Model Flexibility - Takeaways

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- We'll need **some sort of mechanism** to tell which regime we're in
- Examples of different model families
  - Useful tutorial: pyGAM

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- **Reading**: ISLR 2.1, 3.2.1, 3.5

Algorithms: K-Nearest Neighbors (KNN) Regression

- $\boldsymbol{K}$ fixed and given
- **Samples:** $(\boldsymbol{x_i}, \mathbf{y_i})_{\{\mathbf{i=1}\}}^{\mathbf{N}}$
- **Estimate data generating function:** $\hat{\boldsymbol{f}}(\boldsymbol{x}) = \frac{\mathbf{1}}{\boldsymbol{K}} \sum_{\boldsymbol{i} \in \boldsymbol{N_K}(\boldsymbol{x})} \boldsymbol{y_i}$
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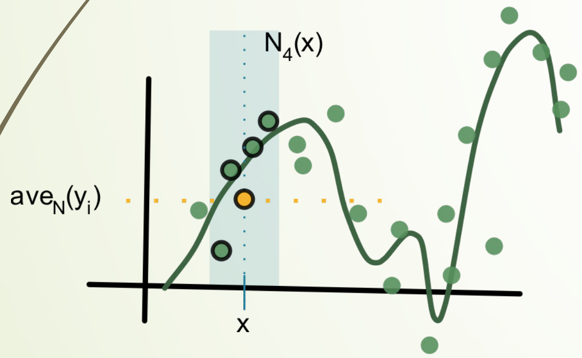

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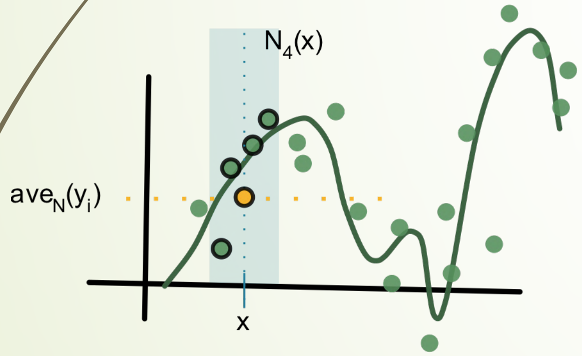

