[SWE2015-41] Introduction to Data Structures (자료구조개론)

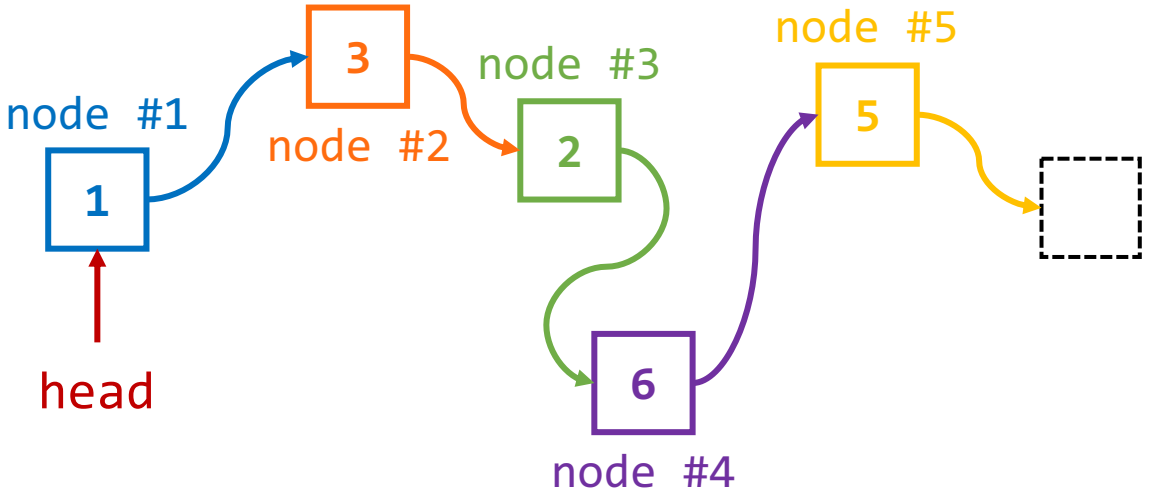
Variants of Linked Lists

Department of Computer Science and Engineering

Instructor: Hankook Lee (이한국)

(Recap) Linked Lists

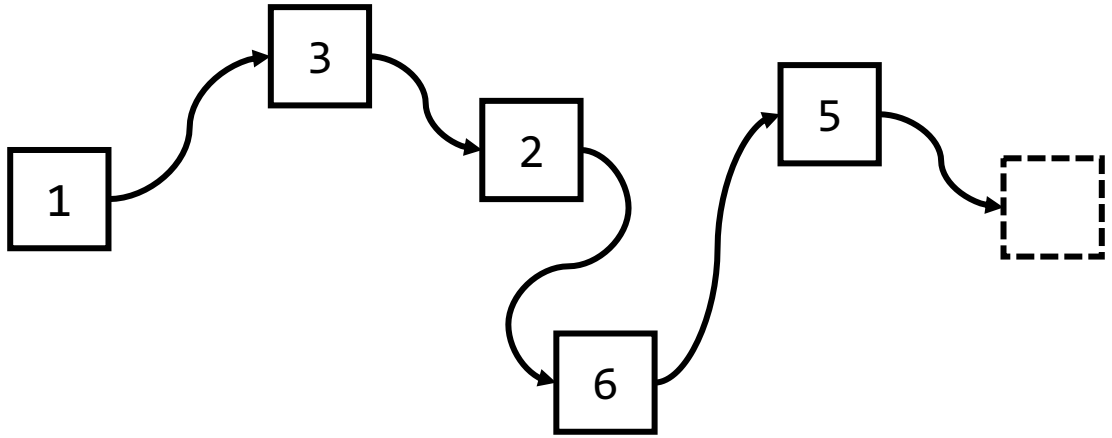
- **A linked list** is a collection of **sequentially-connected** elements
  - The elements are not required to be stored in contiguous memory




```
typedef struct _Node {
    int value;
    struct _Node *next;
} Node;
typedef struct _LinkedList {
    Node *head;
} LinkedList;
```

(Recap) Linked Lists - Insertion

- How to **insert** an item at the middle of the linked list?
  - Example: Insert “4” at the 2nd position



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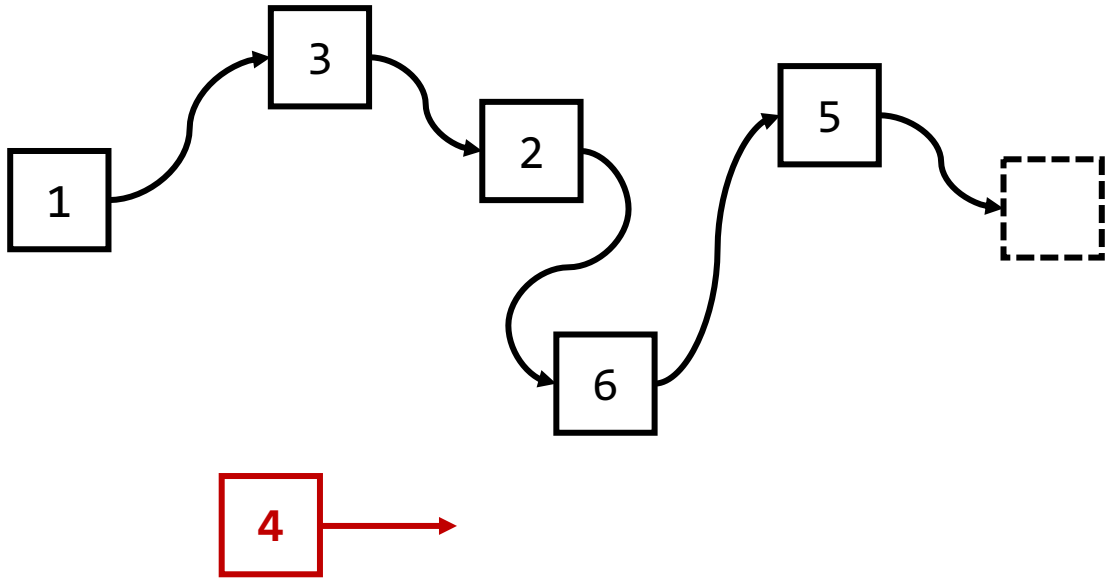

1. Create new node

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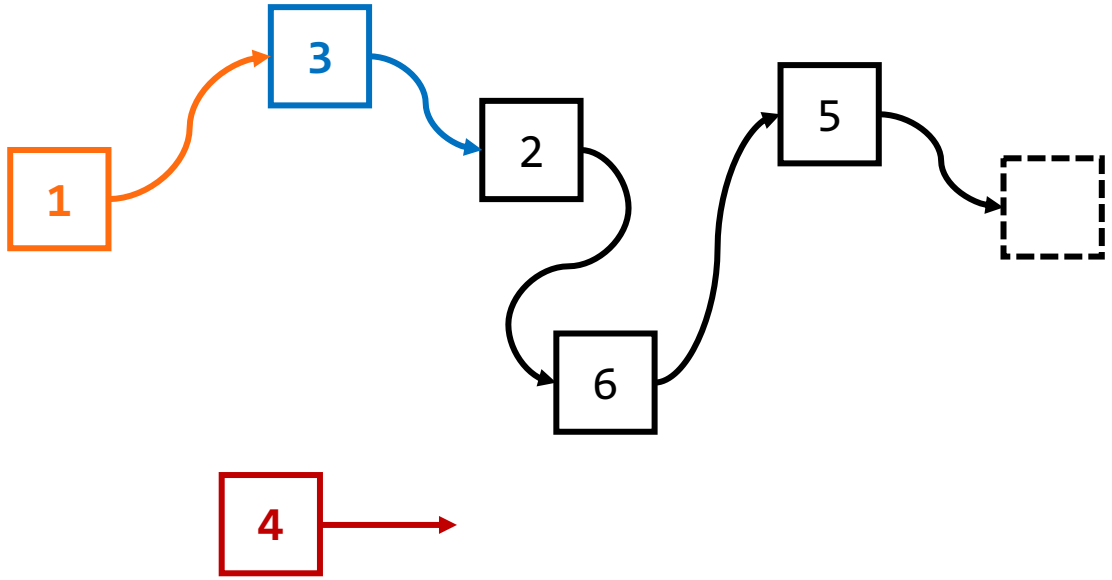

1. Create new node
2. Find the 1st node and its next node

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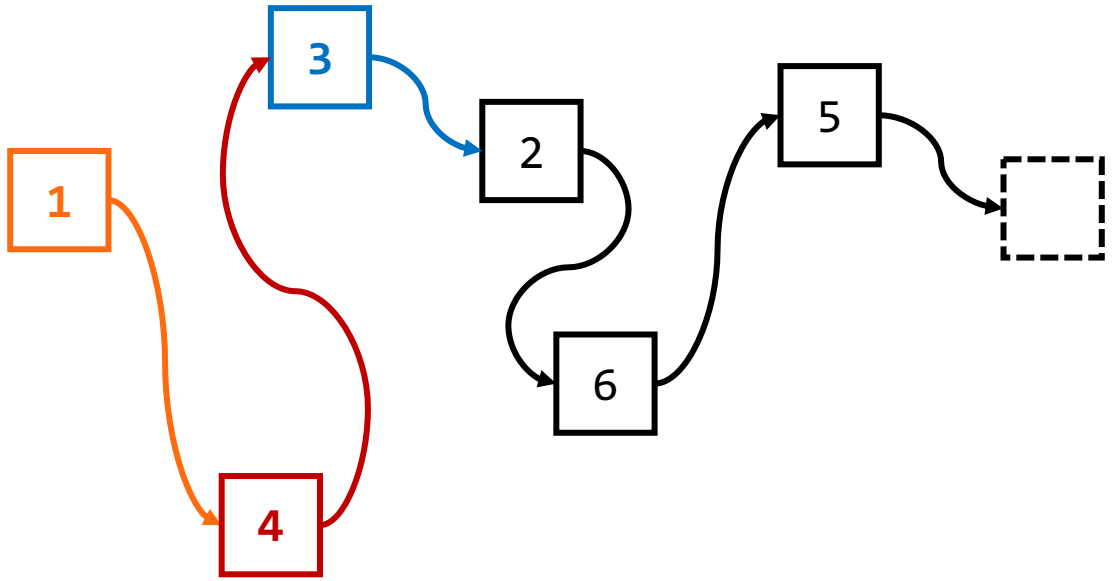
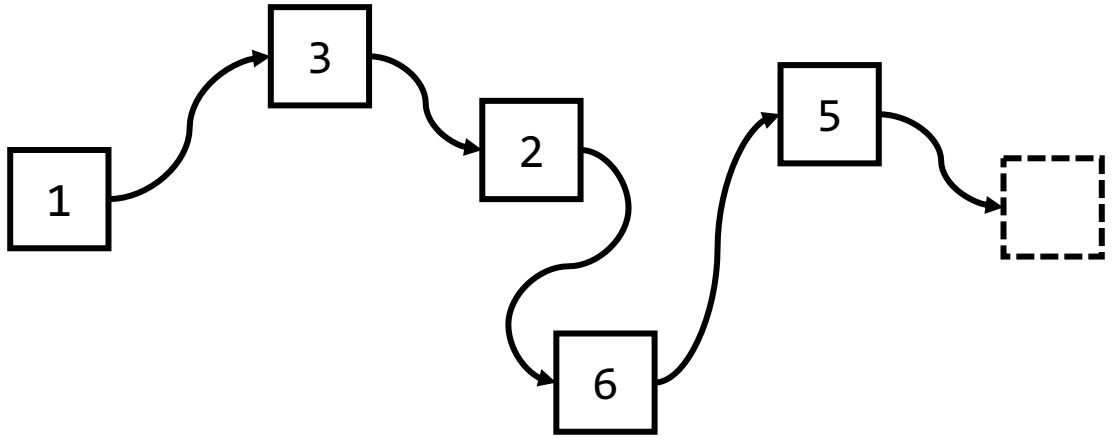


1. Create new node
2. Find the 1st node and its next node
3. Connect them: node → node → node

(Recap) Linked Lists - Deletion

- How to **delete** an item from the linked list?
  - Example: delete “2” at the 3rd position



(Recap) Linked Lists - Deletion

- How to **delete** an item from the linked list?
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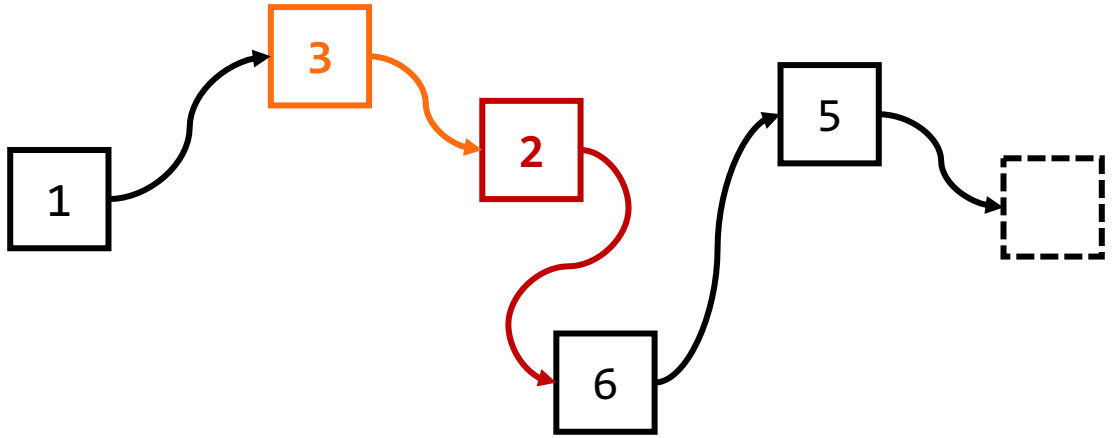
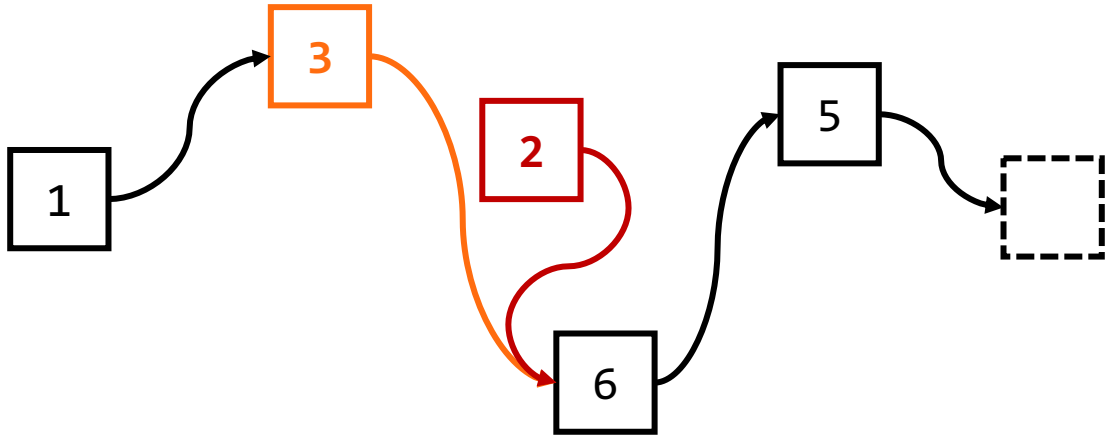


