out[ ]= 1.

Exercice 56 (p.216)

In[ ]:=
```
p = (1 / 365);
Clear[n];
1 - Bin[n, p, 0]
nn = n /. Last[Solve[Rationalize[1 - Bin[n, p, 0] ≤ 0.5 && n > 1], n, Integers]]
```

Out[ ]= $1-\left(\frac{364}{365}\right)^{n}$

Out[ ]= 252

Exercice 61 (p.217)

In[ ]:=
```
p = 14 / 10000;
n = 1000;
1 - Sum[Bin[n, p, i], {i, 0, 1}];
N@%
(*Approximation poissonnienne*)
1 - Sum[Poisson[λ, i], {i, 0, 1}]
1 - Sum[Poisson[n p, i], {i, 0, 1}]
N@%
```

Out[ ]= 0.408264

Out[ ]= $1-e^{-\lambda}-e^{-\lambda}\lambda$

Out[ ]= $1-\frac{12}{5e^{7/5}}$

Out[ ]= 0.408167

Exercice 63 (p.217)

In[•]:=
```
lambda = 1 / 2;
(*A*)
Poisson[5 × 1 / 2, 0]
(*B*)
Clear[λ];
1 - Sum[Poisson[λ, i], {i, 0, 3}]
1 - Sum[Poisson[5 / 2, i], {i, 0, 3}]
```

Out[•]= $\frac{1}{e^{5/2}}$

Out[•]= $1 - e^{-\lambda} - e^{-\lambda}\,\lambda - \frac{1}{2}\,e^{-\lambda}\,\lambda^2 - \frac{1}{6}\,e^{-\lambda}\,\lambda^3$

Out[•]= $1 - \frac{443}{48\,e^{5/2}}$

Exercice 68 (p.218)

In[•]:=
```
anti = 500;
missi = 10;
p = 0.1;
Ee = 1 - Poisson[500 * 1 / 100, 0];
Ee ^ 10
N@%
```

Out[•]= $\left(1 - \frac{1}{e^5}\right)^{10}$

Out[•]= 0.934627

Exercice 78 (p.219)

In[•]:=
```
p = Binomial[4, 2]^2 / Binomial[8, 4]
Clear[n]
Geo[n, p]
Limit[%, n → Infinity];
```

Out[•]= $\frac{18}{35}$

Out[•]= $18 \times 17^{-1+n} \times 35^{-n}$

Exercice 79 (p.219)

In[•]:=
```
n = 10;
Nn = 100;
m = 6;
A = HyperGeo[n, Nn, m, 0];
N@%
B = 1 - Sum[HyperGeo[n, Nn, m, i], {i, 0, 2}];
N@%
```

Out[•]= 0.522305

Out[•]= 0.0125511

Exercice 82 (p .220)

In[•]:=
```
n = 20;
p = 9 / 10;
1 - Bin[4, p, 4]
N@%
```

Out[•]= $\frac{3439}{10000}$

Out[•]= 0.3439

Exercice 83 (p.220)

In[•]:=
```
p1 = 3 / 10;
p2 = 5 / 10;
p3 = 7 / 10;
p1 + p2 + p3
```

Out[•]= $\frac{3}{2}$

Auto-Évaluation

Exercice 9 (p.228)

In[ ]:=
```
(*esp=np*)
(*var-np(1-p)*)
esp = 6;
var = 2.4;
p = 1 - (var / esp);
n = 10;
Bin[n, p, 5]
```

Out[ ]= 0.200658

Exercice 15 (p.229)

In[ ]:=
```
lambda = 52 / 10
Sum[Poisson[lambda, i], {i, 0, 3}]
N@%
```

Out[ ]= $\frac{26}{5}$

Out[ ]= $\frac{16\,183}{375\, e^{26/5}}$

Out[ ]= 0.238065

In[ ]:=
```
p = (14 / 10 000)
Binomial[1000, 2] p ^ 2 (1 - p) ^ 998;
N@%
```

Out[ ]= $\frac{7}{5000}$

Out[ ]= 0.241863

Exercice 12 (p.228)

Exercice 19 (p.229)

a)

In[ ]:=
```
(*P(PPP) + P(FFF)*)
p = 2 (1 / 2)^3
(*P(X=3)*)
A = p^2 (1 - p)
```

Out[ ]= $\frac{1}{4}$

Out[ ]= $\frac{3}{64}$

b) P(X>4)

In[ ]:=
```
1 - Sum[p^(3 - i) * (1 - p), {i, 0, 3}]
N@%
```

Out[ ]= $\frac{1}{256}$

Out[ ]= 0.00390625

Exercice 24 (p.230)

Chapitre 5

Problème

Exercice 2

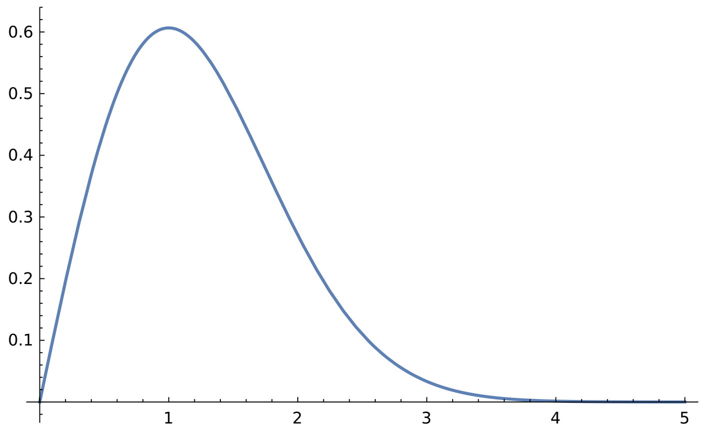
In[ ]:=
```
Clear[c, n, r]
f[x_] := Piecewise[{
    {c x Exp[-(x^2) / 2], x > 0},
    {0, x ≤ 0}}];
Solve[Limit[Integrate[c x Exp[-x^2 / 2], {x, 0, n}], n → Infinity] == 1]
```

