Review in Probability, Intro to RL Concepts

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Probability

- Measurement of **uncertainty**.
  - We ponder on tonight's traffic jam, tomorrow's weather, next week's stock prices, an upcoming election, or where we left our hat, often we do not know an outcome with certainty.
  - Also used in games and simulations.
- Probability tells us **how likely** it is that a particular event will occur.
- Often denoted as $P(A)$ where A is an event or a collection of possible outcomes.
- Conforms to the Discrete Probability Law:

$$P(A) = \frac{Number\ of\ elements\ in\ A}{n\ possible\ outcomes}$$

Probability

What's the probability of getting a 4?

There's only one 4, and six possible numbers total.

So we say the probability is **1 out of 6**, or:
P(4) = 1/6

That matches the formula shown on the slide:

P(A) = (Number of elements in A) / (Total possible outcomes)

Where **A** is the event you're interested in.

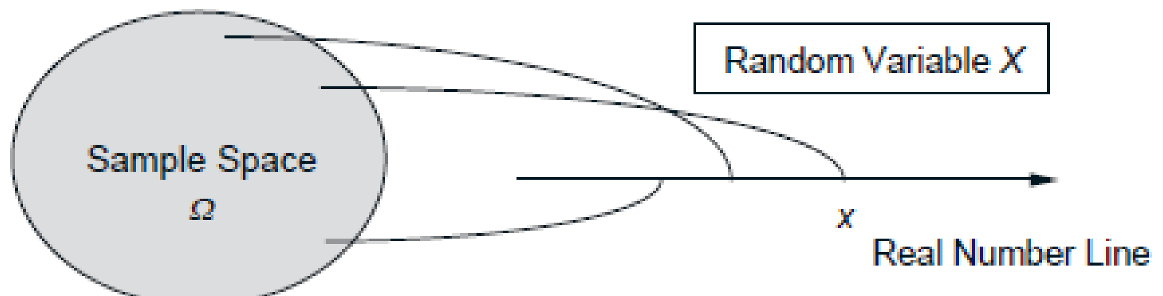
Example

- The most fundamental stochastic experiment is the experiment where a coin is tossed a number of times, say n times.
- Tossing a coin a few times won't immediately give you a 50% chance for all outcomes. But tossing it many, many times will.

*the **Law of Large Numbers.** It tells us that over a large number of trials, the results will settle into predictable patterns.*

Random Variables

- A particular number which is associated with or depends on the outcomes of a random event.
- Mathematically, a random variable is a **real-valued function** of the experimental outcome.
- Denoted as X. Thus, we write $P(X = x)$.



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Random Variables

The notation $P(X = x)$ is read as:
"The probability that the random variable X is equal to x."

Here's a simple breakdown:

- X (capital X) is the **random variable** — think of it as a function that gives a numeric outcome from a random event.
- x (small x) is a specific value that the random variable might take.
- $P(X = x)$ means we're asking: *How likely is it that X ends up being equal to x?*

Example:
If you roll a fair 6-sided die and let **X = the number rolled**, then:

- $P(X = 3)$ means "What's the probability of rolling a 3?"
- The answer would be **1/6**, since all outcomes (1 to 6) are equally likely.

Example

In an experiment involving a sequence of 3 tosses of a coin, the number of getting heads H in the sequence is a random variable X.

$P(X = 0) = 1/8$ (T, T, T)

$P(X = 1) = 3/8$ (H, T, T), (T, H, T), (T, T, H)

$P(X = 2) = 3/8$ (H, H, T), (H, T, H), (T, H, H)

$P(X = 3) = 1/8$ (H, H, H)

$P(X \geq 4) = 0$

Example

The probabilities in this example are all **over 8** because there are **8 possible outcomes** when tossing a fair coin 3 times.

Each toss has 2 possible outcomes: **Head (H)** or **Tail (T)**.

So for 3 tosses, the total number of possible combinations is:
$2 \times 2 \times 2 = 8$

These 8 combinations are:

H, H, H
H, H, T
H, T, H
H, T, T
T, H, H
T, H, T
T, T, H
T, T, T

Example Interpretations

1. **Simple Event:**

    - If A is the event "rolling a 4 on a fair six-sided die," then $\mathrm{P}(\mathrm{A})$ represents the probability of rolling a 4.
    - Since there is one favorable outcome (rolling a 4) out of six possible outcomes (1, 2, 3, 4, 5, 6), the probability is: $\mathrm{P}(\mathrm{A}) = \frac{1}{6} \approx 0.167$

Key idea:
*A **simple event** is when we're focusing on just one specific outcome.*

