Function fitting

Lecture 4; Reading: ISLR sections 2.1, 3.2.1, 3.5

IFT6758, Fall 2020

Leveraging Input → Output relationship in data

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- Person's characteristics → Whether they'll vote
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Mathematical expression

- $x_i = (x_{i_1}, \ldots, x_{i_p})$ ← **inputs**
- $\boldsymbol{y_i} = \boldsymbol{f}(\boldsymbol{x_i})$ ← **inputs to output relationship**

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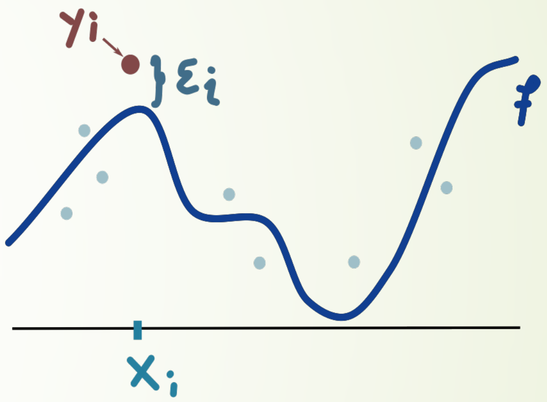

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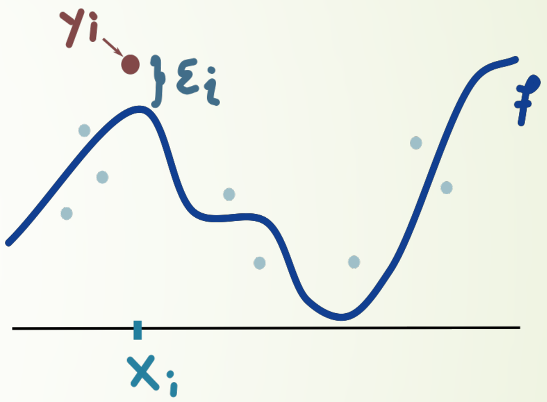

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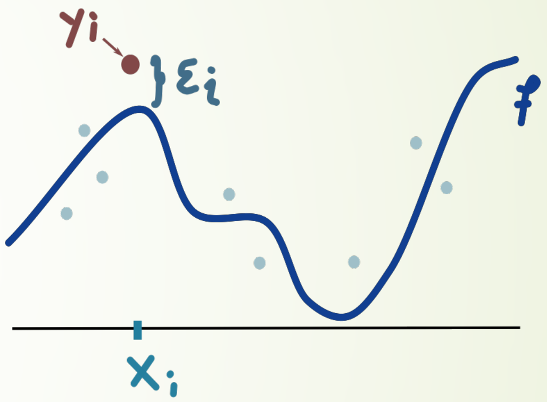

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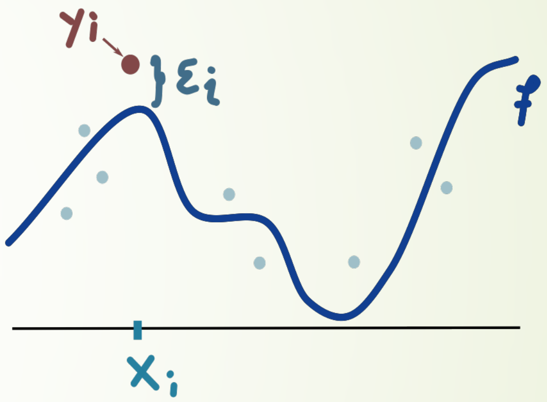

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- $\boldsymbol{f}$ describes **systematic variation** in $\boldsymbol{y_i}$
- $\boldsymbol{\epsilon_i}$ reflects variations whose source is unknown

to us. Consider coin tosses.

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We want **quantitative estimates**, not visual summaries

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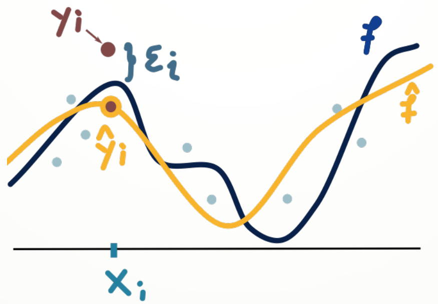

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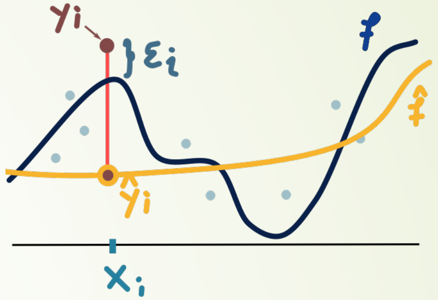