1. Find the 2nd node and its next node

(Recap) Linked Lists - Deletion

- How to **delete** an item from the linked list?
  - Example: delete "2" at the 3rd position



1. Find the 2nd node and its next node
2. Skip the connection between node and node

(Recap) Linked Lists - Deletion

- How to **delete** an item from the linked list?
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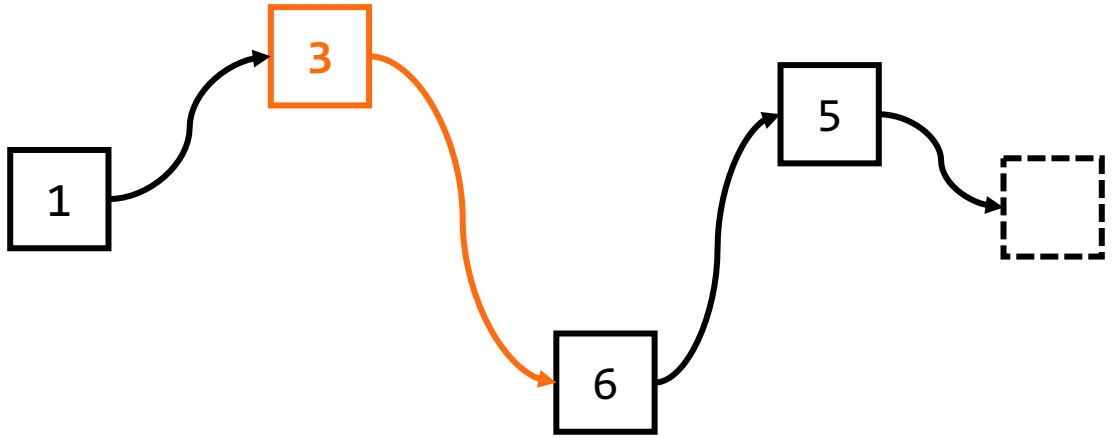
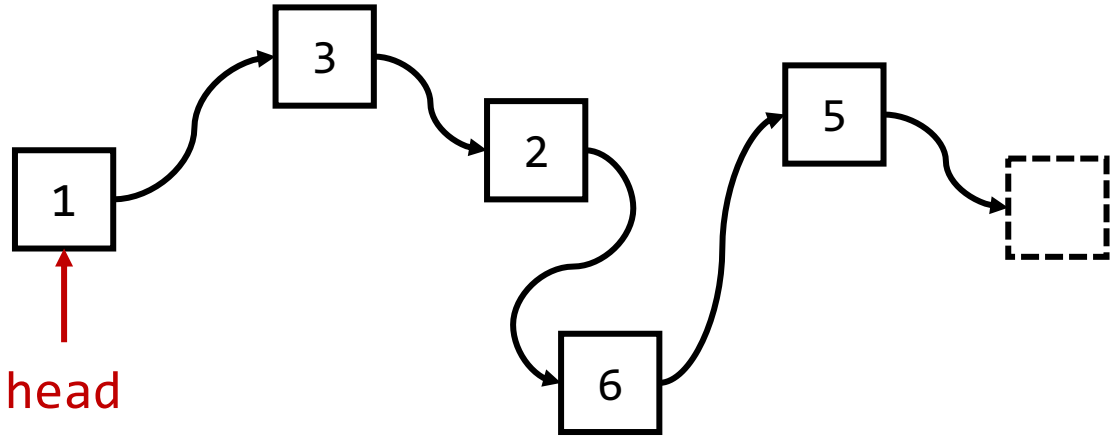


1. Find the 2nd node and its next node
2. Skip the connection between node and node
3. Free the node's memory

Variants of Linked Lists

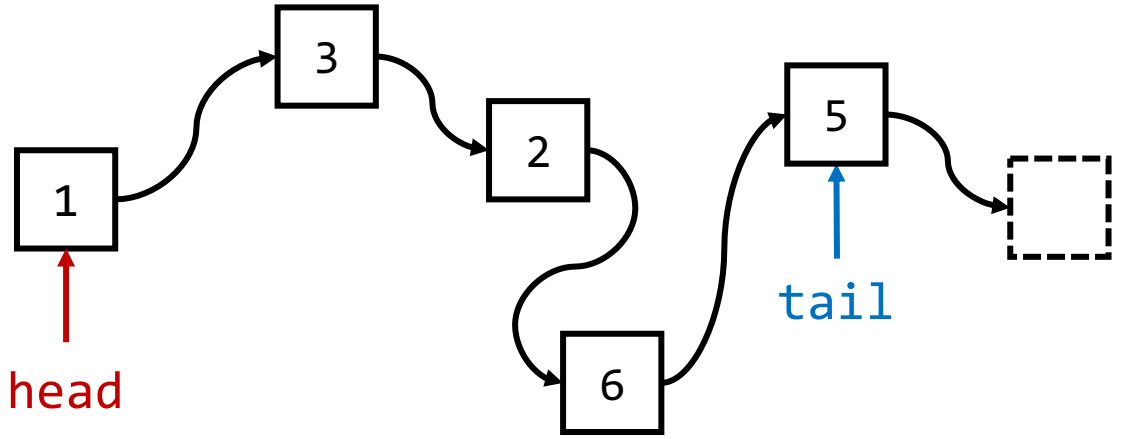
- Singly Linked Lists
  - The simplest linked list structure
  - One-directional → cannot go backward
  - $O(n)$ for insertion at the end



Variants of Linked Lists

- Singly Linked Lists with the tail pointer
  - The simplest linked list structure
  - One-directional → cannot go backward
  - $O(1)$ for insertion at the end

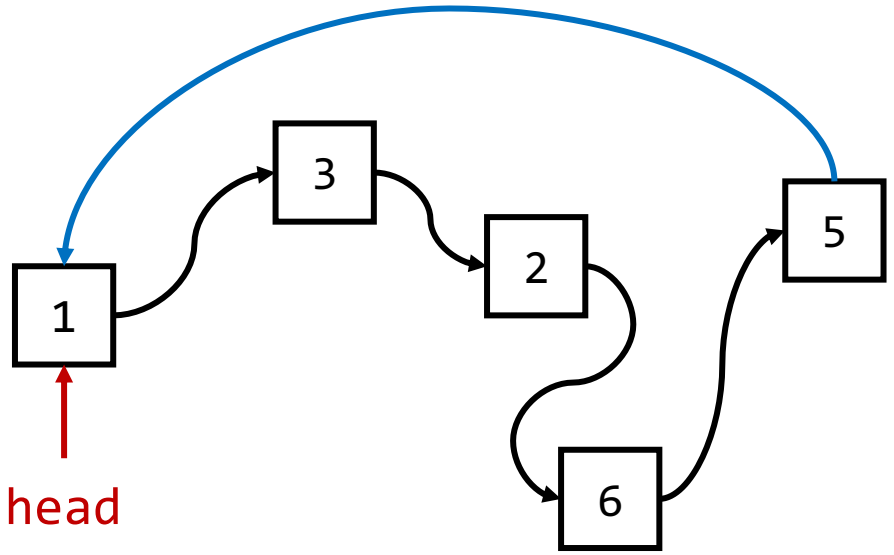
```
typedef struct _LinkedList {
    Node *head, *tail;
} LinkedList;
```



Variants of Linked Lists

- Circular Linked Lists
  - Every node has non-`NULL` `next` pointer → makes handling corner cases easier

```
typedef struct _Node { int value; struct _Node *next; } Node;
typedef struct _LinkedList { Node *head; } LinkedList;
```

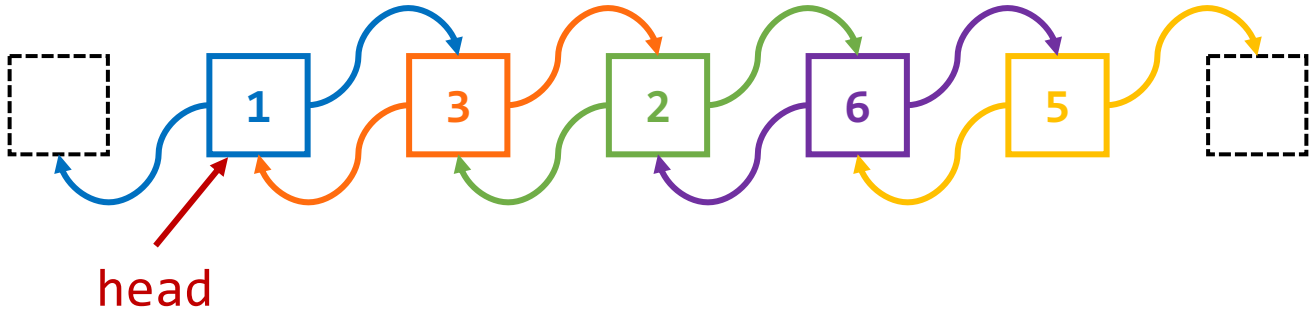


Variants of Linked Lists

- Doubly Linked Lists
  - Every node has `prev` and `next` pointers to point previous and next nodes

```
typedef struct _Node {
    int value;
    struct _Node *prev, *next;
} Node;
```

  - Two-directional → can move forward and backward



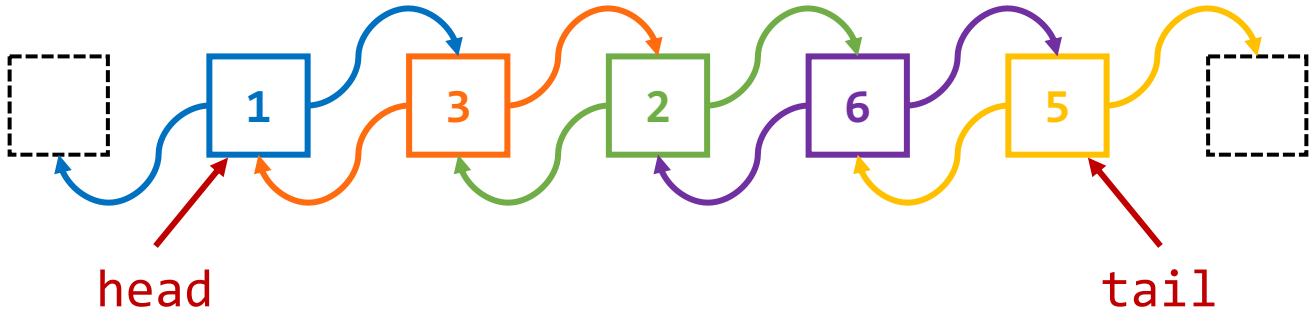
Doubly Linked Lists - Implementation

- Let's implement a doubly linked list with the tail pointer

```
typedef struct _Node {
    int value;
    struct _Node *prev, *next;
} Node;

typedef struct _LinkedList {
    Node *head, *tail;
} LinkedList;

void insert(LinkedList *list, int item, int index);
void delete(LinkedList *list, int index);
```



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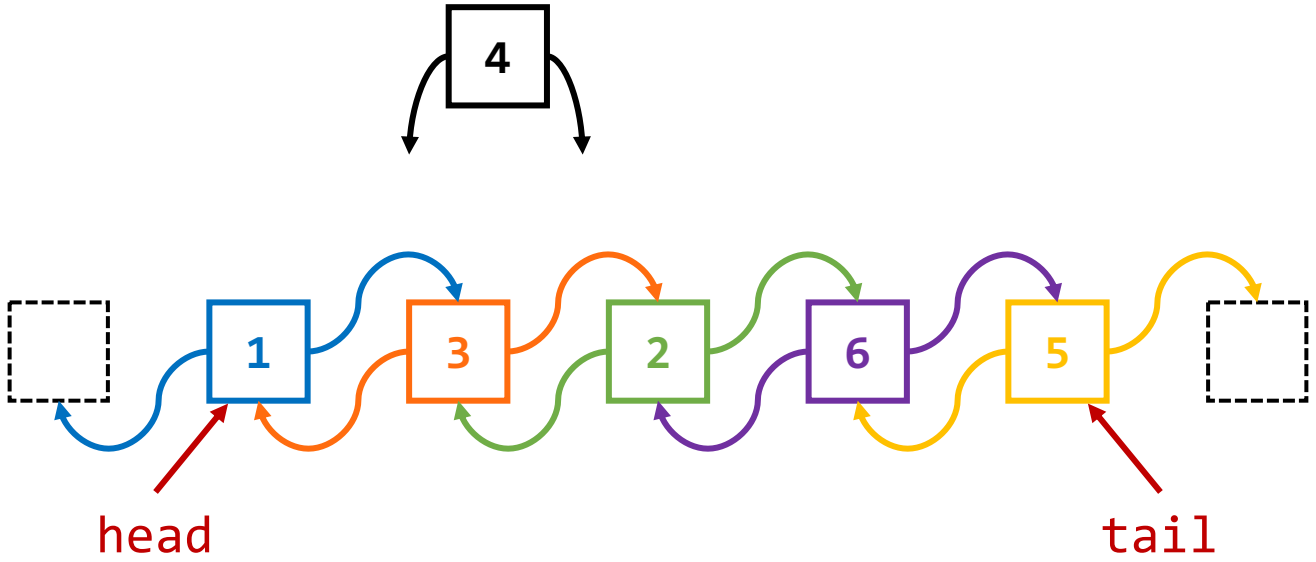
  - How to insert an element **4** between two nodes **1** and **3**?