Out[ ]= {{c → 1}}

In[ ]:=
```
c = 1;
Limit[Integrate[f[x], {x, 5, r}, Assumptions → {r > 5}], r → Infinity]
N@%
Plot[f[x], {x, 0, 5}]
```

Out[ ]= $\frac{1}{e^{25/2}}$

Out[ ]= 3.72665×10^{-6}

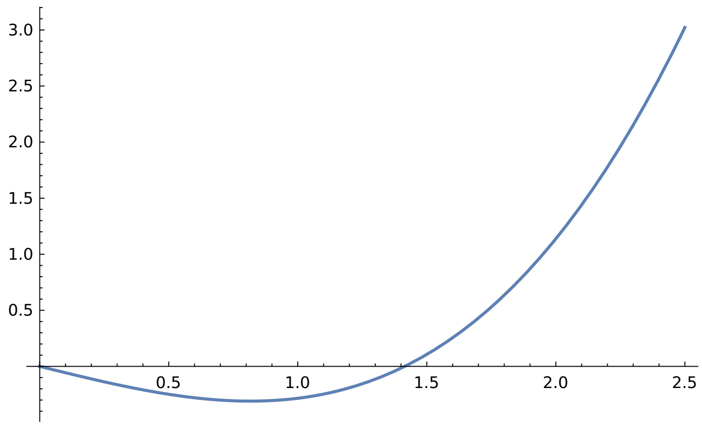
Out[ ]= 

Exercice 3 (p.271)

In[ ]:=
```
Clear[c]
f[x_] := c (2 x - x ^ 3)
Solve[Integrate[f[x], {x, 0, 5 / 2}] == 1, c]
c = c /. %
Plot[f[x], {x, 0, 5 / 2}]
(*Pas une fonction de densite car y<0*)
Clear[c]
f[x_] := c (2 x - x ^ 2)
Solve[Integrate[f[x], {x, 0, 5 / 2}] == 1, c]
c = c /. %
Plot[f[x], {x, 0, 5 / 2}]
```

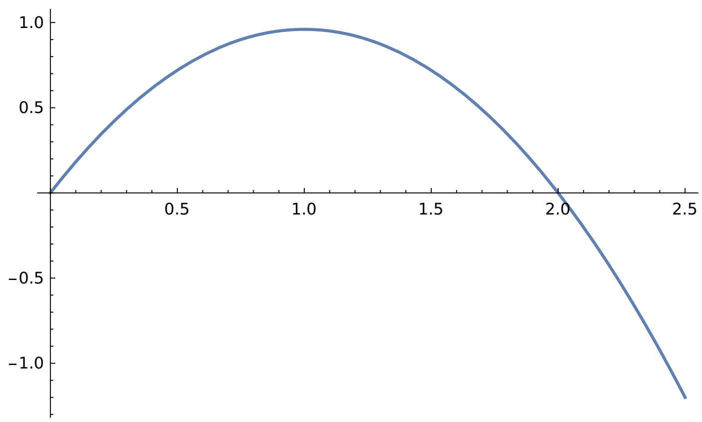
Out[ ]= $\left\{\left\{c \to -\frac{64}{225}\right\}\right\}$

Out[ ]= $\left\{-\frac{64}{225}\right\}$

Out[ ]=


Out[ ]= $\left\{\left\{c \to \frac{24}{25}\right\}\right\}$

Out[ ]= $\left\{\frac{24}{25}\right\}$

Out[ ]=


Exercice 4 (p.271)

In[ ]:=
```
p = 10 / 15;
(*C*)
n = 6;
Sum[Bin[n, p, i], {i, 3, 6}]
N@%
```

Out[ ]= $\frac{656}{729}$

Out[ ]= 0.899863

Exercice 7

In[ ]:=
```
Esp = 3 / 5;
f[x_] := Piecewise[{{a + b x ^ 2, 0 ≤ x ≤ 1}, {0, 0 > x && x > 1}}]
Solve[Integrate[x f[x], {x, 0, 1}] == Esp && Integrate[f[x], {x, 0, 1}] == 1]
Solve[a / 2 + b / 4 == 3 / 5 && a + b / 3 == 1, {a, b}]
```

Out[ ]= $\left\{\left\{a \to \frac{3}{5}, b \to \frac{6}{5}\right\}\right\}$

Out[ ]= $\left\{\left\{a \to \frac{3}{5}, b \to \frac{6}{5}\right\}\right\}$

Exercice 12 (p.272)