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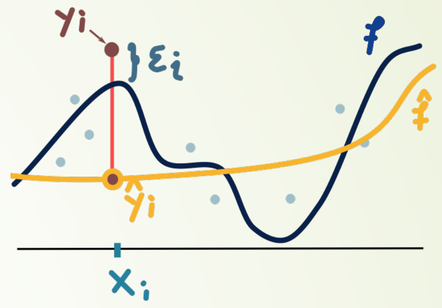

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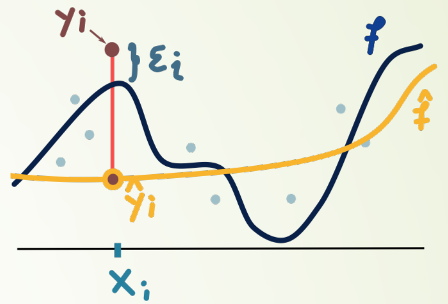

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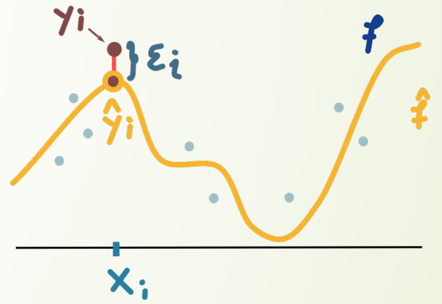

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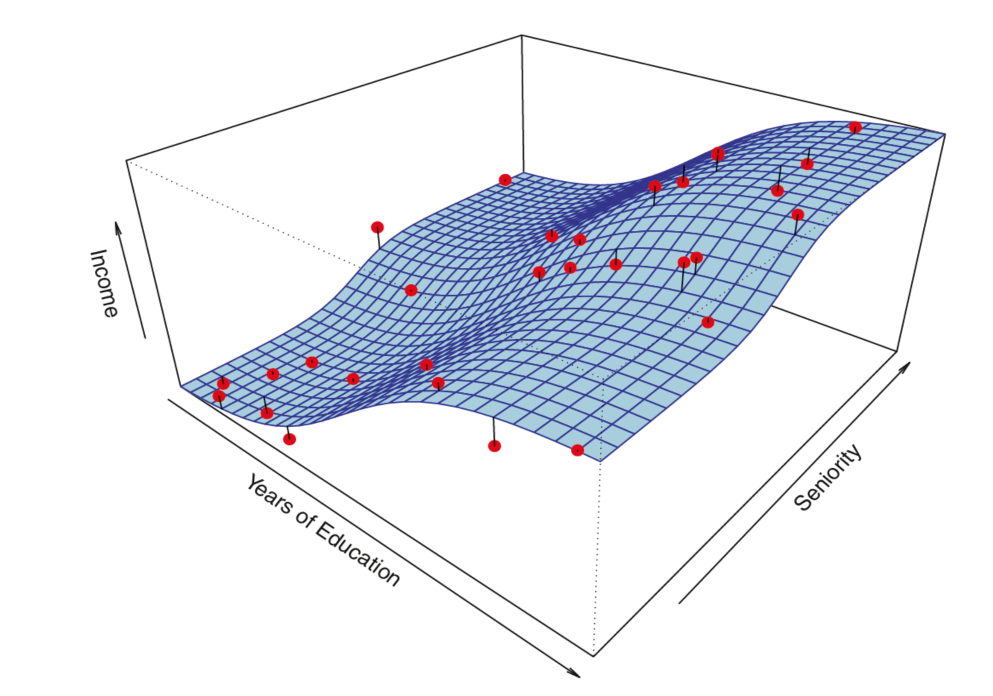