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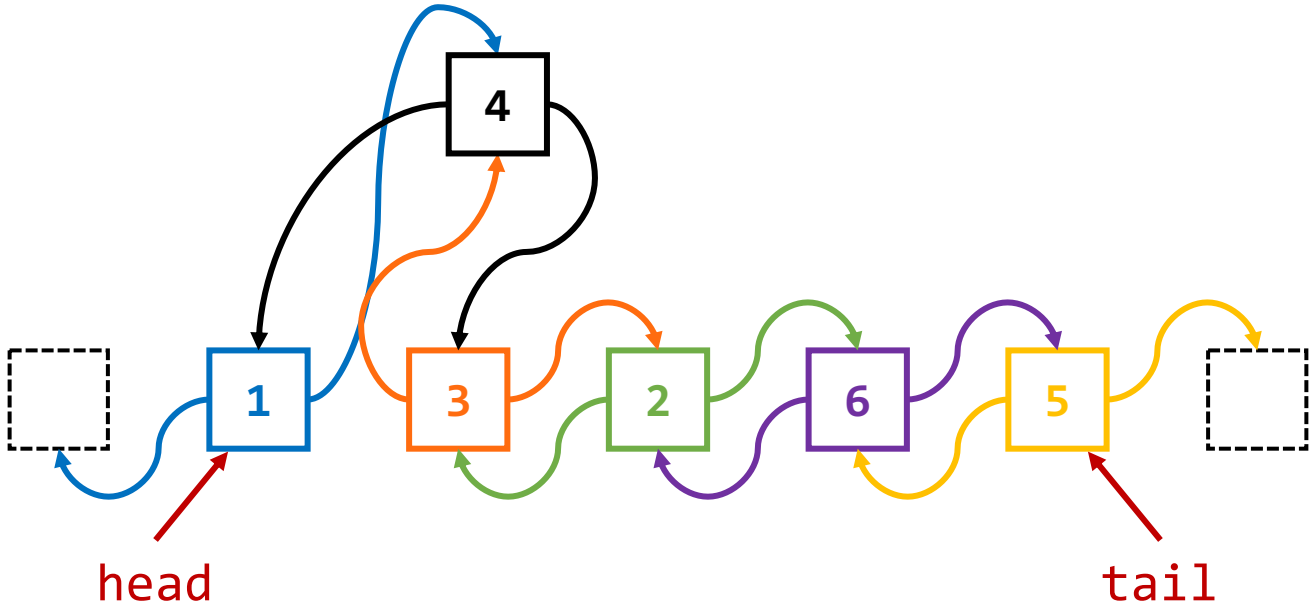
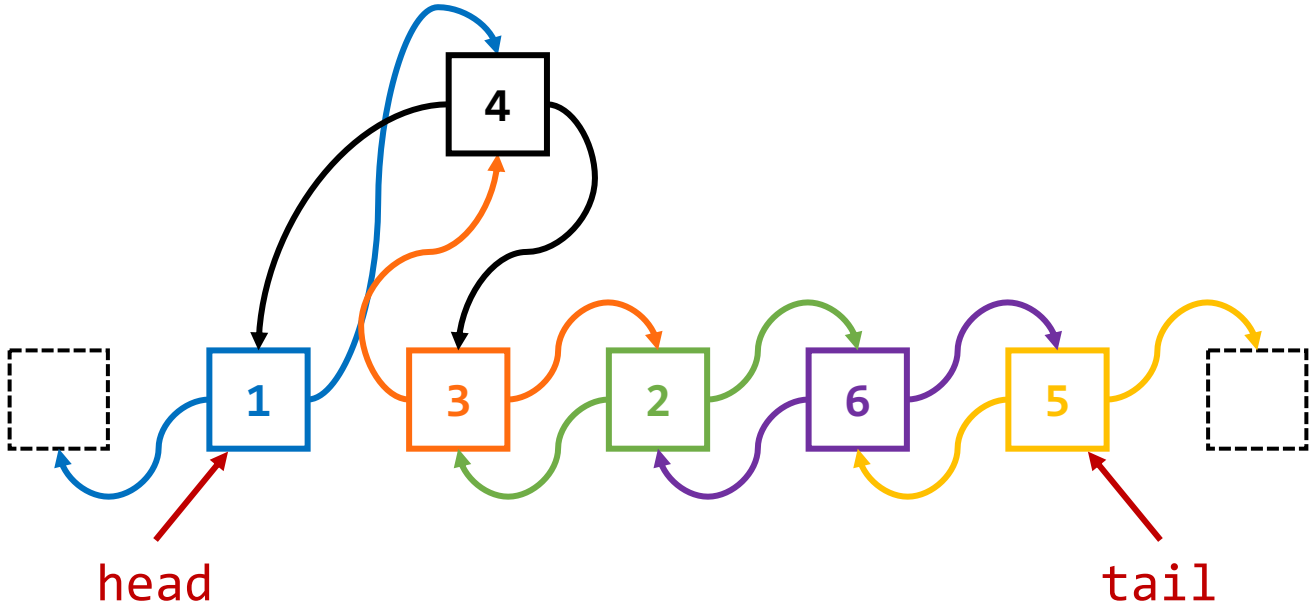


Doubly Linked Lists - Implementation

- Let's implement a doubly linked list with the tail pointer
  - How to insert an element **4** between two nodes **1** and **3**?
  - How to delete the element **2**?



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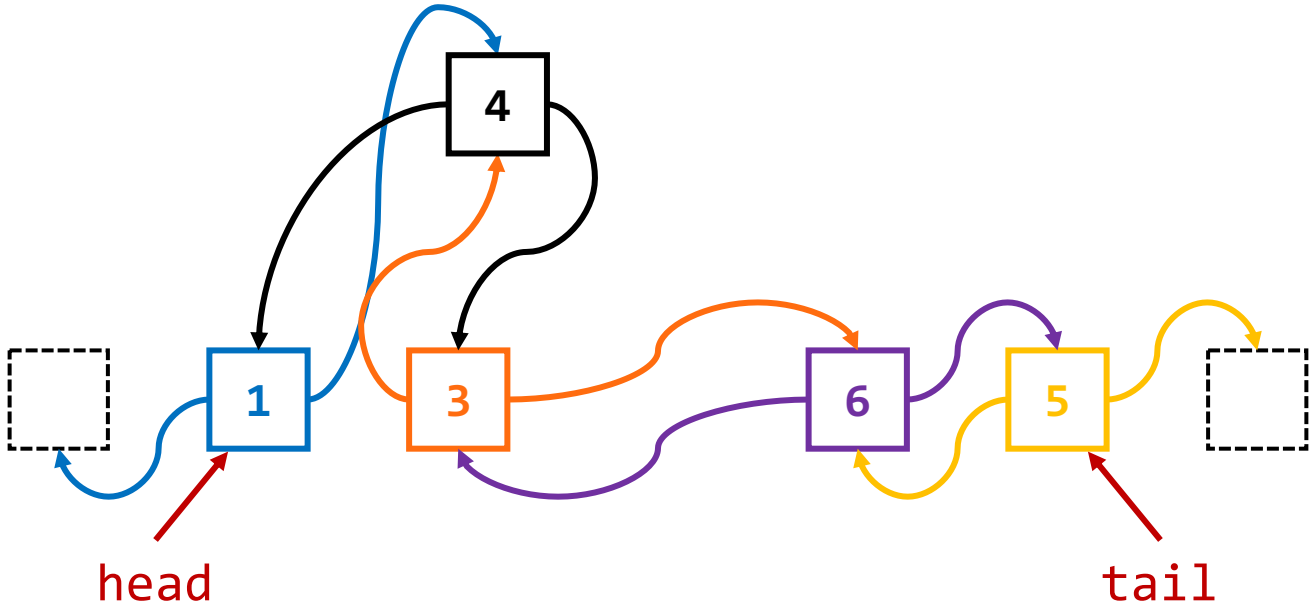
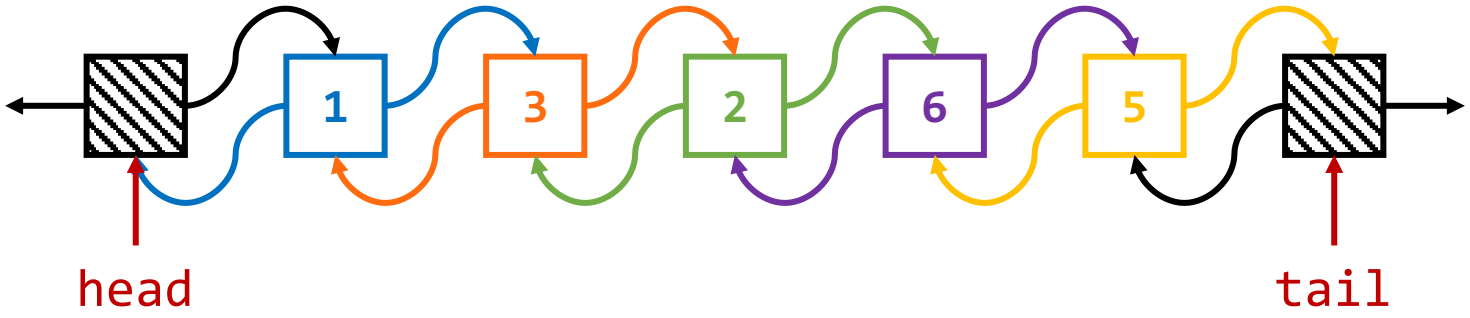
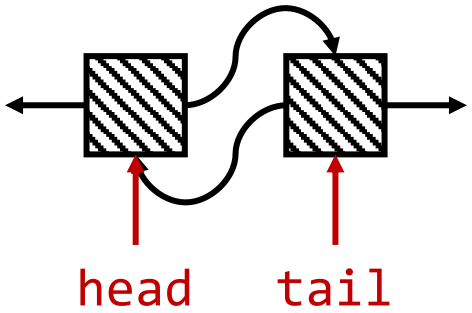


Doubly Linked Lists - Implementation

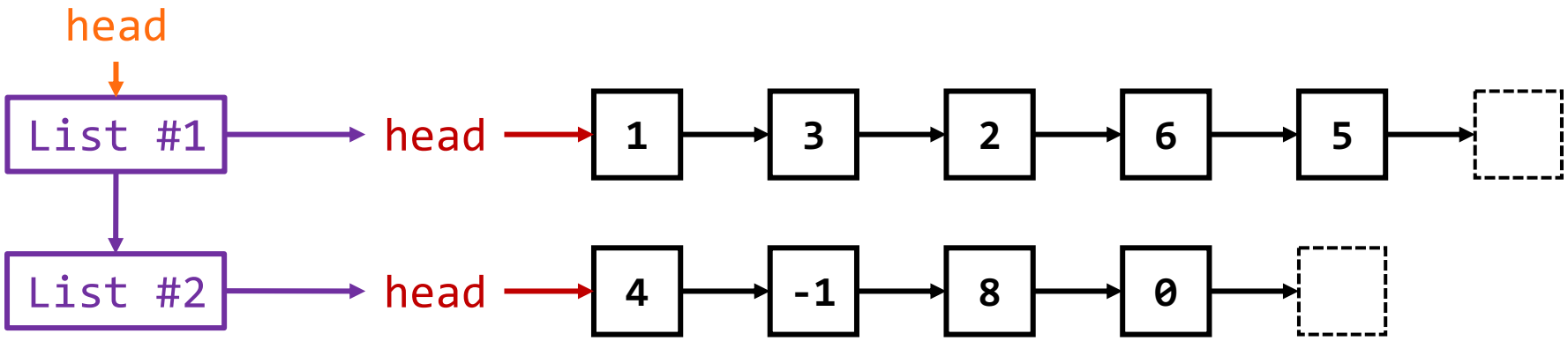
- A technique for easy implementation: **Dummy nodes**
  - This guarantees that `head` and `tail` are always non-`NULL` even if the list is empty
  - This guarantees that intermediate nodes have non-`NULL` `prev` and `next` pointers





Variants of Linked Lists

- Two-dimensional Linked Lists (like 2D arrays)

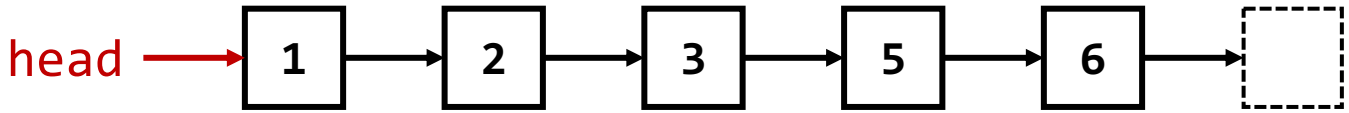



```
typedef struct _Node { int value; struct _Node *next; } Node;
typedef struct _LinkedList {  // This represents a "list" node
    Node *head;
    struct _LinkedList *next; // This connects two linked lists
} LinkedList;

typedef struct _LinkedList2D {
    LinkedList *head;
} LinkedList2D;
```

Variants of Linked Lists

- Sorted Linked Lists
  - A linked list whose elements should be always sorted



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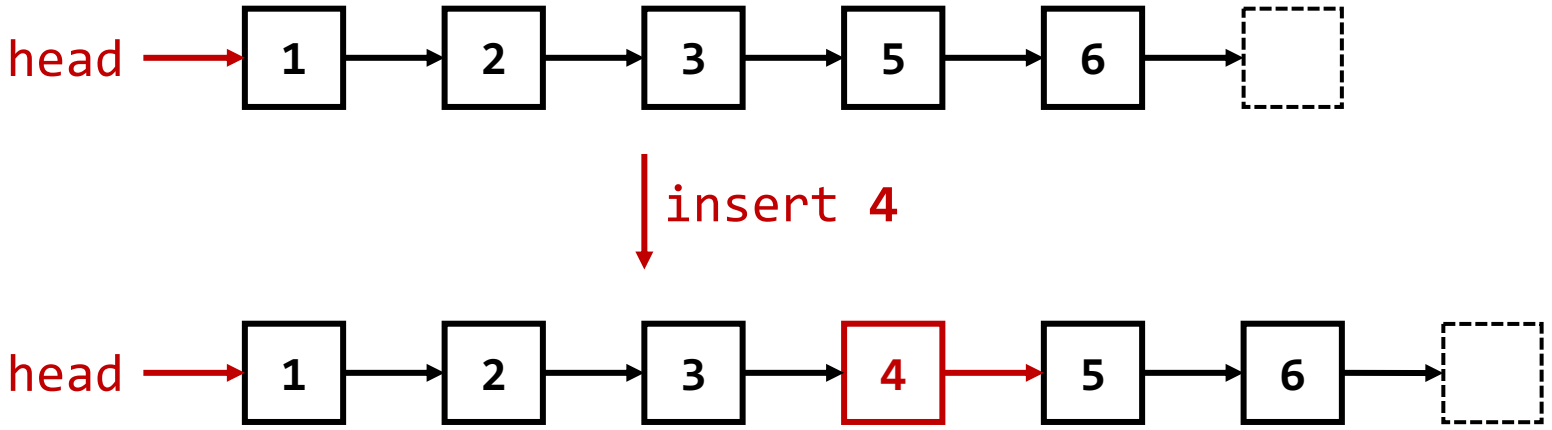