In[ ]:=
```
f[x_] := Piecewise[{
    {x1 - x, 0 < x ≤ x1},
    {x - x1, x1 < x ≤ (x2 + x1) / 2},
    {x2 - x, (x2 + x1) / 2 < x ≤ x2},
    {x - x2, x2 < x ≤ (x3 + x2) / 2},
    {x3 - x, (x3 + x2) / 2 < x ≤ x3},
    {x - x3, x3 < x ≤ 100}}];
FindMinimum[{0.01 Integrate[f[x], {x, 0, 100}], {x1 ≥ 0 && x2 ≥ 0 && x3 ≥ 0}}, {x1, x2, x3}];
nm = {x1, x2, x3} /. Last[%];
x1 = 0;
x2 = 50;
x3 = 100;
A = 0.01 Integrate[f[x], {x, 0, 100}]
x1 = 25;
x2 = 50;
x3 = 75;
B = 0.01 Integrate[f[x], {x, 0, 100}]
x1 = nm[[1]];
x2 = nm[[2]];
x3 = nm[[3]];
Cc = 0.01 Integrate[f[x], {x, 0, 100}]
```

Out[ ]= 12.5

Out[ ]= 9.375

Out[ ]= 8.33333

Exercice 16 (p.273)

In[ ]:=
```
Clear[x, p]
mu = 140;
sigma = 4;
(**)
p = 1 - Pnorm[Centre[150, mu, sigma]]
N@Probability[x > 150, x ≈ NormalDistribution[mu, sigma]]
(1 - p) ^ 10
1 - Sum[Geo[n, p], {n, 1, 10}]
```

Out[ ]= 0.00620967

Out[ ]= 0.00620967

Out[ ]= 0.93961

Out[ ]= 0.93961

Exercice 17 (p.273)

In[ ]:=
```
mu = (320 + 180) / 2;
Clear[sigma]
sigma = sigma /. Last[
   NSolve[Centre[320, mu, sigma] == InverseCDF[NormalDistribution[0, 1], 0.75], sigma]]
(*A*)
CDF[NormalDistribution[0, 1], Centre[200, mu, sigma]]
(*B*)
CDF[NormalDistribution[0, 1], Centre[{320, 280}, mu, sigma]]
%[[1]] - %[[2]]
```

Out[ ]= 103.782

Out[ ]= 0.314982

Out[ ]= {0.75, 0.613735}

Out[ ]= 0.136265

Exercice 20 (p.273)

In[ ]:=
```
n = 100;
p = 65 / 100;
(*A*)
Sum[Bin[n, p, i], {i, 50, 100}];
N@%
(*Approximation par une normale*)
(*P(a-0.5<Y<b+0.5)=P(Y<b+0.5)-P(a-0.5<Y)*)
a = 49.5;
mu = n p;
sigma = Sqrt[n p (1 - p)];
N@Centre[a, mu, sigma]
A = 1 - N@CDF[NormalDistribution[0, 1], Centre[a, mu, sigma]]
(*B*)
a = 60;
b = 70;
Sum[Bin[n, p, i], {i, 60, 70}];
N@%;
```

Out[ ]= 0.999262

Out[ ]= -3.24968

Out[ ]= 0.999422

Exercice 22 (p.273)

In[ ]:=
```
p = 4 / 10;
n = 100;
1 - Sum[Bin[n, p, i], {i, 50, 100}];
N@%
(*Approximation normale avec correction de continuite*)
mu = n p;
sigma = Sqrt[n p (1 - p)];
Centre[49.5, mu, sigma]
CDF[NormalDistribution[0, 1], Centre[49.5, mu, sigma]]
Clear[x]
Print["Poisson"]
N@Probability[x < 50, x ≈ PoissonDistribution[n p]]
Clear[x]
Print["Binomial"]
N@Probability[x < 50, x ≈ BinomialDistribution[n, p]]
Clear[x]
Print["Normale"]
N@Probability[x ≤ 49.5, x ≈ NormalDistribution[mu, sigma]]
```

Out[ ]= 0.972901

Out[ ]= 1.93918

Out[ ]= 0.97376

Poisson

Out[ ]= 0.929665

Binomial

Out[ ]= 0.972901

Normale

Out[ ]= 0.97376

Exercice 23 (p.274)

In[ ]:=
```
n = 1000;
p = 1 / 6;
Centre[{150, 200}, n p, Sqrt[n p (1 - p)]]
N@CDF[NormalDistribution[0, 1], %]
%[[2]] - %[[1]]
(**)
n = 800;
p = 1 / 5;
(*Approximation normale*)
mu = n p;
sigma = Sqrt[n p (1 - p)];
(*Centre reduit*)
N@Centre[150, mu, sigma]
N@CDF[NormalDistribution[0, 1], %]
```

Out[ ]= $\left\{-\sqrt{2}, 2\sqrt{2}\right\}$

Out[ ]= {0.0786496, 0.997661}

Out[ ]= 0.919012

Out[ ]= -0.883883

Out[ ]= 0.18838

Exercice 24 (p.274)

In[ ]:=
```
mu = 1.4;
sigma = 0.3;
Pnorm[Centre[1.8, mu, sigma]]
p = Probability[x < 1.8, x ≈ NormalDistribution[mu, sigma]]
(*np > 5 =>> Normale approx Bin *)
n = 100;
mu2 = n p;
sigma2 = n p (1 - p);
(*Bin*)
1 - Sum[Bin[n, p, i], {i, 0, 19}]
(*approx*)
1 - Pnorm[Centre[19.5, mu2, sigma2]]
```

Out[ ]= 0.908789

Out[ ]= 0.908789

Out[ ]= 1.