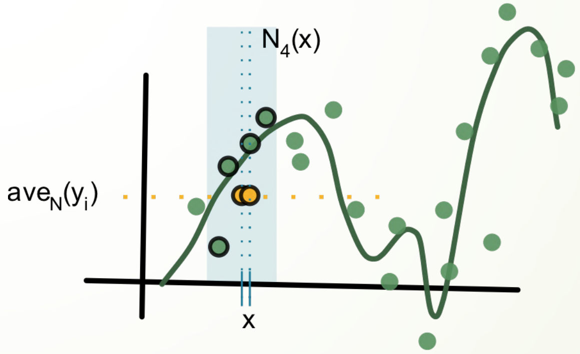

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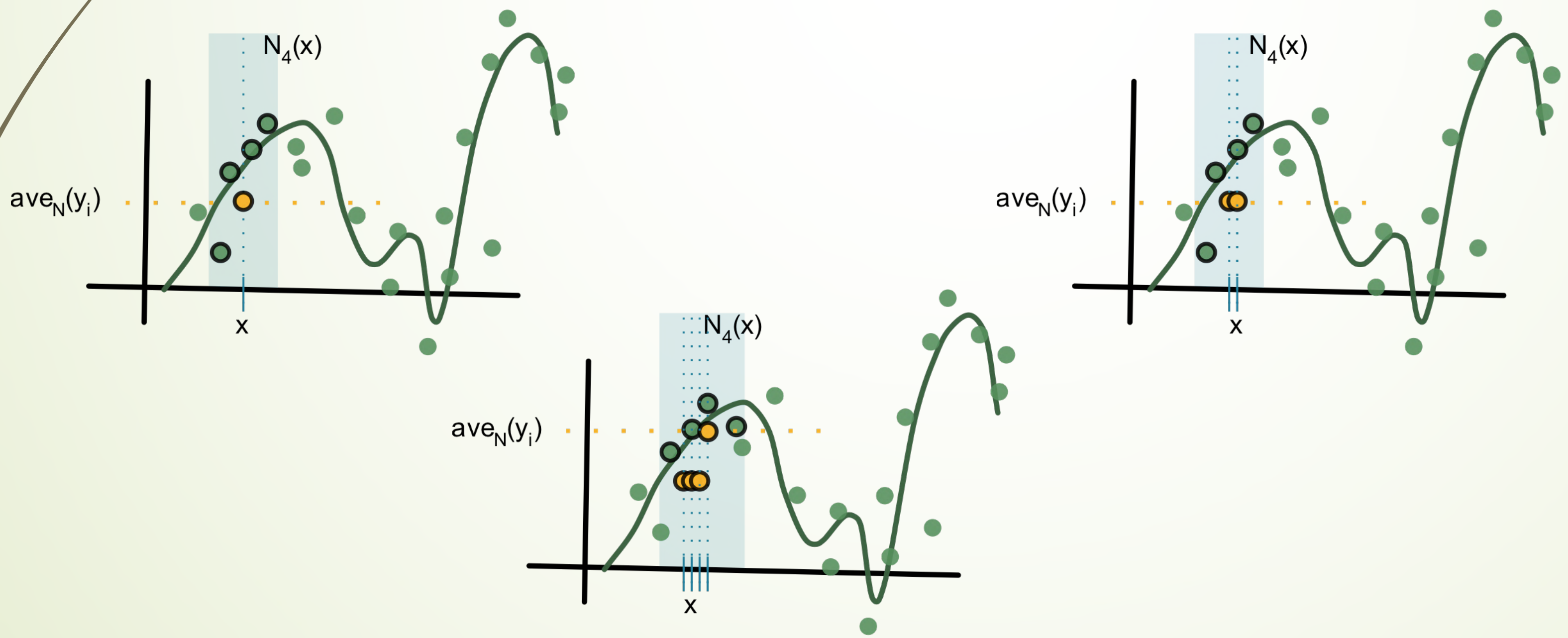

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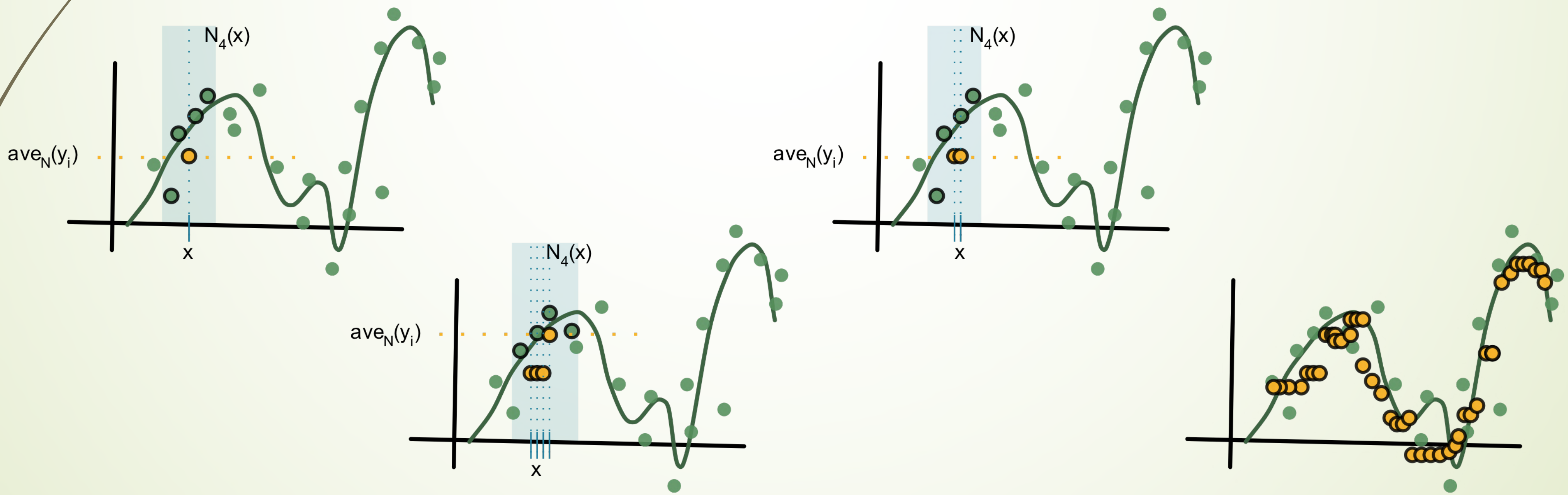

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  - Larger $\boldsymbol{K}$ learns smoother functions; smaller $\boldsymbol{K}$ can match more complex functions when the sampling density is high enough

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