Ex 4.17 $P(X=1) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$P(X=2) = \frac{11}{12} - \left(\frac{1}{2} + \frac{2-1}{4}\right)$

$= \frac{11}{12} - \frac{3}{4} = \frac{11}{12} - \frac{9}{12} = \frac{1}{6}$

$P(X=3) = 1 - \frac{11}{12} = \frac{1}{12}$

b) $P\left(\frac{1}{2} < x < \frac{3}{2}\right) = F\left(\frac{3}{2}^-\right) - F\left(\frac{1}{2}^+\right)$

$= \frac{1}{2} + \frac{3/2 - 1}{4} - \left(\frac{1/2}{4}\right)$

$= 1/2 + \frac{1}{8} - \frac{1}{8} = 1/2$

Ex 4.38 $E(X) = 1$ $\quad Var(X) = 5$

a) $E[(2+X)^2]$

$= E[4 + 4x + x^2]$

$= 4 + 4E(x) + E(x^2)$

$= 4 + 4 + E(x^2)$

$= 8 + Var(X) + [E(x)]^2$

$= 8 + 5 + 1^2 = 14$

$E[ax+b] = aE(x) + b$

$Var X = E(x^2) - [E(x)]^2$

$E(x^2) = Var X + [E(x)]^2$

b) $Var(4+3X)$ $= 9\,Var(X) = 45$

$Var(aX+b) = a^2\,Var(X)$

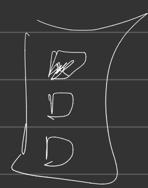
Ex 4.40

Examen $\rightarrow$ 5 questions à 3 choix multiple

$P(X \geq 4)$

1 question $\rightarrow$

2 questio



$$P(X=0) \quad \binom{2}{0}\left(\frac{1}{3}\right)^0 \times \left(\frac{2}{3}\right)^2$$

$$P(X=1) \quad \binom{2}{1}\left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)$$

$$P(X=2) \quad \binom{2}{2}\left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^0$$

$$= \frac{4}{9} + 2 \times \frac{2}{9} + \frac{1}{9} = 1$$

$$P(X=4) = \binom{5}{4} \left(\frac{1}{3}\right)^4 \times \left(\frac{2}{3}\right)^1$$

$+$

$$P(X=5) = \binom{5}{5} \left(\frac{1}{3}\right)^5 \times \left(\frac{2}{3}\right)^0$$

$$= 5 \times \frac{1}{3^4} \times \frac{2}{3} + 1 \cdot \frac{1}{3^5}$$

$$= \frac{11}{3^5} = \frac{11}{243} = 4{,}53\%$$

Ex 4.46

n sous-systèmes

OK si k sous-système

$$P(n_i \mid r) = p_1$$

$$P(n_i \mid r^c) = p_2$$

$r =$ "rain" $\qquad r^c =$ "sec"

$$P(X = k) = \binom{n}{k} p_1^{k} (1 - p_1)^{n-k}$$

$$= P$$

$$P(X=k) = \binom{n}{k} p_2^{k} (1-p_2)^{n-k}$$

$$= S$$

$$P(X=k) = \alpha P + (1-\alpha) S$$

$$P(X \geq k) = \binom{n}{k} \cdots + \binom{n}{k+i}$$

$$+ \cdots \binom{n}{n} p$$

$$= \sum_{i=k}^{n} \binom{n}{i} p_1^{i} (1-p_1)^{n-i}$$

$$= P$$

$$\alpha \sum_{i=k}^{n} \binom{n}{i} p_1^{i} (1-p_1)^{n-i}$$

$$+ (1-\alpha) \sum_{i=k}^{n} \binom{n}{i} p_2^{i} (1-p_2)^{n-i}$$

~~Ex 5.57~~ $\quad \lambda = 3$

$$P(X=i) = e^{-\lambda} \frac{\lambda^i}{i!}$$

$$P(X=3) = e^{-3} \frac{3^3}{3!}$$

$$= \frac{9}{2e^3}$$

$$\approx 22,40\%$$

$$\cancel{P(X \leq 3) = P(X=0) + P(X=2) + P(X=3)}$$

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2).$$

$$= e^{-3} \times \frac{3^0}{0!} + e^{-3} \times \frac{3^1}{1!} + e^{-3} \cdot \frac{3^2}{2!}$$

$$= e^{-3}\left(1 + 3 + \frac{9}{2}\right) \qquad = \frac{17}{2e^3}$$

$$= e^{-3}\left(\frac{2 + 6 + 9}{2}\right)$$

$$= \frac{17}{2e^3} \approx 42{,}32\%$$

$$1 - \frac{17}{2e^3} \simeq 57,68\%$$

— o —

b) a) $P(X \geq 3)$

$$1 - [P(X=0) + P(X=1) + P(X=2])$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$A : \cancel{X} \geq 3$