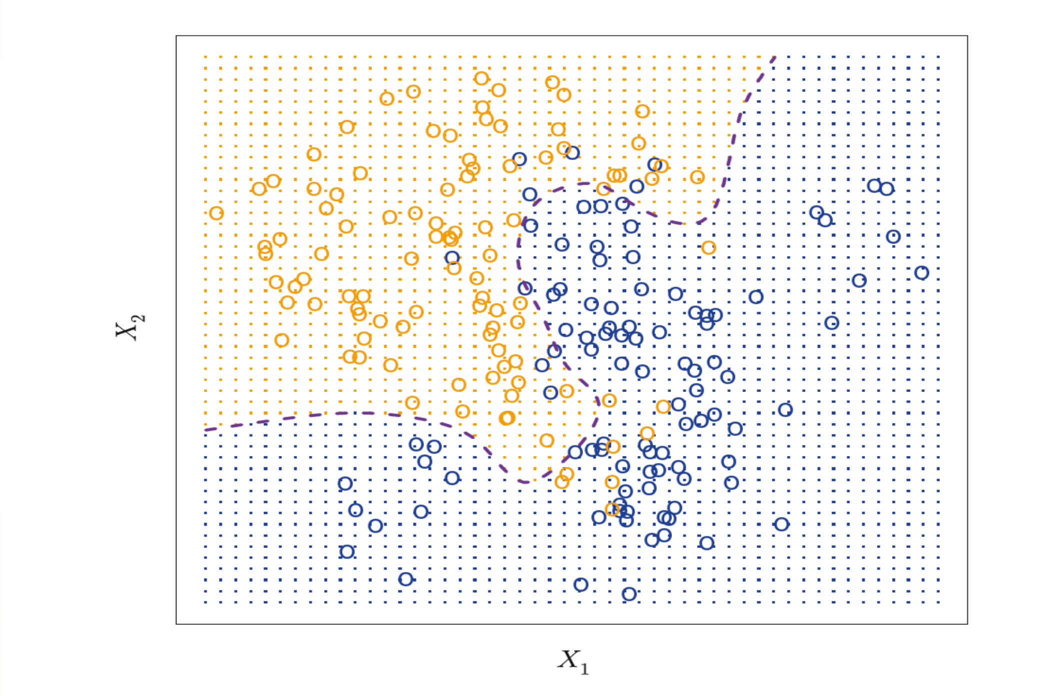

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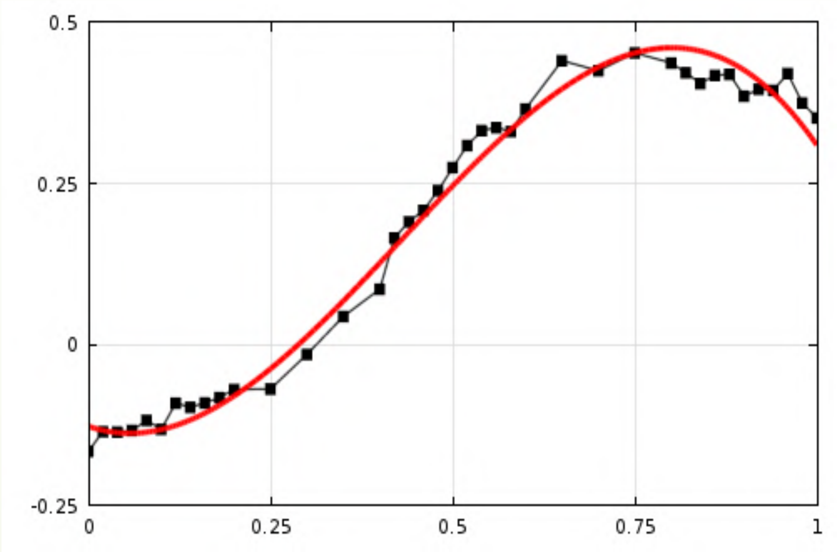

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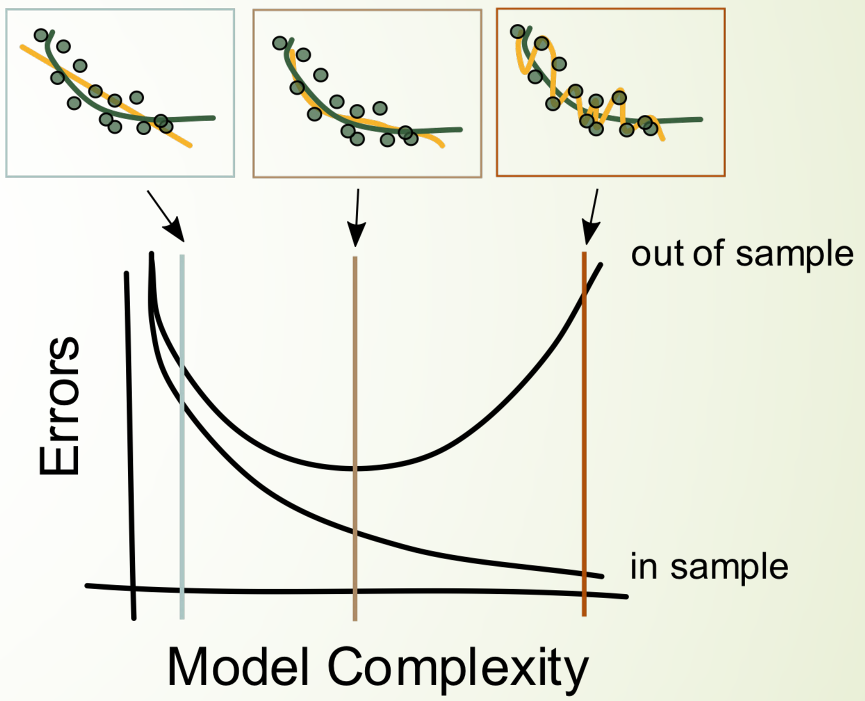