Out[ ]= 1.

Exercice 25 (p.274)

In[ ]:=
```
n = 150;
p = 95 / 100;
Sum[Bin[n , 1 - p, i], {i, 0, 10}];
N@%
(*Approximation par la normale avec correction*)
mu = n (1 - p);
sigma = Sqrt[n p (1 - p)];
Centre[10.5, mu, sigma]
CDF[NormalDistribution[0, 1], Centre[10.5, mu, sigma]]
```

Out[ ]= 0.867785

Out[ ]= 1.1239

Out[ ]= 0.869473

Exercice 26 (p.274)

In[ ]:=
```
pnh = 55 / 100;
ph = 1 / 2;
n = 1000;
(*Approximation par normale*)
(*P(X≥ 525) |homogene*)
muh = n ph;
sigmah = Sqrt[n ph (1 - ph)];
1 - CDF[NormalDistribution[0, 1], Centre[524.5, muh, sigmah]]
(*P(X≥ 525) |pas homogene*)
munh = n pnh;
sigmanh = Sqrt[n pnh (1 - pnh)];
CDF[NormalDistribution[0, 1], Centre[525.5, munh, sigmanh]]
```

Out[ ]= 0.0606289

Out[ ]= 0.059697

Exercice 29 (p.274)

In[ ]:=
```
n = 1000;
p = 0.52;
u = 1.012;
d = 0.990;
NSolve[u ^ x d ^ (1000 - x) == 1.3, x, Reals]
Log[1.3 / (d) ^ 1000] / Log[u / (d)]
N@Probability[x ≥ 470, x ≈ NormalDistribution[n p, Sqrt[n p (1 - p)]]]
```

Out[ ]= {{x → 469.209}}

Out[ ]= 469.209

Out[ ]= 0.999224

Exercice 30 (p.274)

In[ ]:=
```
Clear[alpha]
mu1 = 4;
sigma1 = 2;
mu2 = 6;
sigma2 = 3;
a = N@Pnorm[Centre[{5, 5}, {mu1, mu2}, {sigma1, sigma2}]]
a[[2]] / (a[[1]] + a[[2]])
alpha = alpha /. NSolve[alpha a[[2]] / (alpha a[[2]] + (1 - alpha) a[[1]]) == 1 / 2, alpha]
1 - alpha
```

Out[ ]= {0.691462, 0.369441}

Out[ ]= 0.348233

Out[ ]= {0.651767}

Out[ ]= {0.348233}

Exercice 34 (p.275)

In[ ]:=
```
lambda = 1 / 20;
FullSimplify[1 - Integrate[Expo[lambda, x], {x, 0, 20}]]
N@%
FullSimplify[1 - Integrate[Expo[lambda, x], {x, 0, 20}]]
```

Out[ ]= $\frac{1}{e}$

Out[ ]= 0.367879

Out[ ]= $\frac{1}{e}$

Exercice 36 (p.275)

```
In[ ]:= λ[t_] := t^3;
(*A*)
1 - (1 - Exp[-Integrate[λ[t], {t, 0, 2}]])
(*B*)
(1 - Exp[-Integrate[λ[t], {t, 0, 14/10}]]) - (1 - Exp[-Integrate[λ[t], {t, 0, 4/10}]])
N@%
(*C*)
(Exp[-Integrate[λ[t], {t, 0, 2}]]) / (Exp[-Integrate[λ[t], {t, 0, 1}]])
N@%
```

Out[ ]= $\frac{1}{e^4}$

Out[ ]= $-\frac{1}{e^{2401/2500}} + \frac{1}{e^{4/625}}$

Out[ ]= 0.610881

Out[ ]= $\frac{1}{e^{15/4}}$

Out[ ]= 0.0235177

Exercice 38 (p.276)

```
In[ ]:= Clear[x, Y]
A = Solve[4 x^2 + 4 x Y + Y + 2 == 0, x]
x1 = x /. A[[1]]
x2 = x /. A[[2]]
(Length@Solve[-2 - Y + Y^2 >= 0 && 0 ≤ Y ≤ 5, Y, Integers] - 1) / 5
```

Out[ ]= $\left\{\left\{x \to \frac{1}{2}\left(-Y - \sqrt{-2 - Y + Y^2}\right)\right\}, \left\{x \to \frac{1}{2}\left(-Y + \sqrt{-2 - Y + Y^2}\right)\right\}\right\}$

Out[ ]= $\frac{1}{2}\left(-Y - \sqrt{-2 - Y + Y^2}\right)$

Out[ ]= $\frac{1}{2}\left(-Y + \sqrt{-2 - Y + Y^2}\right)$

Out[ ]= $\frac{3}{5}$

Auto-Évaluation

Exercice 4 (p.281)

In[ ]:=
```
Clear[a, b];
f[x_] := a x + b x^2;
(**)
Solve[Integrate[f[x], {x, 0, 1}] == 1 && Integrate[x f[x], {x, 0, 1}] == 6 / 10, {a, b}]
(**)
u = {a, b} /. %[[1]];
a = u[[1]];
b = u[[2]];
(*A*)
With[{a = a, b = b}, Integrate[f[x], {x, 0, 1 / 2}]]
(*B*)
With[{a = a, b = b}, Integrate[x^2 f[x], {x, 0, 1}]] - (6 / 10)^2
```