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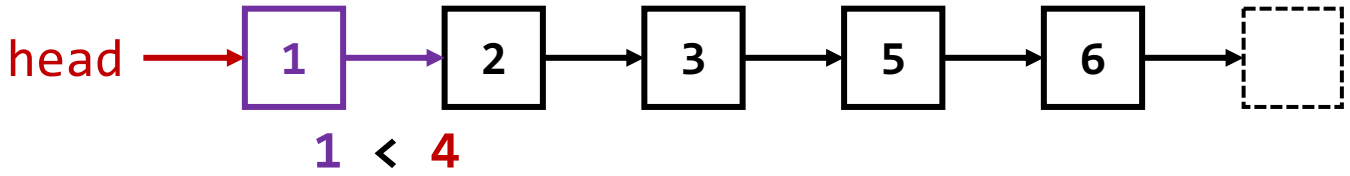

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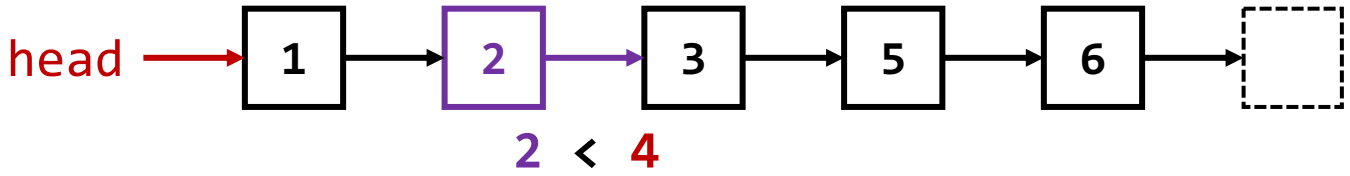

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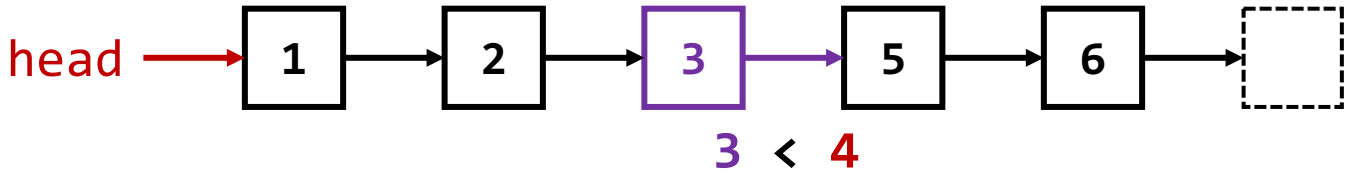

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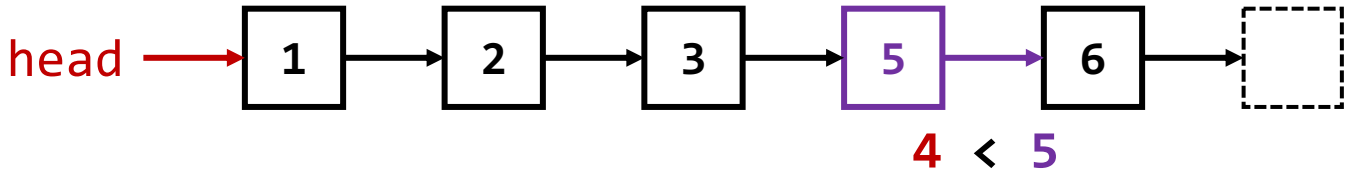

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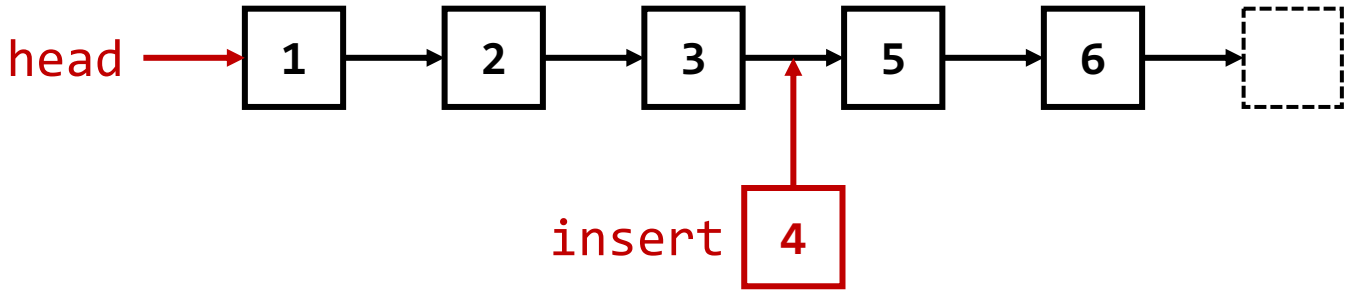

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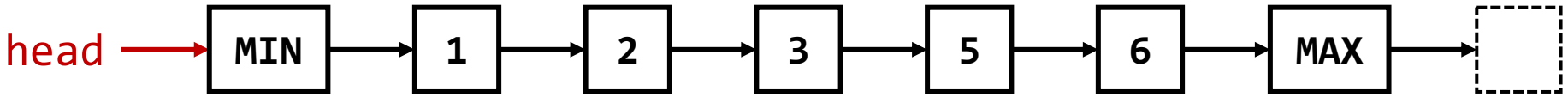

  - For easy implementation, one can consider dummy nodes, `MIN` and `MAX`

```
typedef struct _Node { int value; struct _Node *next; } Node;
typedef struct _LinkedList { Node *head; } LinkedList;

void search(LinkedList *list, int value);
void insert(LinkedList *list, int value);
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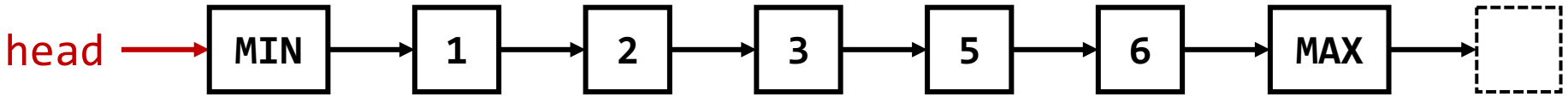

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```
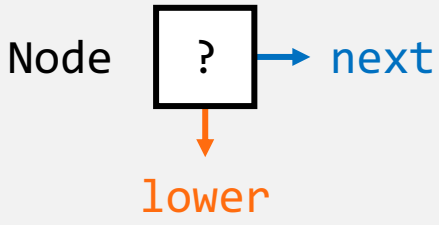

  - The search, insertion, deletion operations require $O(n)$ time complexity

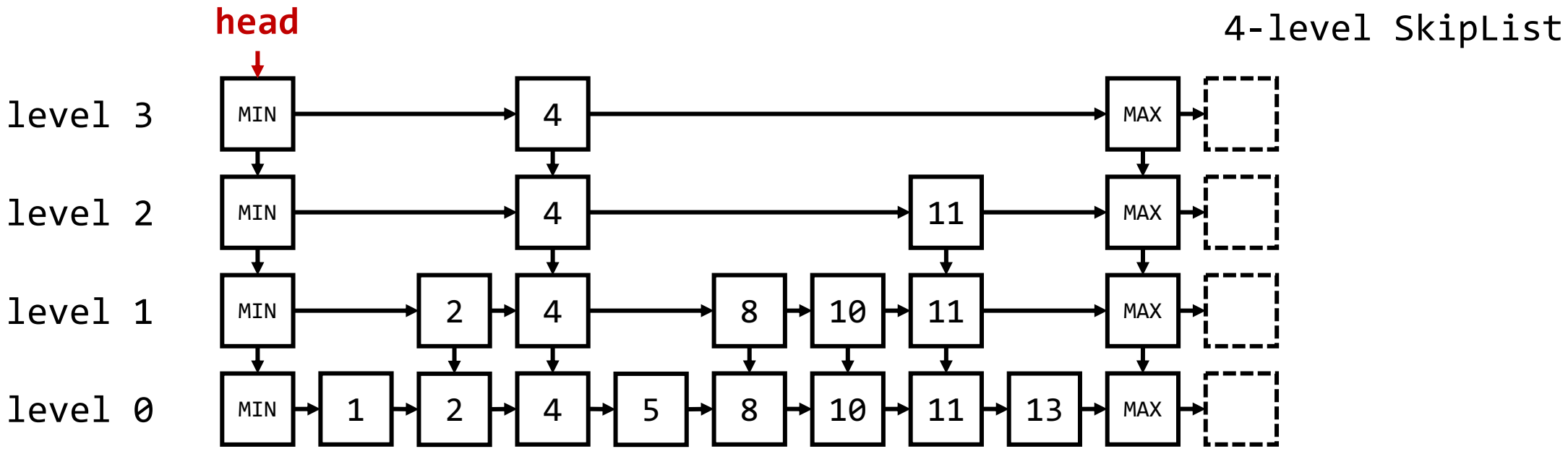
Variants of Linked Lists - Skip List

- A **Skip List** is an advanced variant of **ordered/sorted** linked lists
  - This can be implemented by a two-dimensional linked list

```
typedef struct _Node {
    int value;
    struct _Node *next, *lower;
} Node;
```

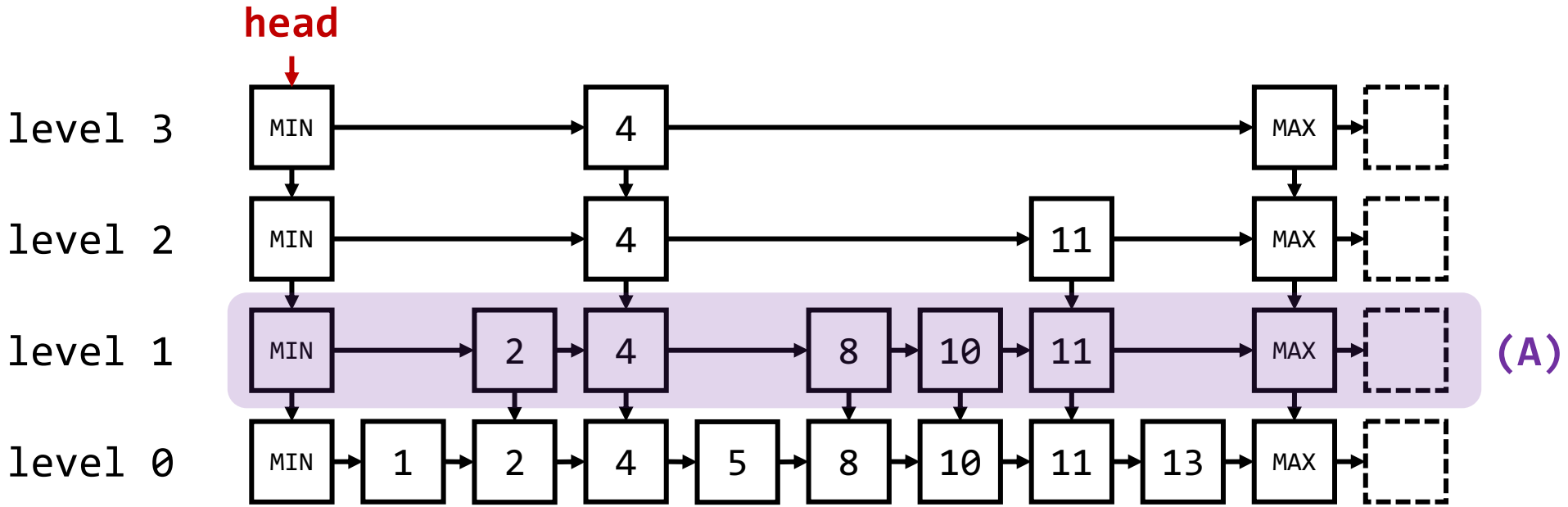