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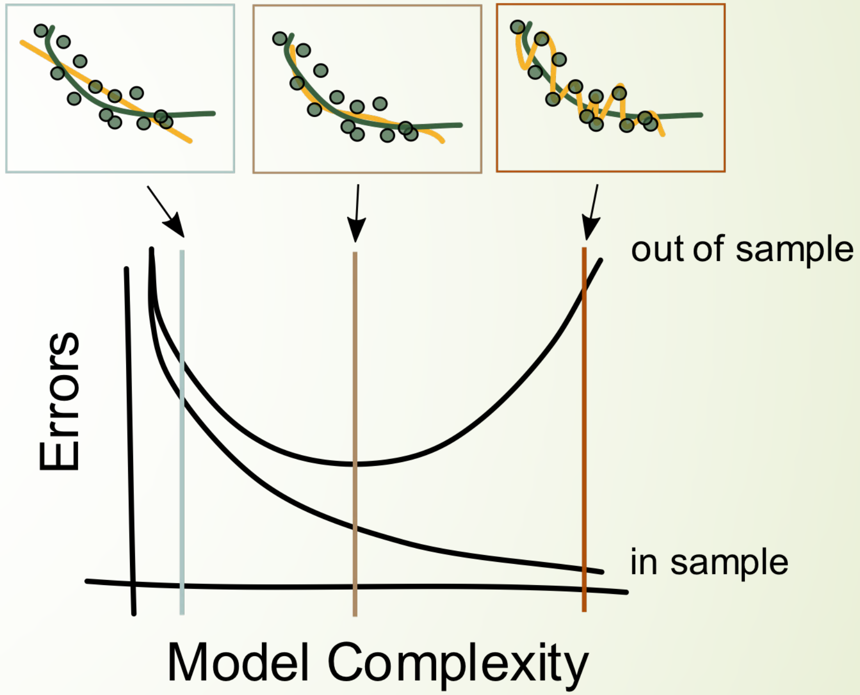

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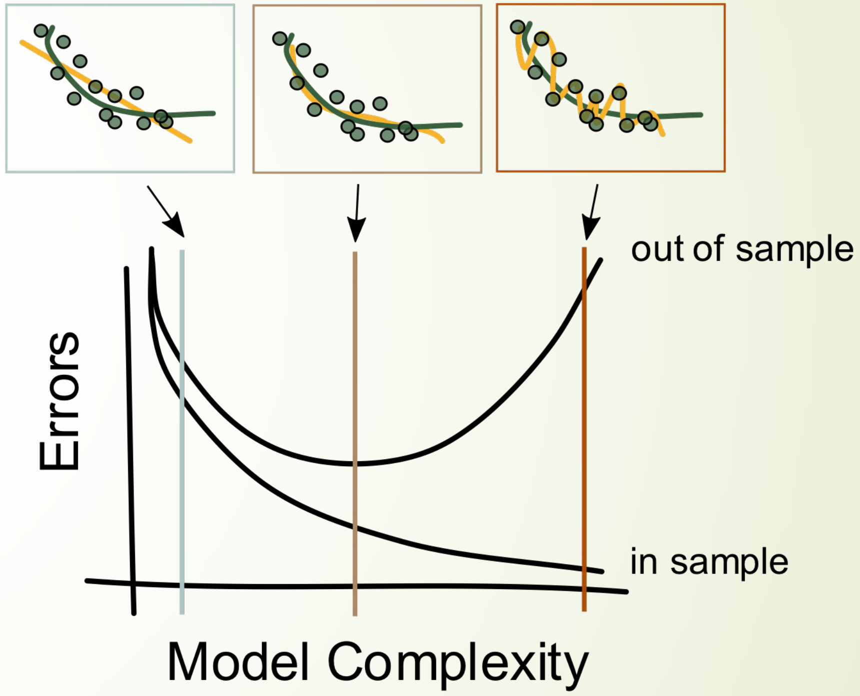
- **Incurring some bias can** lead to **better stability** and overall **better predictions**



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- If you have **many samples**, you can afford a **more complex model**
- We'll need **some sort of mechanism** to tell which regime we're in
- Examples of different model families