Out[ ]= $\left\{\left\{a \to \frac{18}{5}, b \to -\frac{12}{5}\right\}\right\}$

Out[ ]= $\frac{7}{20}$

Out[ ]= $\frac{3}{50}$

Exercice 11 (p.283)

In[ ]:=
```
Centre[44, 40.2, 8.4];
1 - CDF[NormalDistribution[0, 1], %]
Binomial[7, 3] %^3 (1 - %)^4
```

Out[ ]= 0.325497

Out[ ]= 0.24983

Exercice 15 (p.283)

In[ ]:=
```
λ[t_] := Piecewise[{{0.2, 0 < t < 2}, {0.2 + 0.3 (t - 2), 2 ≤ t < 5}, {1.1, t > 5}}];
(*A*)
A = 1 - (1 - Exp[-Integrate[λ[t], {t, 0, 6}]])
(*B*)
(1 - Exp[-Integrate[λ[t], {t, 6, 8}]]) / A
```

Out[ ]= 0.0317456

Out[ ]= 28.01

Chapitre 6

Problème

Exercice 8 (p.340)

In[ ]:=
```
(*A*)
Clear[c, a, x, y];
f[x_, y_] := c (y ^ 2 - x ^ 2) Exp[-y];
Integrate[f[x, y], {y, 0, Infinity}, {x, -y, y}];
c = c /. Solve[% == 1, c][[1]]
(*B*)
(*f_x[x]*)
fx = Integrate[f[x, y], {y, Abs[x], Infinity}]
(*f_y[y]*)
Integrate[f[x, y], {x, -y, y}]
(*C*)
Integrate[x fx, {x, -Infinity, Infinity}]
```

Out[ ]= $\frac{1}{8}$

Out[ ]= $\frac{1}{8} e^{-\text{Abs}[x]} \left(2 - x^2 + 2\,\text{Abs}[x] + \text{Abs}[x]^2\right)$

Out[ ]= $\frac{1}{6} e^{-y}\, y^3$

Out[ ]= 0

Exercice 9 (p.340)

In[ ]:=
```
f[x_, y_] := (6 / 7) * (x ^ 2 + x y / 2);
(*A*)
Integrate[Integrate[f[x, y], {x, 0, 1}], {y, 0, 2}]
(*B*)
fX[x_] := FullSimplify@Integrate[f[x, y], {y, 0, 2}];
fX[x]
fY[y_] := FullSimplify@Integrate[f[x, y], {x, 0, 1}];
fY[y];
(*Pas independant*)
fX[x] × fY[y];
(*C*)
(*P(x>y)*)
Integrate[Integrate[f[x, y], {y, 0, x}], {x, 0, 1}]
(*P(x<y)*)
Integrate[Integrate[f[x, y], {y, x, 2}], {x, 0, 1}]
(*D*)
(*P(A inter B)*)
Integrate[Integrate[f[x, y], {x, 0, 1 / 2}], {y, 1 / 2, 2}]
% / (Integrate[fX[x], {x, 0, 1 / 2}])
(*E*)
Integrate[x fX[x], {x, 0, 1}]
(*F*)
Integrate[y fY[y], {y, 0, 2}]
```

Out[ ]= 1

Out[ ]= $\frac{6}{7} x (1 + 2 x)$

Out[ ]= $\frac{15}{56}$

Out[ ]= $\frac{41}{56}$

Out[ ]= $\frac{69}{448}$

Out[ ]= $\frac{69}{80}$

Out[ ]= $\frac{5}{7}$

Out[ ]= $\frac{8}{7}$

Exercice 10 (p.340)

Exercice 11 (p.341)

In[ ]:=
```
p1 = 45 / 100;
p2 = 15 / 100;
p3 = 40 / 100;
Multinomial[2, 1, 2] p1 ^ 2 p2 p3 ^ 2
N@%
```

Out[ ]= $\frac{729}{5000}$

Out[ ]= 0.1458

Exercice 18 (p.342)

In[ ]:=
```
Clear[x, y, l]
Clear["Global`*"]
Probability[y - x > 1 / 3,
  ProductDistribution[{x, y} ≈ UniformDistribution[{{0, 1 / 2}, {1 / 2, 1}}]]]
```

Out[ ]= Probability$\left[-x + y > \frac{1}{3},\right.$
ProductDistribution$\left[\{x, y\} \approx \text{UniformDistribution}\left[\left\{\left\{0, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 1\right\}\right\}\right]\right]\Big]$

Exercice 19 (p.342)

In[ ]:=
```
ClearAll[f, fX, fY]
f[x_, y_] := 1 / x;
(*A*)
fY[y_] := Integrate[1 / x, {x, y, 1}, Assumptions → 0 < y < x < 1];
fY[y]
(*B*)
fX[x_] := Integrate[1 / x, {y, 0, x}, Assumptions → 0 < y < x < 1];
fX[x]
(*C*)
espX = Integrate[x fX[x], {x, 0, 1}, Assumptions → 0 < y < x < 1]
(*D*)
espY = Integrate[y fY[y], {y, 0, 1}, Assumptions → 0 < y < x < 1]
```

Out[ ]= −Log[y]

Out[ ]= 1

Out[ ]= $\frac{1}{2}$

Out[ ]= $\frac{1}{4}$

In[ ]:=
```
WolframAlpha["int -ylog(y) over y",
 PodStates -> {"IndefiniteIntegral__Step-by-step solution"}]
```