```
typedef struct _SkipList {
    Node *head;
    int num_levels;
} SkipList;
```



Variants of Linked Lists - Skip List

- A **Skip List** is an advanced variant of **ordered/sorted** linked lists
  - This can be implemented by a two-dimensional linked list
  - Two properties:
    (A) Elements at each level (i.e., row) are sorted



Variants of Linked Lists - Skip List

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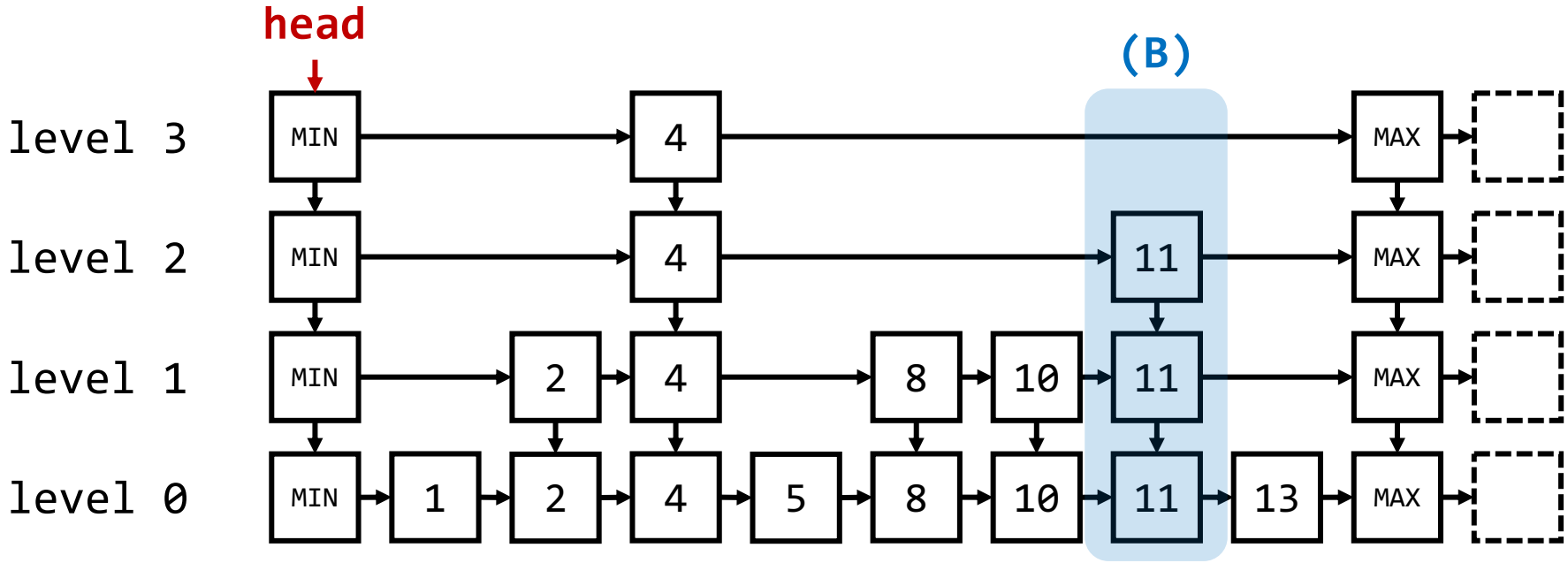
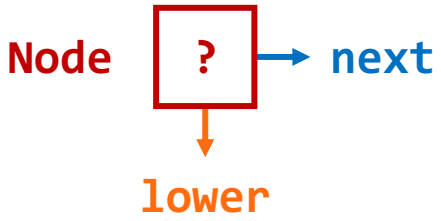
  - This can be implemented by a two-dimensional linked list
  - Two properties:
    - **(A)** Elements at each level (i.e., row) are sorted
    - **(B)** Elements at each column exist consecutively

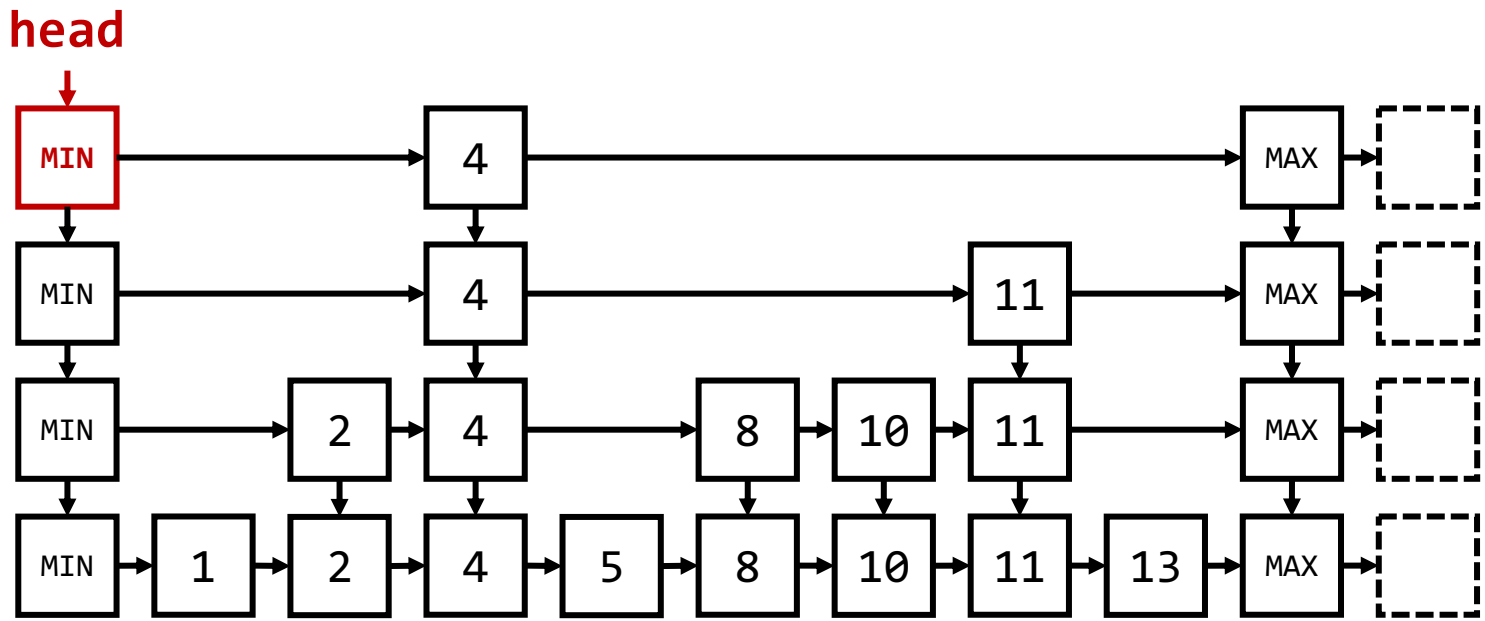


Variants of Linked Lists - Skip List

- The search procedure in **Skip Lists**
  1. Starting from **head**,

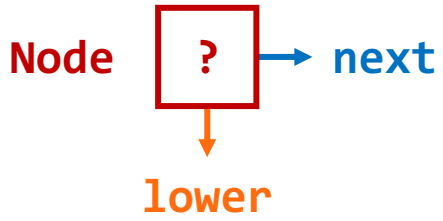


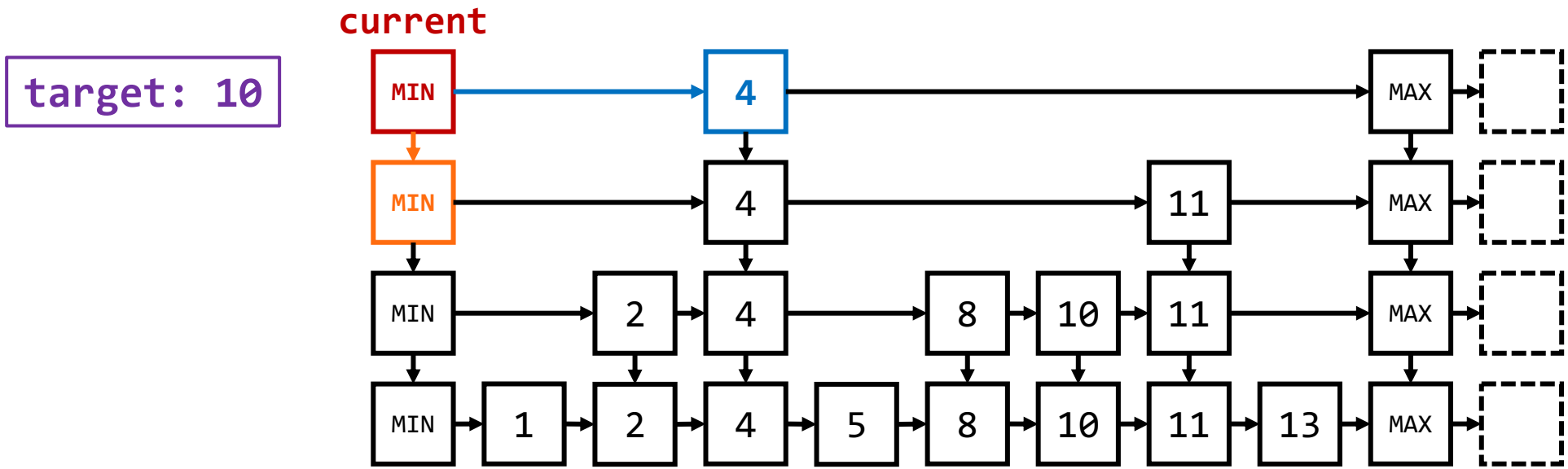
target: 10



Variants of Linked Lists - Skip List

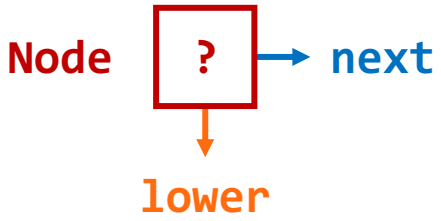
- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node

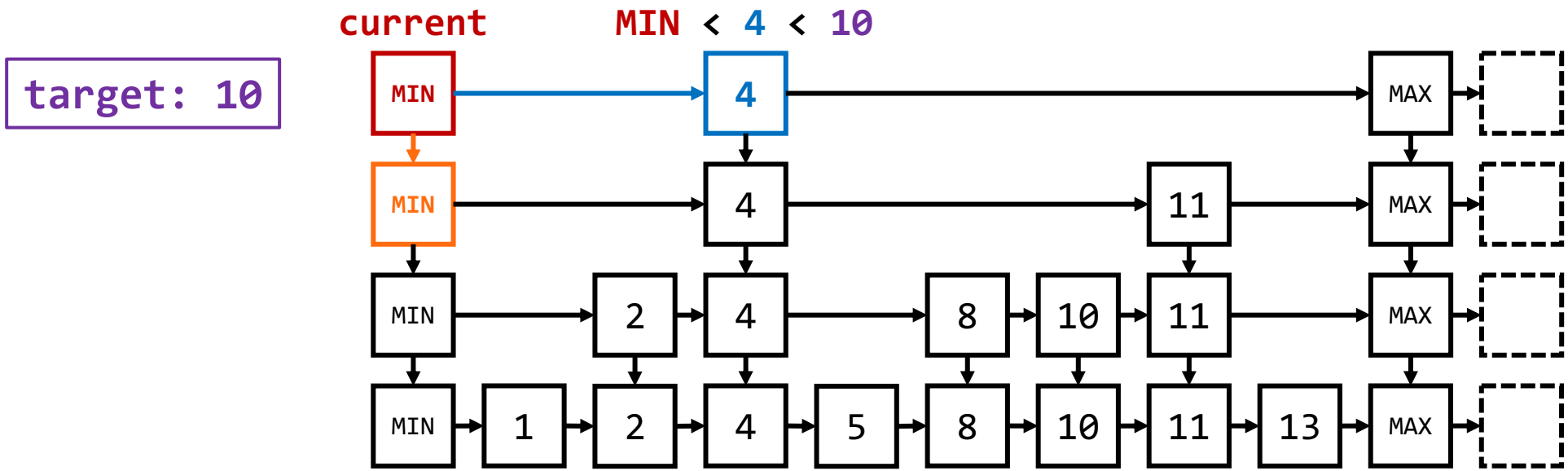




Variants of Linked Lists - Skip List

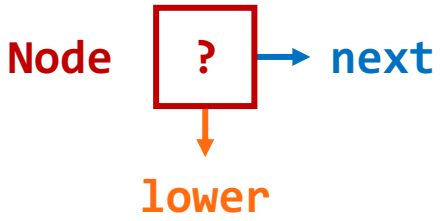
- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node
     1. If **current** < **next** <= **target**, then move to the **next** node

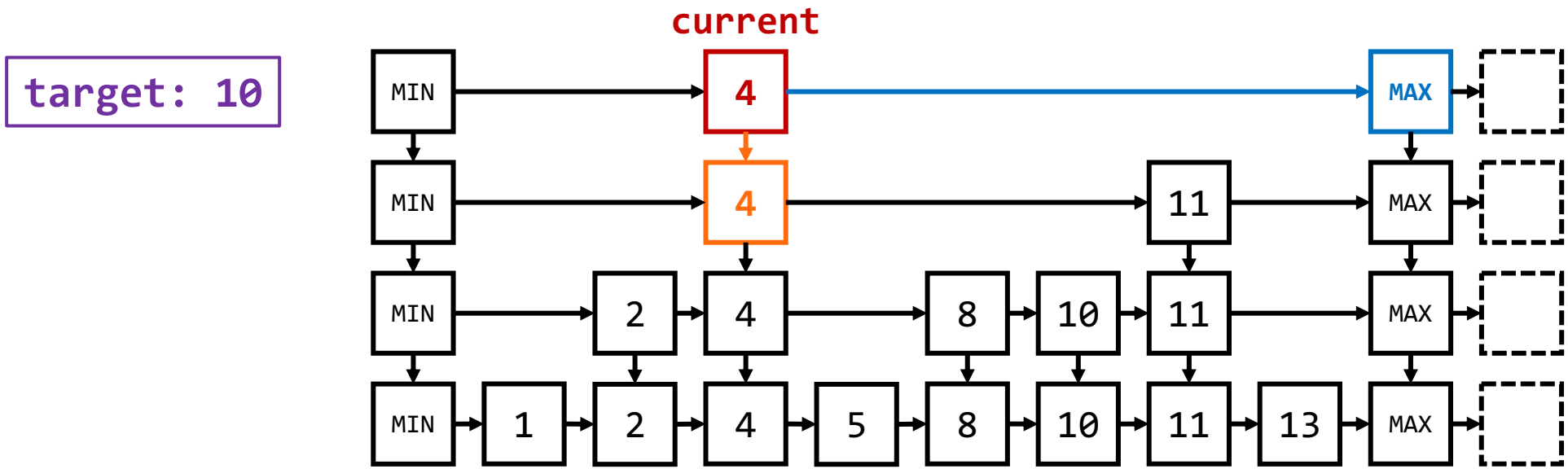




Variants of Linked Lists - Skip List

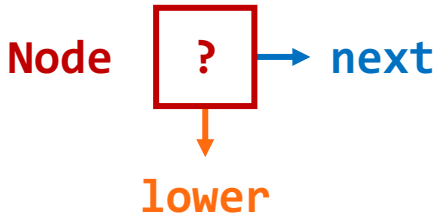
- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node
     1. If **current** < **next** <= **target**, then move to the **next** node

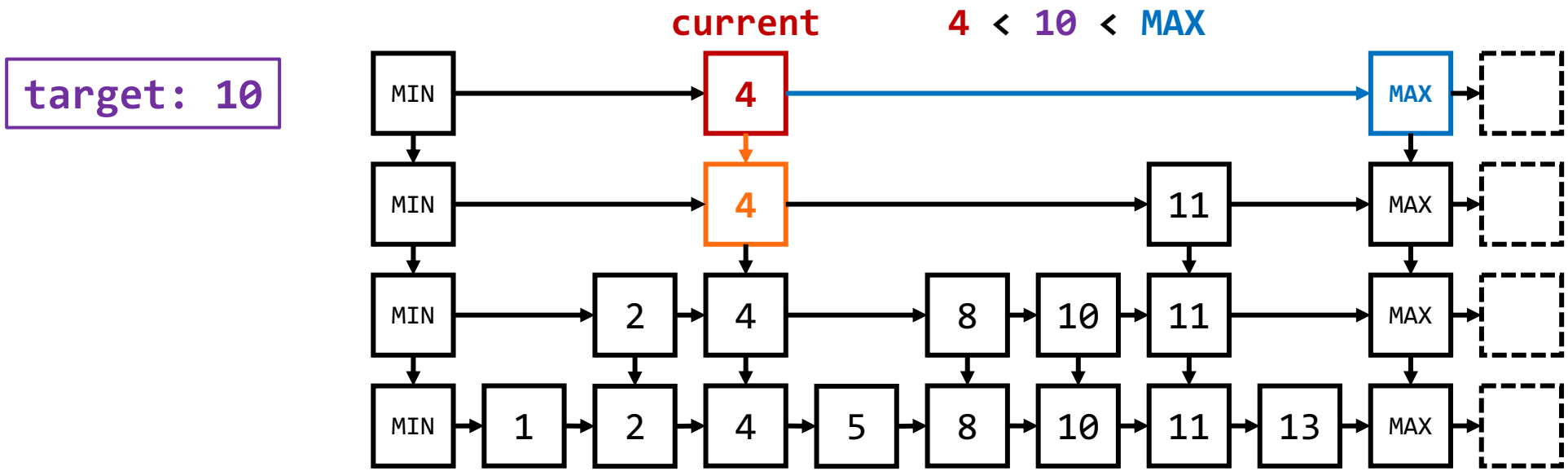




Variants of Linked Lists - Skip List

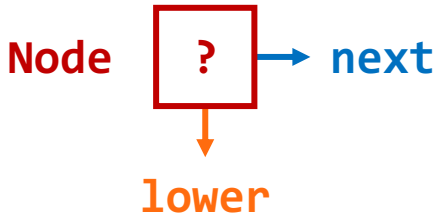
- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node
     1. If **current** < **next** <= **target**, then move to the **next** node
     2. If **current** <= **target** < **next**, then move to the **lower** node



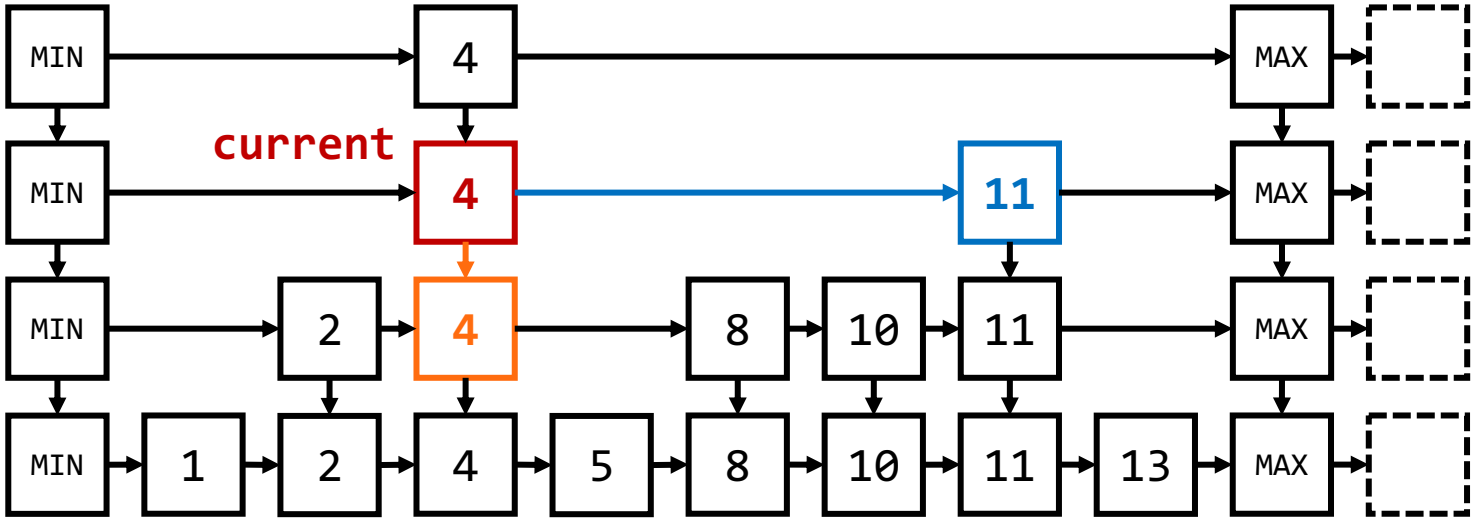


Variants of Linked Lists - Skip List

- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node
     1. If **current** < **next** <= **target**, then move to the **next** node
     2. If **current** <= **target** < **next**, then move to the **lower** node

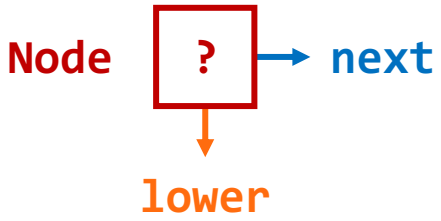


target: 10

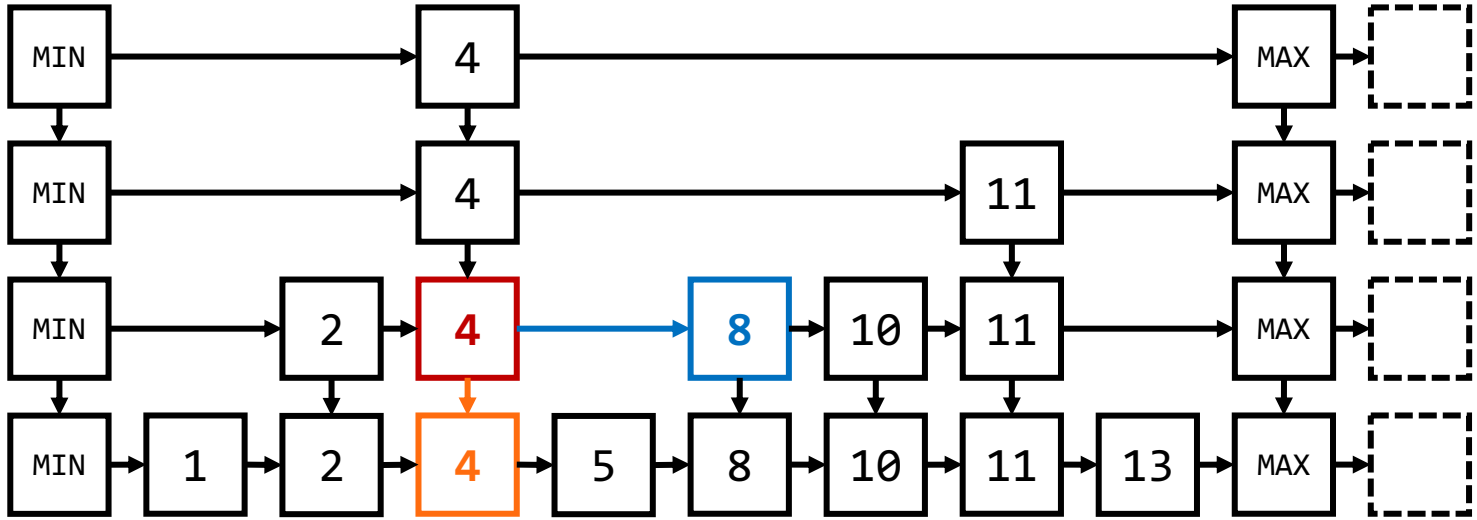


Variants of Linked Lists - Skip List

- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node
     1. If **current** < **next** <= **target**, then move to the **next** node
     2. If **current** <= **target** < **next**, then move to the **lower** node

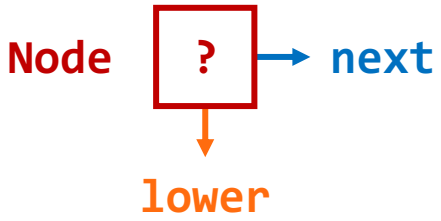


target: 10

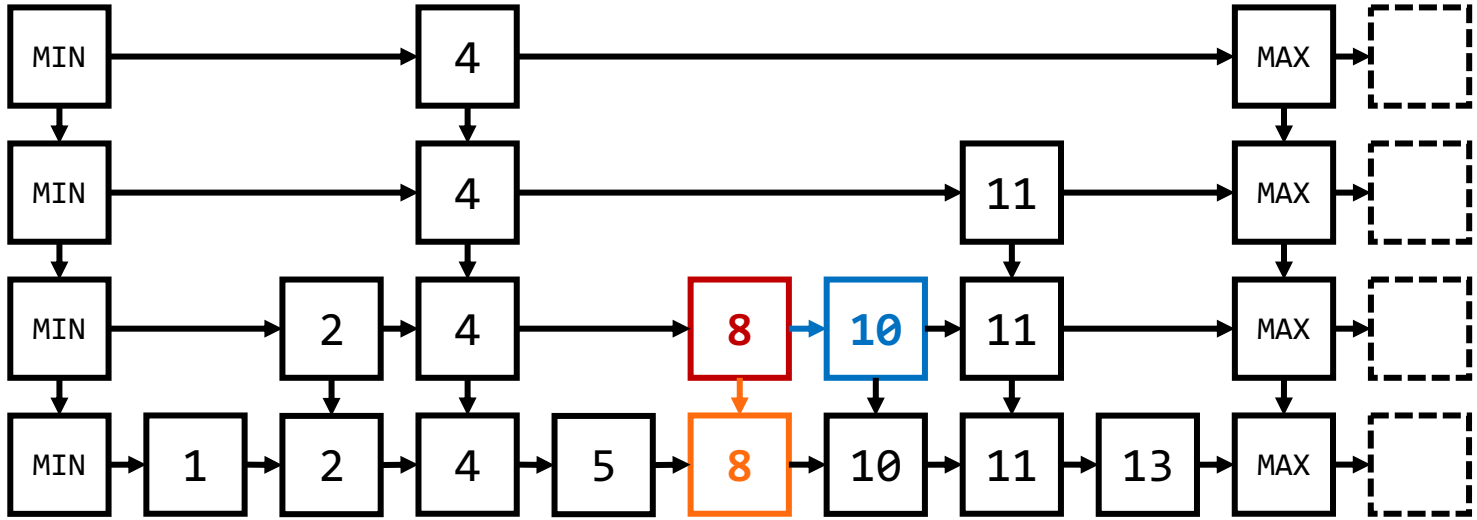


Variants of Linked Lists - Skip List

- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node
     1. If **current** < **next** <= **target**, then move to the **next** node
     2. If **current** <= **target** < **next**, then move to the **lower** node

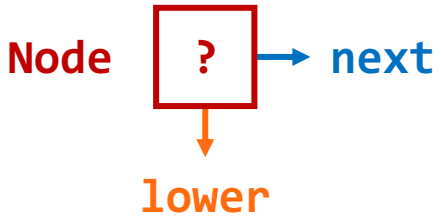


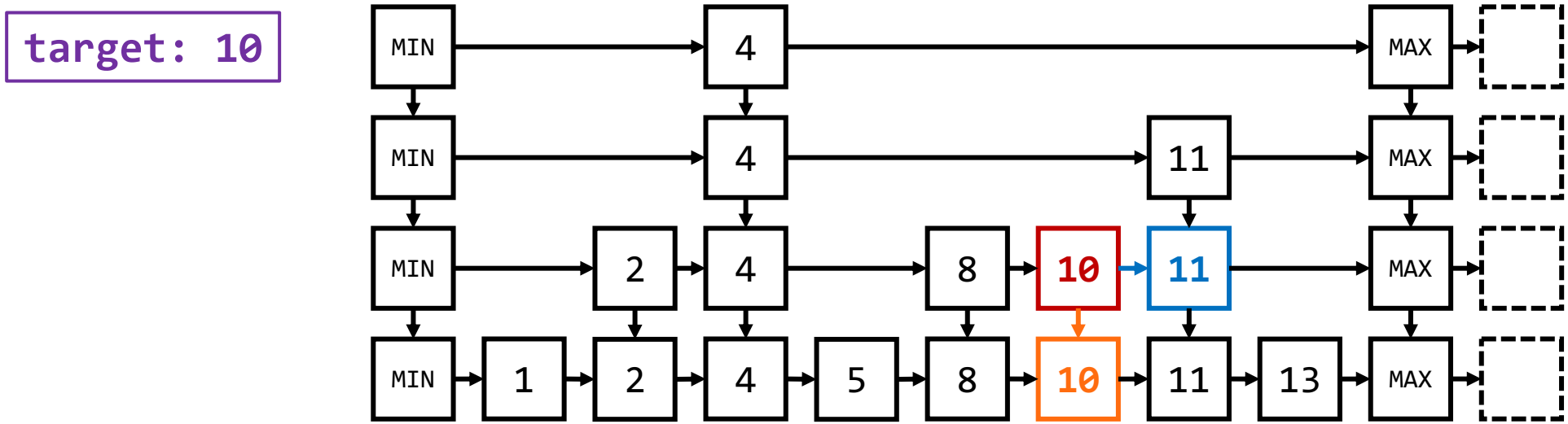
target: 10



Variants of Linked Lists - Skip List

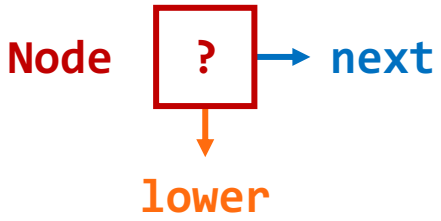
- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node
     1. If **current** < **next** <= **target**, then move to the **next** node
     2. If **current** <= **target** < **next**, then move to the **lower** node



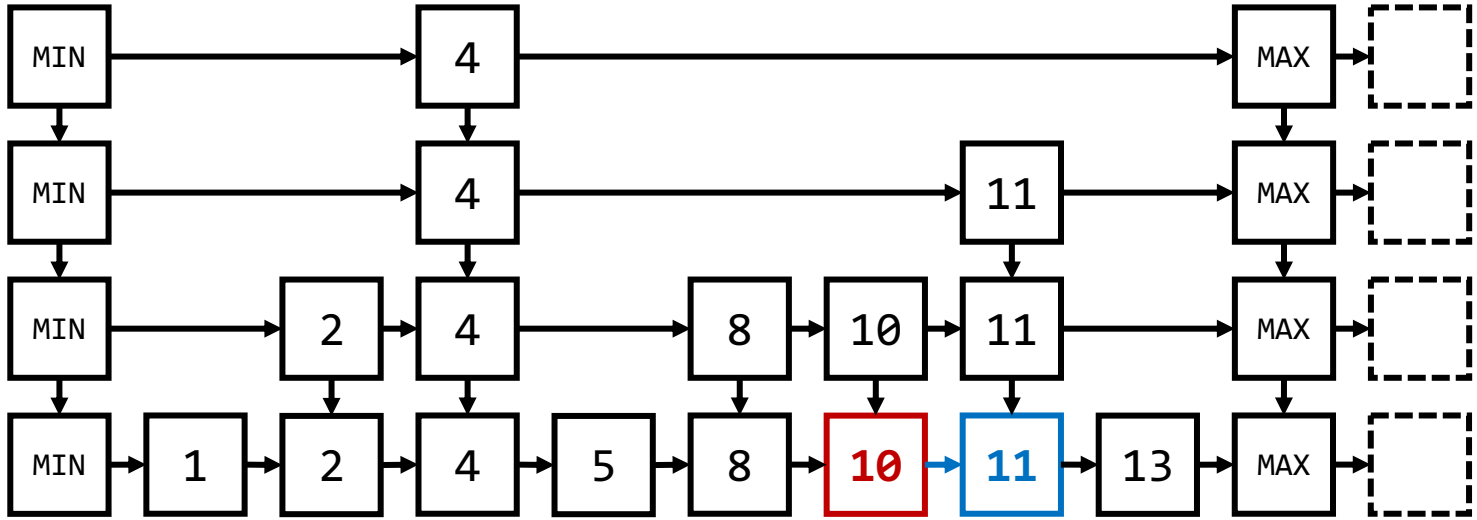


Variants of Linked Lists - Skip List

- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node
     1. If **current** < **next** <= **target**, then move to the **next** node
     2. If **current** <= **target** < **next**, then move to the **lower** node

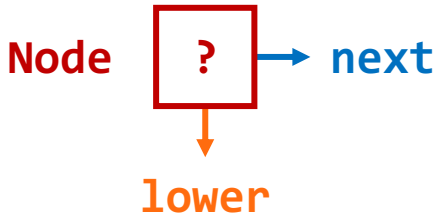


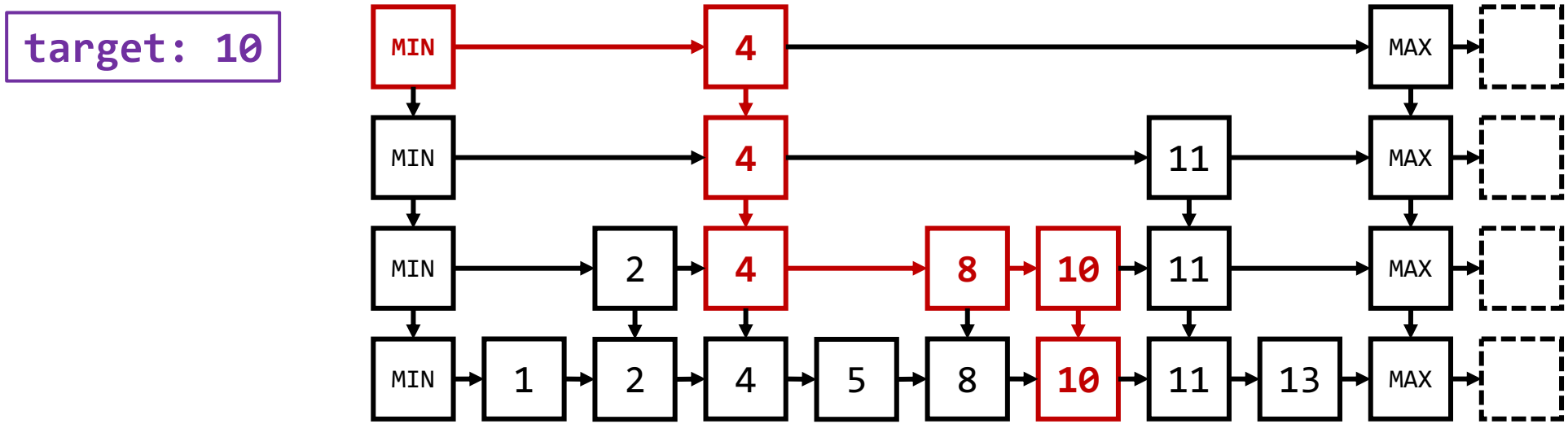
target: 10



Variants of Linked Lists - Skip List

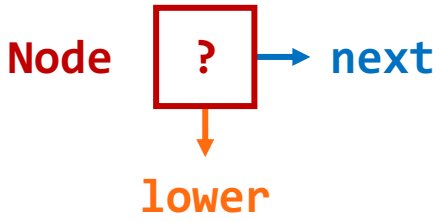
- The search procedure in **Skip Lists**
  1. Starting from **head**,
  2. Compare the **target** value with the **next** node
     1. If **current** < **next** <= **target**, then move to the **next** node
     2. If **current** <= **target** < **next**, then move to the **lower** node

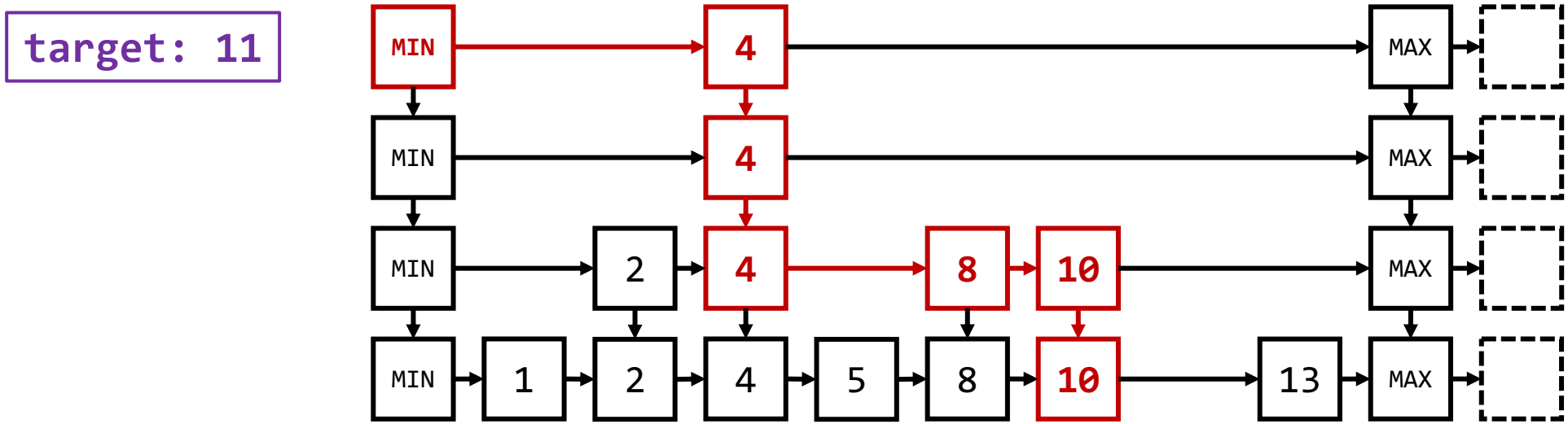




Variants of Linked Lists - Skip List

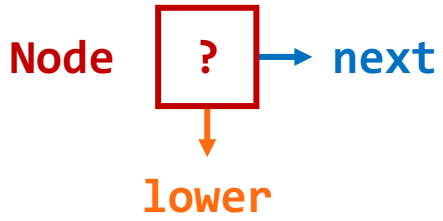
- The **insertion** procedure in **Skip Lists**
  1. For all levels, find rightmost **nodes** < **target**



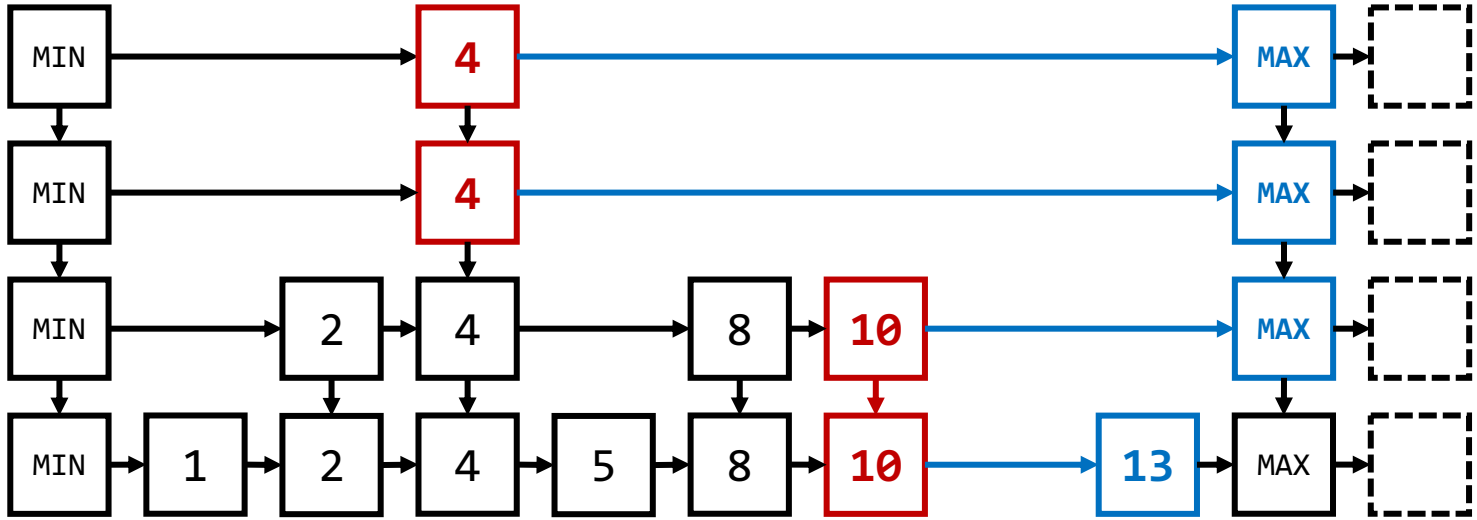


Variants of Linked Lists - Skip List

- The **insertion** procedure in **Skip Lists**
  1. For all levels, find rightmost **nodes** < **target**

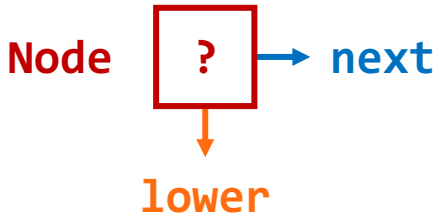


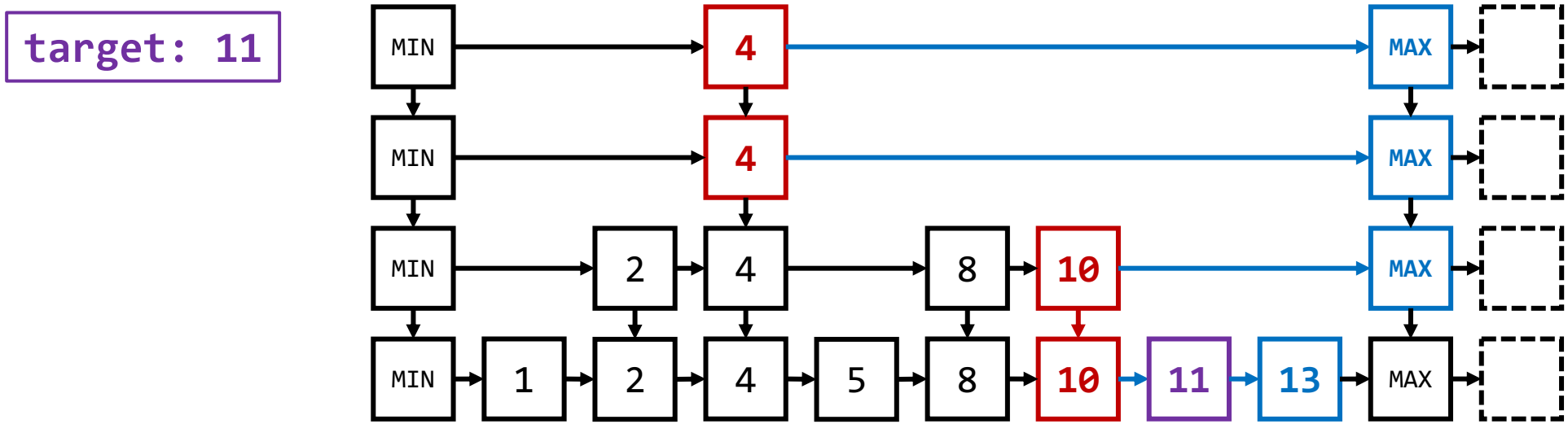
target: 11



Variants of Linked Lists - Skip List

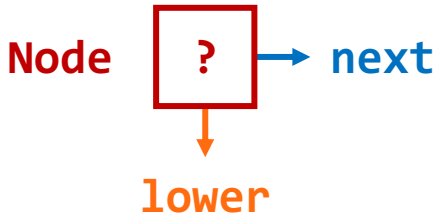
- The **insertion** procedure in **Skip Lists**
  1. For all levels, find rightmost **nodes** < **target**
  2. Insert **nodes** from the bottom to the top





Variants of Linked Lists - Skip List

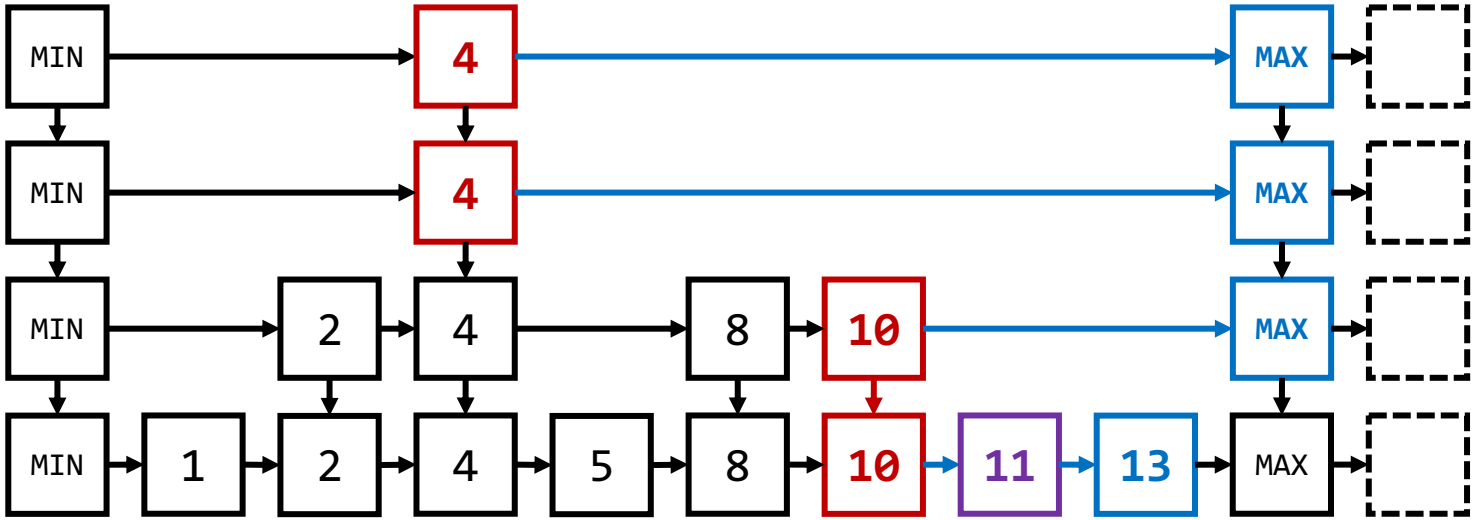
- The **insertion** procedure in **Skip Lists**
  1. For all levels, find rightmost **nodes** < **target**
  2. Insert **nodes** from the bottom to the top
     1. If the flipped coin shows head, continue to insert