$B : \cancel{X} \geq 1$

$P(A \cap B) = P(A)$

$\underline{P(X \geq 3)}$ → on l'a

$P(X \geq 1)$

↳ $1 - P(X = 0)$

$$1 - \boxed{\frac{1}{e^3}} \; ?$$

$$\frac{1 - \dfrac{17}{2e^3}}{1 - \dfrac{1}{e^3}} \approx 60,70\%$$

Ex 4.63 $5 \to 2,5$ person

a) $X \sim Po(2,5)$

$$P(X=0) = e^{-\lambda} \frac{\lambda^{\cancel{i}\,0}}{i!}$$

$$= e^{-2,5} \cdot \frac{2,5^0}{0!}$$

$$= \frac{1}{e^{2,5}} = 8,21\%$$

b) $\cancel{P} - (P(X=0) + P(X=1) + P(X=3))$

$$= 1 - \frac{1}{e^{2,5}}\left[\frac{443}{48}\right]$$

$$\approx 0,2424 = 24,24\%$$

Ex 4.65 $n = 500$ $p = \frac{1}{1000}$

a) $\lambda = np = \frac{1}{2}$ (approximation de Bernoulli par Poisson)

$P(X=0) = e^{-\lambda} \frac{\lambda^i}{i!}$

$= \frac{1}{e^{1/2}} = \frac{1}{\sqrt{e}} \rightarrow 1 - \frac{1}{\sqrt{e}}$

b) $X \geq 1$

$\cancel{P(X \geq 2) = 1 - P(X=0)}$

$\cancel{- P(X=1)}$

$\cancel{= 1 - \frac{1}{\sqrt{e}} \cdot \frac{3}{2} = 0{,}0902}$

c) $P(X \geq 2 \mid X = 1)$

$$= \frac{(X \geq 2) \cap (X \geq 1)}{P(X = 1)}$$

$$= \frac{P(X \geq 2)}{P(X = 1)}$$

$$= \frac{1 - \frac{1}{\sqrt{e}} \times \frac{3}{2}}{\frac{1}{\sqrt{e}} \times \frac{1}{2}} = 2\left(1 - \frac{3}{2\sqrt{e}}\right)\sqrt{e}$$

$$= 0,2974$$

b) $P(X > 1 \mid X \geq 1) =$

$$\frac{P(X > 1 \cap X \geq 1)}{P(X \geq 1)}$$

$$= \frac{P(X > 1)}{P(X \geq 1)} \Rightarrow \text{notre}$$

c) $\lambda = np = \frac{500}{1000}$

$\lambda = \frac{499}{1000} \Rightarrow$ c) a)

d)

$p = \frac{1}{1000}$

$$\sum_{j=k}^{500} \binom{n}{j} p^j (1-p)^{500-j}$$

500

$500 - (i-1) + 1$

$= 500 - i$

$1 \ldots\ldots (i-1) \; i$

negatif 1

+

~~termes = 500 - i + 1~~

~~= 501 - i~~

$$\underbrace{1 \cdots\cdots i}_{\text{on}} \ \boxed{i+1} \cdots 500$$

$$\underbrace{\underbrace{\hphantom{xxxx}}_{500-(i+1)} + 1}_{500 - i}$$

$$1 - P(X = 0) \rightarrow n = \cancel{500}$$

$$n = 500 - i$$

$$\sum_{j=500-i}^{500} \binom{500}{j} p^{j} (1-p)^{500-j}$$

Ex 4.71

- 0
- 00
- 1 à 36

a) Smith il met des jetons sur les cases 1 à 12

$$P_{succes} = \frac{12}{38}$$

$$P_{echec} = \frac{26}{38}$$

a) $$\left(P_{echec}\right)^5 = \left(\frac{26}{38}\right)^5$$

b) $$\left(\frac{26}{38}\right)^3 \left(\frac{12}{38}\right)^1$$

Ex 4.75

Obtenir 2 Piles

X = nombre de face

$$P(X=0) = \left(\frac{1}{2}\right)^2$$

$$P(X=1) = \binom{2}{1}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2$$

$$P(X=2) = \binom{3}{1} \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$$

$$P(X=i) = \binom{i+1}{1}\left(\frac{1}{2}\right)^i\left(\frac{1}{2}\right)^2$$

Exc 4.78

4 B

4 N

on tire 4 boule

~~2~~ 2 B + 2 N → STOP

A = " "

$$P(A) = \frac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}} = p$$

$$(1-p)^{n-1} \quad (p)$$