CECS 451
Assignment 4
Total: 45 Points

General Instruction

- Submit uncompressed file(s) in the Dropbox folder via Canvas (Not email).
- Use `Python 3`, any other programming language is not acceptable.
- You can import modules in the following list of libraries (please check the full list *here*). If you want to use any other library, please consult with the instructor.

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1. Refer to Figure 1 (note that some probabilities differ from those in the textbook). Implement a program to answer the query $\vec{P}(C|\neg s, w)$ using MCMC sampling. The program should generate n samples to estimate the probability. For parts (b) and (c), you can prepare the answers on scratch paper and print them out (hard coding is acceptable). However, for part (d), you are required to implement a simulation program. Please name your file `mcmc.py`.

   (a) (1 point) To evaluate your simulation program, install `sorobn` and `pandas` libraries on your workstation. Please refer this *site*.

```
import sorobn as hh
import pandas as pd

bn = hh.BayesNet(
... ('C', ['S', 'R']),
... ('S', 'W'),
... ('R', 'W'))

bn.P['C'] = pd.Series({True: 0.5, False:0.5})
bn.P['S'] = pd.Series({
... (True, True): 0.1, (True, False): 0.9,
... (False, True): 0.5, (False, False): 0.5})
bn.P['R'] = pd.Series({
... (True, True): 0.8, (True, False): 0.2,
... (False, True): 0.2, (False, False): 0.8})
bn.P['W'] = pd.Series({
... (True, True, True): 0.99, (True, True, False): 0.01,
... (True, False, True): 0.9, (True, False, False): 0.1,
... (False, True, True): 0.95, (False, True, False): 0.05,
... (False, False, True): 0.05, (False, False, False): 0.95})

bn.prepare()
bn.query('C', event={'S': False, 'W': True})
```

(b) (8 points) Show $\vec{P}(C|\neg s, r), \vec{P}(C|\neg s, \neg r), \vec{P}(R|c, \neg s, w), \vec{P}(R|\neg c, \neg s, w)$.

(c) (16 points) Show the transition probability matrix $Q \in \mathbb{R}^{4\times 4}$ where q_{ij} = transition probability from S_i to S_j in Figure 2.

(d) (20 points) For given n, show the estimated probability of the query $\vec{P}(C|\neg s, w)$

(e) Please follow the output format. (Fix precisions using `"{:.xf}".format`)

```
Part A. The sampling probabilities
P(C|-s,r) = <0.xxxx, 0.xxxx>
P(C|-s,-r) = <0.xxxx, 0.xxxx>
P(R|c,-s,w) = <0.xxxx, 0.xxxx>
P(R|-c,-s,w) = <0.xxxx, 0.xxxx>

Part B. The transition probability matrix
       S1      S2      S3      S4
S1      .       .       .       .
S2      .       .       .       .
S3      .       .       .       .
S4      .       .       .       .

Part C. The probability for the query P(C|-s,w)
Exact probability: <0.xxxx, 0.xxxx>
n = 10 ^ 3: <0.xxxx, 0.xxxx>, error =  x.xx %
n = 10 ^ 4: <0.xxxx, 0.xxxx>, error =  x.xx %
n = 10 ^ 5: <0.xxxx, 0.xxxx>, error =  x.xx %
n = 10 ^ 6: <0.xxxx, 0.xxxx>, error =  x.xx %
```

The error can be calculated using the formula:

$$error(\%) = |\frac{p - \hat{p}}{p}| \times 100,$$

where $\hat{p}$ is the estimated probability and p is the exact probability of the event $C = True$.

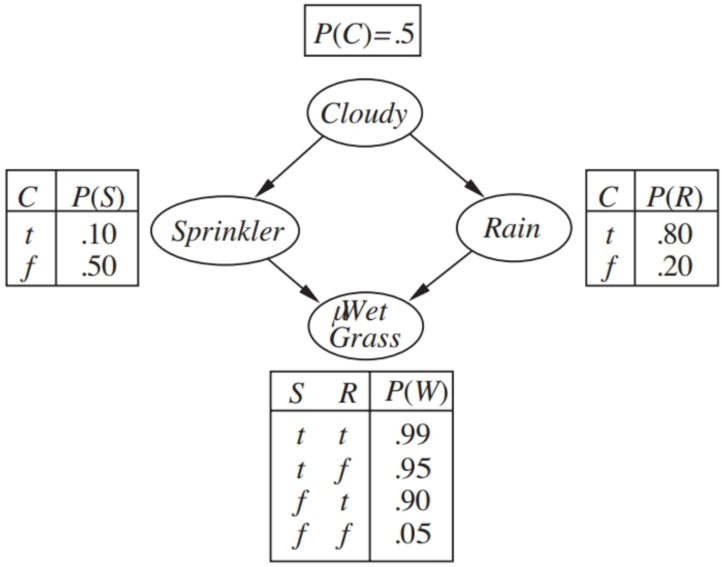

Figure 1: A multiply connected network with conditional probability tables. Note that the probabilities are slightly different than the lecture notes and the text book example.

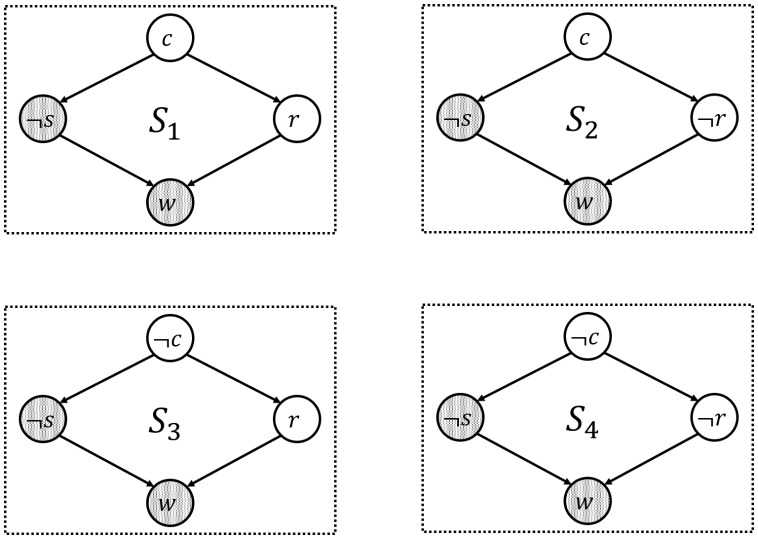

Figure 2: Possible states diagram