Assuming “int” is an integral | Use as a function property instead
Assuming “log” is the natural logarithm | Use the base 10 logarithm instead

Indefinite integrals:

Hide steps

$$\int -(y\log(y))\,dy = \frac{1}{4}\,y^2\,(1 - 2\log(y)) + \text{constant}$$

Possible intermediate steps:

Take the integral:

$$\int -y\log(y)\,dy$$

Factor out constants:

$$= -\int y\log(y)\,dy$$

For the integrand $y \log(y)$, integrate by parts, $\int f \, dg = f g - \int g \, df$, where

$$f = \log(y), \quad dg = y \, dy,$$

$$df = \frac{1}{y} \, dy, \quad g = \frac{y^2}{2}:$$

$$= -\frac{1}{2} y^2 \log(y) + \frac{1}{2} \int y \, dy$$

The integral of y is $\frac{y^2}{2}$:

$$= \frac{y^2}{4} - \frac{1}{2} y^2 \log(y) + \text{constant}$$

Which is equal to:

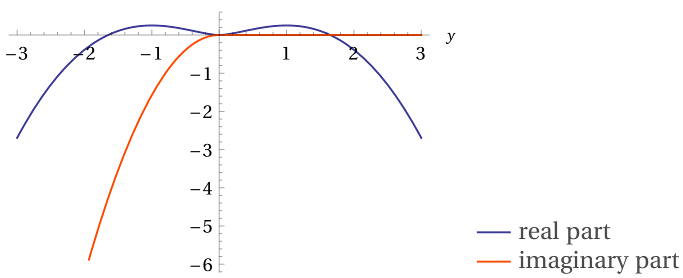
Answer:

$$= \frac{1}{4} y^2 (1 - 2 \log(y)) + \text{constant}$$

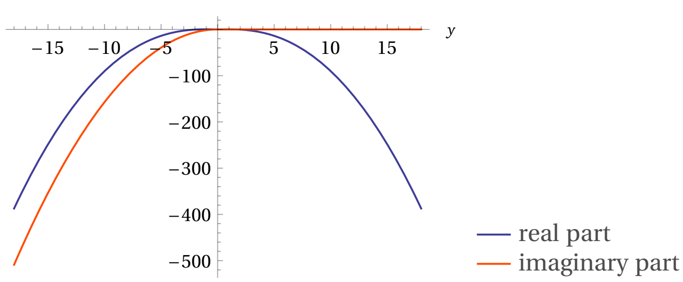
Out[∘]=

Plots of the integral:

Complex-valued plots



min max



min max

Alternate forms of the integral:

$$-\frac{1}{4} y^2 (2 \log(y) - 1) + \text{constant}$$

$$y^2 \left(\frac{1}{4} - \frac{\log(y)}{2}\right) + \text{constant}$$

$$\frac{y^2}{4} - \frac{1}{2} y^2 \log(y) + \text{constant}$$

Definite integral:

$$\int_0^1 -y \log(y)\, dy = \frac{1}{4} = 0.25$$

WolframAlpha

Exercice 22 (p.342)

In[ ]:=
```
Quit
```

In[ ]:=
```
Clear[a]
f[x_, y_] := x + y
(*A*)
fY[y_] := Integrate[f[x, y], {x, 0, 1}, Assumptions → {0 < y < 1, 0 < x < 1}];
fX[x_] := Integrate[f[x, y], {y, 0, 1}, Assumptions → {0 < y < 1, 0 < x < 1}];
BooleanQ[f[x, y] == fX[x] × fY[y]]
(*C*)
```

Out[ ]= False

Exercice 23 (p.343)

```
Quit
```

In[ ]:=
```
Clear[x, y]
Clear["Global`*"]
f[x_, y_] := 12 x y (1 - x)
(*A*)
fX[x_] := Integrate[f[x, y], {y, 0, 1}];
fX[x]
fY[y_] := FullSimplify@Integrate[f[x, y], {x, 0, 1}];
fY[y]
(*independant*)
fX[x] × fY[y] == f[x, y]
(*B*)
espx = Integrate[x fX[x], {x, 0, 1}]
(*C*)
espy = Integrate[y fY[y], {y, 0, 1}]
(*D*)
Integrate[x^2 fX[x], {x, 0, 1}] - espx^2
(*E*)
Integrate[y^2 fY[y], {y, 0, 1}] - espy^2
```

Out[ ]= $6 (1 - x) x$

Out[ ]= $2 y$

Out[ ]= True

Out[ ]= $\frac{1}{2}$

Out[ ]= $\frac{2}{3}$

Out[ ]= $\frac{1}{20}$

Out[ ]= $\frac{1}{18}$

Exercice 26 (p.343)

In[ ]:=
```
Integrate[1, {y, 2 Sqrt[x z], 1}]
Integrate[%, {x, 0, 1}]
Integrate[%, {z, 0, 1}]
```

Out[ ]= $1 - 2\sqrt{x\,z}$

Out[ ]= $1 - \frac{4\sqrt{z}}{3}$

Out[ ]= $\frac{1}{9}$

In[ ]:=
```
Clear[x, y]
Integrate[-Log[x], {x, 0, (y ^ 2) / 4}]
Integrate[%, {y, 0, 1}]
N@%
N[Log[2] / 6 + 5 / 36]
```

