Quiz 2 - MAT1720

#1

a) $D = \det\begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} = r\cos^2\theta + r\sin^2\theta = r$

b) $f_{R,\Theta}(r,\theta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(r\cos\theta)^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(r\sin\theta)^2} \cdot |r|$ car X_1 et X_2 sont indép.

$= re^{-\frac{1}{2}r^2} \cdot \frac{1}{2\pi} \quad (r>0,\ \theta \in (0,2\pi))$ car $\cos^2\theta + \sin^2\theta = 1$

c) Oui car $f_R(r) = re^{-\frac{1}{2}r^2}$, $f_\Theta(\theta) = \frac{1}{2\pi}$ et $f_{R,\Theta}(r,\theta) = f_R(r) f_\Theta(\theta)$ par b).

#2

a) $J = \det\begin{pmatrix} z & y \\ \frac{1}{z} & -\frac{y}{z^2} \end{pmatrix} = -\frac{y}{z} - \frac{y}{z} = -2\frac{y}{z} = -2t$

b) $f_{S,T}(s,t) = \frac{1}{(\sqrt{st})^2\left(\sqrt{\frac{s}{t}}\right)^2} \cdot \frac{1}{|2t|} = \frac{1}{2s^2t} \quad (s>1,\ s^{-1}<t<s)$

c) $f_S(s) = \int_{\frac{1}{s}}^{s} \frac{1}{2s^2t}\,dt = \frac{1}{2s^2}\left[\ln t\right]\Big|_{1/s}^{s} = \frac{1}{s^2}\ln(s) \quad (s>1)$.

<u>BONUS</u> : $f_T(t) = \int_{\max\left(\frac{1}{t},t\right)}^{\infty} \frac{1}{2s^2t}\,ds = \begin{cases} \int_{1/t}^{\infty} \frac{1}{2s^2t}\,ds & \text{si } t\in(0,1) \\ \int_{t}^{\infty} \frac{1}{2s^2t}\,ds & \text{si } t\geq 1 \end{cases}$

$= \begin{cases} \frac{1}{2t}\left[-\frac{1}{s}\right]\Big|_{1/t}^{\infty} & \text{si } t\in(0,1) \\ \frac{1}{2t}\left[-\frac{1}{s}\right]\Big|_{t}^{\infty} & \text{si } t\geq 1 \end{cases} = \begin{cases} \frac{1}{2} & \text{si } t\in(0,1) \\ \frac{1}{2t^2} & \text{si } t\geq 1 \end{cases}$