2. **Compound Event:**

   - If A is the event "rolling an even number on a fair six-sided die," then $\mathrm{P}\left(\mathrm{A}\right)$ represents the probability of rolling an even number (2, 4, or 6).
   - There are three favorable outcomes (2, 4, 6) out of six possible outcomes, so the probability is: $\mathrm{P}\left(\mathrm{A}\right) = \frac{3}{6} = \frac{1}{2} = 0.5$

3. **General Case:**

   - For any event A, the probability $\mathrm{P}\left(\mathrm{A}\right)$ is calculated as: $\mathrm{P}\left(\mathrm{A}\right) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$

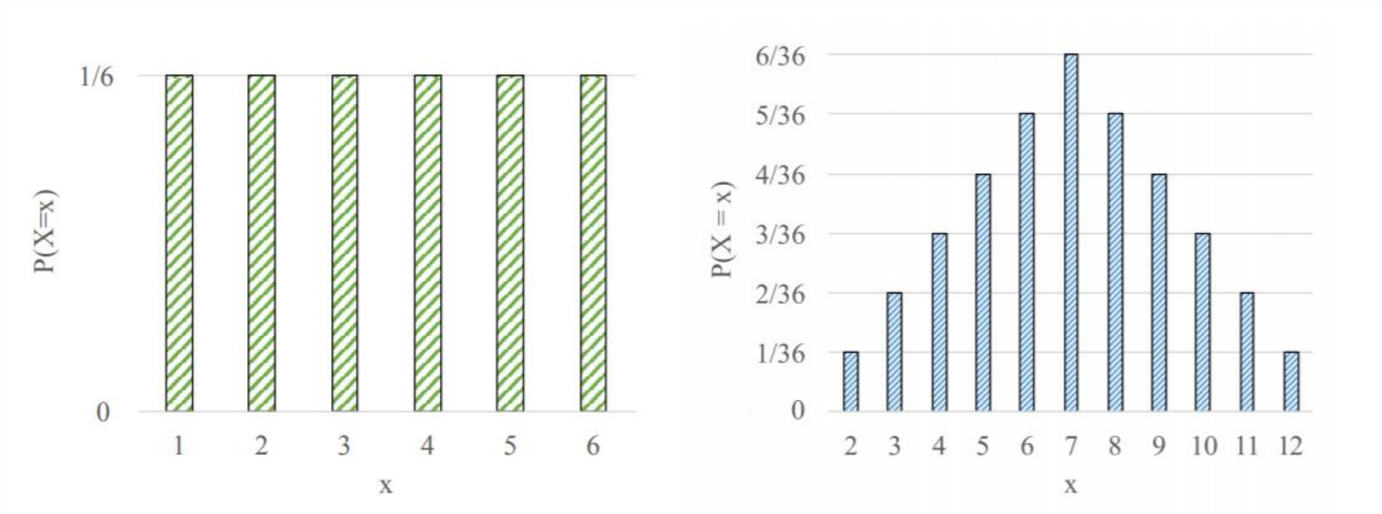
Probability Mass Function (PMF)

- PMF of a random variable is a function that maps possible outcomes of a random variable to the corresponding probabilities.

PMF = How likely each outcome is

- We can represent PMFs as a formula, a graph, or a table.

P(X = 0)	1/8
P(X = 1)	3/8
P(X = 2)	3/8
P(X = 3)	1/8
P(X >= 4)	0



Probability Mass Function (PMF)

Properties of PMF

1. **Non-negativity:**
   - For any value x, the PMF is always non-negative: $p(x) \geq 0$
2. **Normalization:**
   - The sum of the probabilities for all possible values of X must equal 1: $\sum_x p(x) = 1$
3. **Specific Values:**
   - The PMF is defined only for the specific values that the discrete random variable can take. For values not in the support of X, the PMF is zero: $p(x) = 0 \quad \text{if } x \notin \text{Support of X}$

If you're tossing a coin 3 times, you can't get 5 heads. So P(X = 5) would be 0

1/8 for 0 heads
3/8 for 1 head
3/8 for 2 heads
1/8 for 3 heads
Add them up:
1/8+3/8+3/8+1/8=1

VALID

Probability Mass Function (PMF)