Out[ ]= $-\frac{1}{4}\,y^2\left(-1 + \mathrm{Log}\left[\frac{y^2}{4}\right]\right)$

Out[ ]= $\frac{1}{36}\,(5 + \mathrm{Log}[64])$

Out[ ]= 0.254413

Out[ ]= 0.254413

Exercice 27 (p.344)

In[ ]:=
```
Clear[a, b, z];
Clear["Global`*"]
fx[x_] := a Exp[-a x]
fy[y_] := b Exp[-b y]
FZ[z_] := Integrate[Integrate[fx[x] × fy[y], {x, 0, y z}], {y, 0, Infinity},
  Assumptions → {a ∈ PositiveReals && b ∈ PositiveReals, z ∈ PositiveReals}]
FZ[1]
D[FZ[z], z]
```

Out[ ]= $\frac{a}{a+b}$

Out[ ]= $-\frac{a^2 z}{(b+a z)^2}+\frac{a}{b+a z}$

Exercice 33 (p.344)

In[ ]:=
```
lambda = 1 / 5;
(*A*)
Poisson[lambda, 0]
N@%
Poisson[10 lambda, 0]
(*B*)
1 - Sum[Poisson[lambda, i], {i, 0, 1}]
N@%
1 - Sum[Poisson[10 lambda, i], {i, 0, 1}]
```

Out[ ]= $\frac{1}{e^{1/5}}$

Out[ ]= 0.818731

Out[ ]= $\frac{1}{e^2}$

Out[ ]= $1-\frac{6}{5 e^{1/5}}$

Out[ ]= 0.0175231

Out[ ]= $1-\frac{3}{e^2}$

Exercice 34 (p.345)

In[ ]:=
```
lambda = 2.2;
(*A*)
1 - Sum[Poisson[lambda, i], {i, 0, 2}]
(*B*)
1 - Sum[Poisson[2 lambda, i], {i, 0, 4}]
(*C*)
1 - Sum[Poisson[3 lambda, i], {i, 0, 5}]
```

Out[ ]= 0.377286

Out[ ]= 0.448816

Out[ ]= 0.645327

Exercice 40 (p.345)

Exercice 41 (p.345)

In[ ]:=
```
Quit
```

In[ ]:=
```
f[x_, y_] := x Exp[-x (y + 1)];
(*A*)
(*fX|Y*)
Integrate[f[x, y], {x, 0, Infinity}, Assumptions → {x > 0 && y > 0}]
a = f[x, y] / Integrate[f[x, y], {x, 0, Infinity}, Assumptions → {x > 0 && y > 0}]
(*fY|X*)
Integrate[f[x, y], {y, 0, Infinity}, Assumptions → {x > 0 && y > 0}]
b = f[x, y] / Integrate[f[x, y], {y, 0, Infinity}, Assumptions → {x > 0 && y > 0}]
BooleanQ[a b == f[x, y]]
(*B*)
Integrate[Integrate[f[x, y], {x, 0, z / y},
   Assumptions → {y > 0 && z ∈ PositiveReals}], {y, 0, Infinity}]
D[
 %,
 z]
```

Out[ ]= $\frac{1}{(1+y)^2}$

Out[ ]= $e^{-x(1+y)}\, x\, (1+y)^2$

Out[ ]= e^{-x}

Out[ ]= $e^{x-x(1+y)}\, x$

Out[ ]= False

Out[ ]= ConditionalExpression[$1 - e^{-z}$, Re[z] ≥ 0]

Out[ ]= ConditionalExpression[e^{-z}, Re[z] ≥ 0]

In[ ]:=
```
WolframAlpha["int x Exp[-x(y+1)] over x",
 PodStates -> {"IndefiniteIntegral__Step-by-step solution"}]
```

Assuming “int” is an integral | Use as a function property instead

Indefinite integrals: Approximate form Hide steps

$$\int x \exp(-(x(y+1)))\, dx = -\frac{e^{-x(y+1)}(x\, y + x + 1)}{(y+1)^2} + \text{constant}$$

Possible intermediate steps:

Take the integral:

$$\int x\, e^{-x(y+1)}\, dx$$

For the integrand $x\,e^{x(-y-1)}$, integrate by parts, $\int f\,dg = f g - \int g\,df$, where

$f = x, \quad dg = e^{x(-y-1)}\,dx,$

$df = dx, \quad g = \dfrac{e^{x(-y-1)}}{-y-1}$:

$$= \frac{x\,e^{x(-y-1)}}{-y-1} - \frac{1}{-y-1}\int e^{x(-y-1)}\,dx$$

For the integrand $e^{x(-y-1)}$, substitute $u = x(-y-1)$ and $du = (-y-1)\,dx$:

$$= \frac{x\,e^{x(-y-1)}}{-y-1} - \frac{1}{(-y-1)^2}\int e^{u}\,du$$

The integral of e^u is e^u:

$$= \frac{x\,e^{x(-y-1)}}{-y-1} - \frac{e^{u}}{(-y-1)^2} + \text{constant}$$

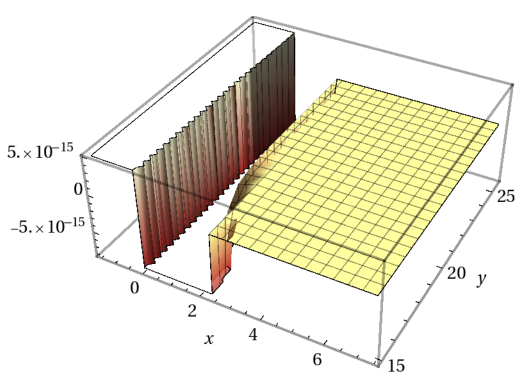
Substitute back for $u = x(-y-1)$:

Answer:

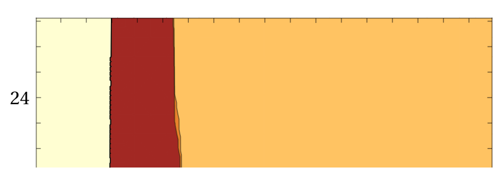
$$= -\frac{e^{-x(y+1)}\,(x\,y + x + 1)}{(y+1)^2} + \text{constant}$$

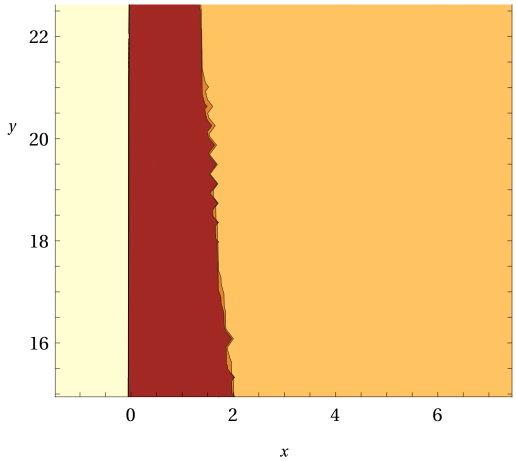
3D plot:

Out[ ]=



Contour plot:





Alternate form of the integral:

$$-\frac{e^{x(-y-1)}(x(y+1)+1)}{(y+1)^2} + \text{constant}$$

Expanded form of the integral: Step–by–step solution

$$-\frac{x\,e^{-x(y+1)}}{(y+1)^2} - \frac{x\,y\,e^{-x(y+1)}}{(y+1)^2} - \frac{e^{-x(y+1)}}{(y+1)^2} + \text{constant}$$

Series expansion of the integral at x = 0:

$$-\frac{1}{(y+1)^2} + \frac{x^2}{2} + \frac{1}{3}x^3(-y-1) + \frac{1}{8}x^4(y+1)^2 + O(x^5)$$

(Taylor series)

Big-O notation »

Series expansion of the integral at x = ∞:

$$e^{-x(y+1)}\left(-\frac{x}{y+1} - \frac{1}{(y+1)^2} + O\left(\left(\frac{1}{x}\right)^{13}\right)\right)$$

Big-O notation »

WolframAlpha

In[ ]:=
```
D[Exp[-x^3], x]
```

Out[ ]= $-3\,e^{-x^3}\,x^2$

Exercice 53 (p.347)

In[ ]:=
```
r[x_, y_] := Sqrt[x^2 + y^2];
theta[x_, y_] := ArcTan[y / x];
(*Jacobien*)
MatrixForm[FullSimplify@D[{r[x, y], theta[x, y]}, {{x, y}}]]
FullSimplify@Det@%
Solve[r == r[x_, y_] && theta == theta[x, y], {x, y}]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} \\ -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{pmatrix}$$

Out[ ]= $\frac{1}{\sqrt{x^2+y^2}}$

Out[ ]= $\text{Solve}\left[r == \sqrt{x\_^2 + y\_^2} \text{ \&\& } \text{theta} == \text{ArcTan}\left[\frac{y}{x}\right], \{x, y\}\right]$

Exercice 54 (p.347)

In[ ]:=
```
Clear[x, y, z, u]
x[u_, z_] := Sqrt[2 z] Cos[u]
y[u_, z_] := Sqrt[2 z] Sin[u]
MatrixForm[FullSimplify@D[{x[u, z], y[u, z]}, {{u, z}}]]
FullSimplify@Det@%
y[u, z] / x[u, z]
Cos[ArcTan[y / x]]
Sin[ArcTan[y / x]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -\sqrt{2}\,\sqrt{z}\,\text{Sin}[u] & \frac{\text{Cos}[u]}{\sqrt{2}\,\sqrt{z}} \\ \sqrt{2}\,\sqrt{z}\,\text{Cos}[u] & \frac{\text{Sin}[u]}{\sqrt{2}\,\sqrt{z}} \end{pmatrix}$$

Out[ ]= -1

Out[ ]= Tan[u]

Out[ ]= $\frac{1}{\sqrt{1+\frac{y^2}{x^2}}}$

Exercice 58 (p.341)

In[4]:=
```
g1[x1_, x2_] := x1 + x2
g2[x1_, x2_] := Exp[x1]
MatrixForm[FullSimplify@D[{g1[x1, x2], g2[x1, x2]}, {{x1, x2}}]]
FullSimplify@Det@%
```

Out[6]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ e^{x1} & 0 \end{pmatrix}$$

Out[7]= $-e^{x1}$

Auto-évaluation

In[ ]:=
```
InverseFunction[# c &]
```

Out[ ]= $\frac{\#1}{c}$ &

In[3]:=
```
Integrate[Integrate[Log[z], {z, b^2 / 4, 0}] 1, {b, 0, 1}]
```

Out[3]= $\frac{1}{36}$ (5 + Log[64])