Example

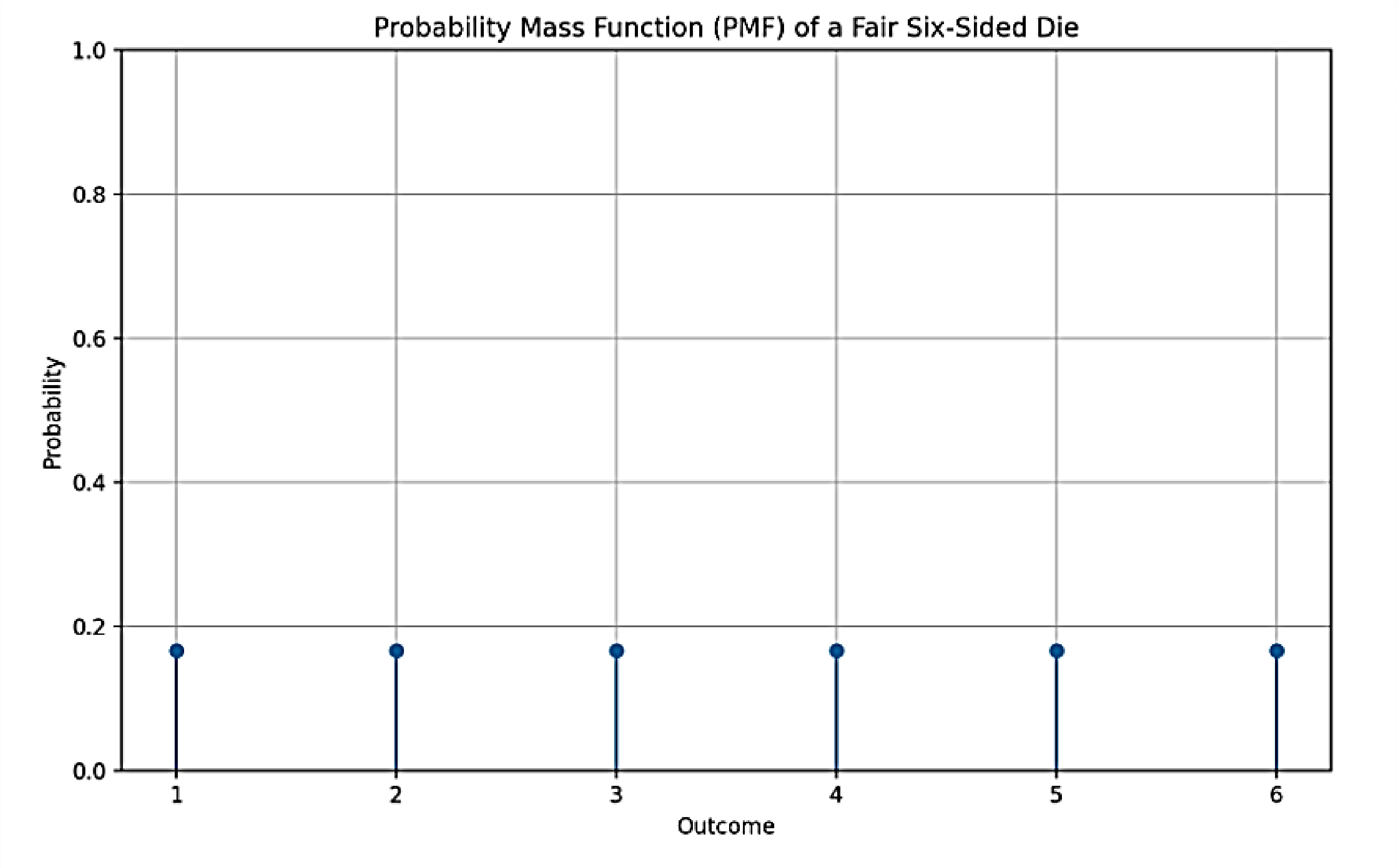
Consider a simple example of a fair six-sided die. Let X be the random variable representing the outcome of a die roll. The possible values of X are $\{1, 2, 3, 4, 5, 6\}$.

The PMF for this random variable X is:

$$p(x) = \begin{cases} \frac{1}{6} & \text{if } x \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases}$$

This means that each outcome (1 through 6) has an equal probability of $\frac{1}{6}$.

Probability Mass Function (PMF)



Assignment: **Rolling Two Dice – PMF Practice**

Instructions:
You are to simulate and analyze the **sum** of the outcomes when **rolling two fair six-sided dice**.

A. Define the Random Variable

Let **X** be the **sum of the numbers** rolled on two dice.
List all possible values of **X**.

B. List the Sample Space

List at least **five** different pairs of dice rolls that give different values of X.
(e.g., (1,1) → X = 2)

C. Fill in the PMF Table

X (Sum)	P(X = x)
2	___
3	___
4	___
5	___
6	___
7	___
8	___
9	___
10	___
11	___
12	___

D. Draw a PMF Graph

Plot your values of X on the x-axis and their probabilities on the y-axis.
Label your graph properly.