Coin Flip

$p(\text{head}) = p(\text{tail}) = 0.5$

target: 11



Variants of Linked Lists - Skip List

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  1. For all levels, find rightmost **nodes** < **target**
  2. Insert **nodes** from the bottom to the top
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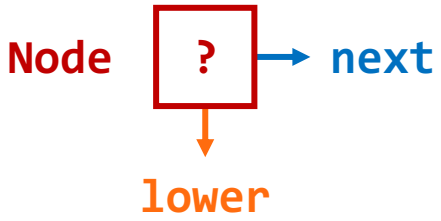

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$p(\text{head}) = p(\text{tail}) = 0.5$

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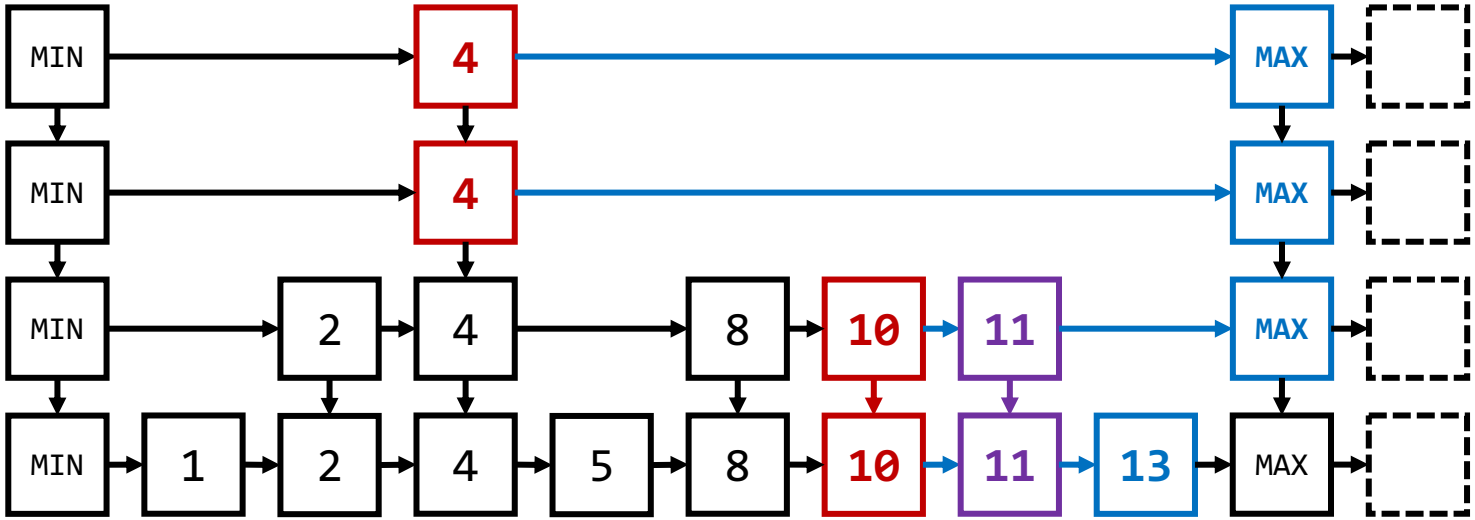

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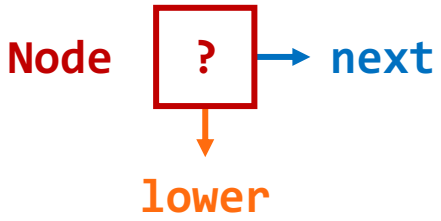
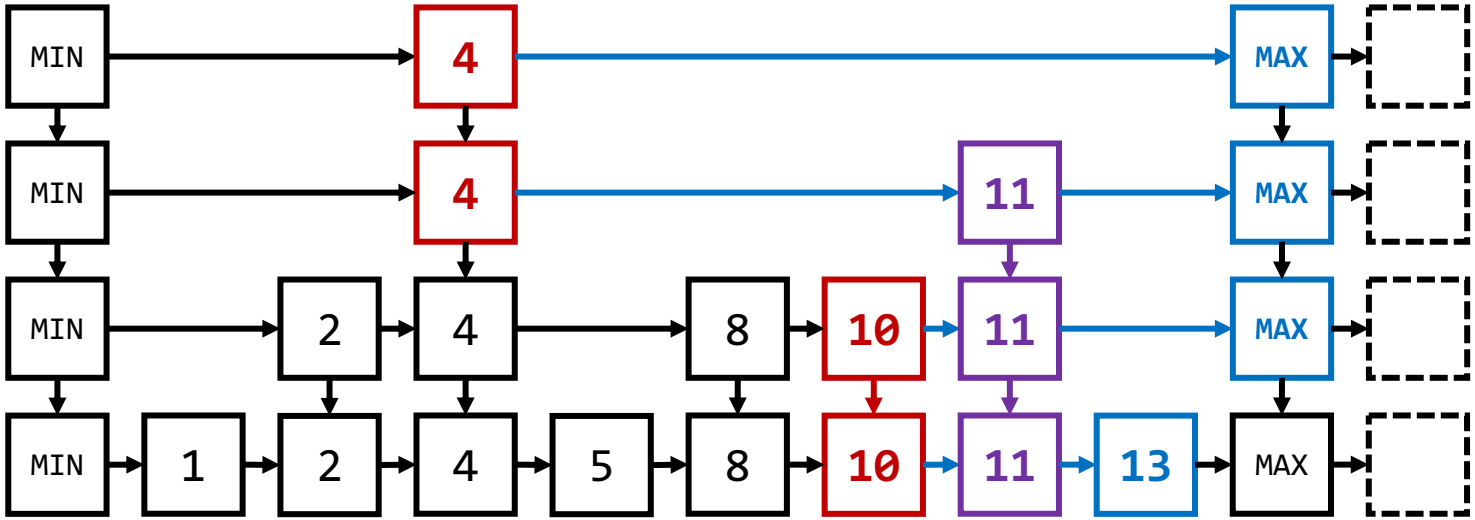


Coin Flip

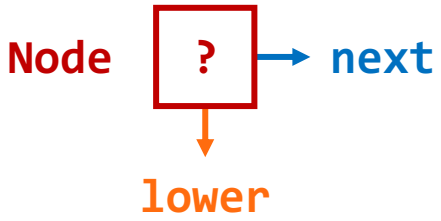
$p(\text{head}) = p(\text{tail}) = 0.5$

target: 11



Variants of Linked Lists - Skip List

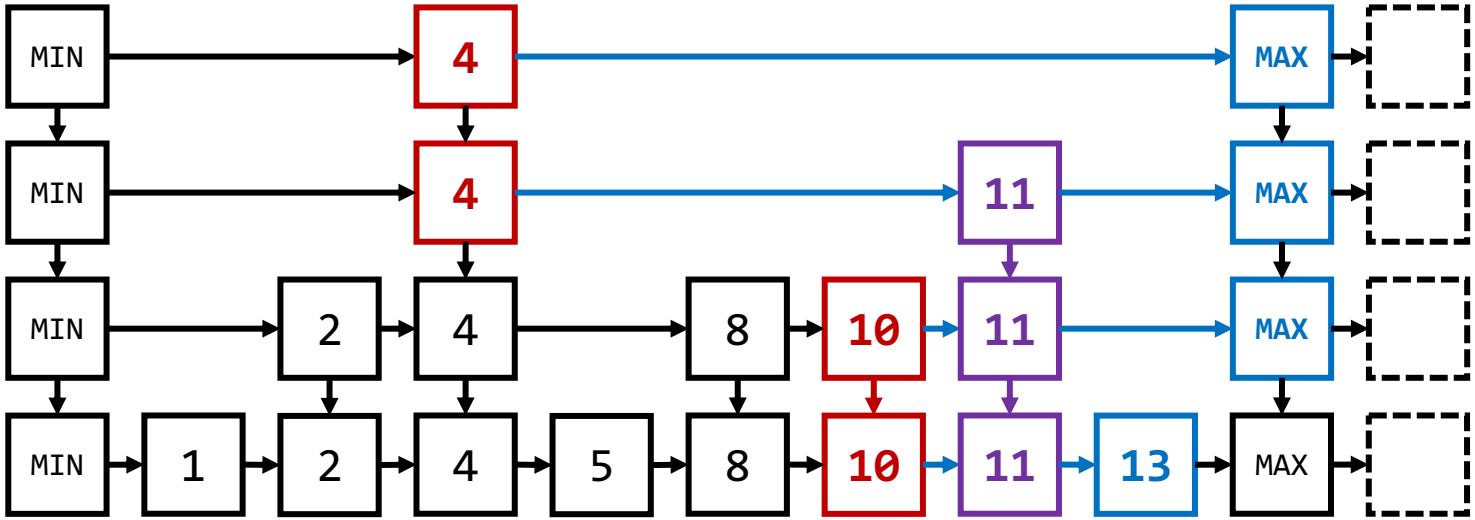
- The **insertion** procedure in **Skip Lists**
  1. For all levels, find rightmost **nodes** < **target**
  2. Insert **nodes** from the bottom to the top
     1. If the flipped coin shows head, continue to insert
     2. If the flipped coin shows tail, stop insertion



Coin Flip

$p(\text{head}) = p(\text{tail}) = 0.5$

target: 11



Variants of Linked Lists - Skip List

- The **insertion** procedure in **Skip Lists**
  1. For all levels, find rightmost **nodes** < **target**
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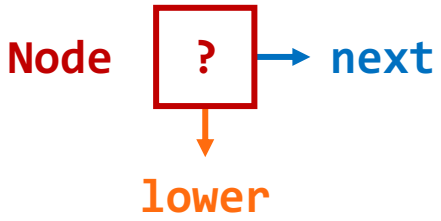
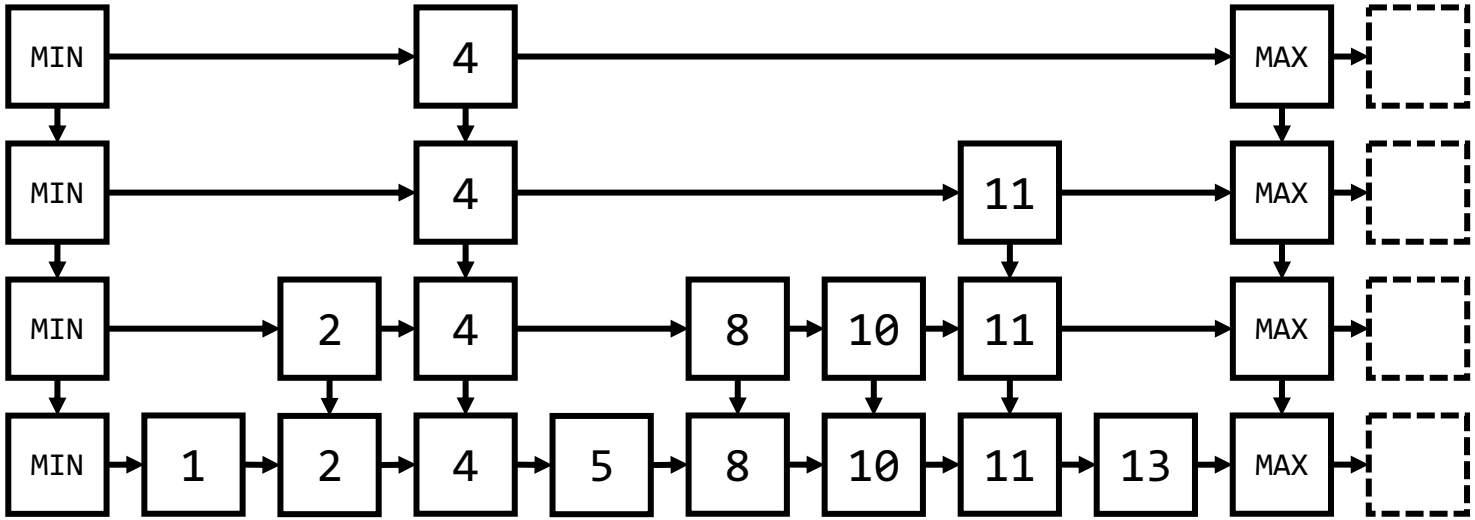
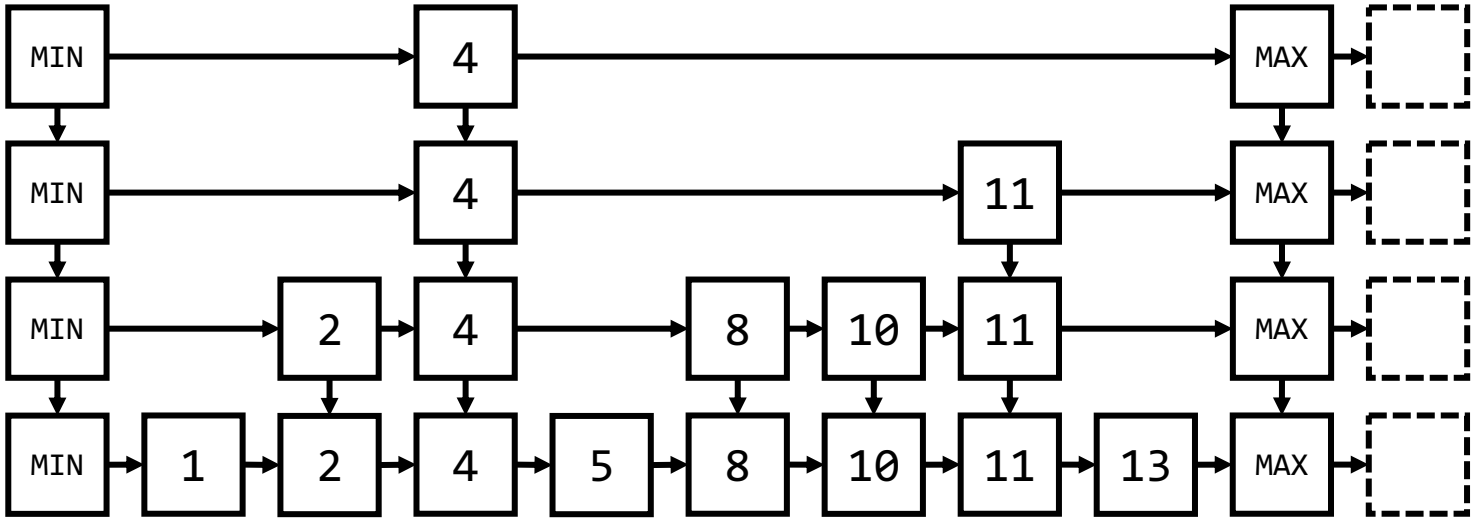




Variants of Linked Lists - Skip List

- Skip Lists - A **probabilistic** data structure for an ordered sequence
  - This allows $O(\log n)$ average time complexity for search
  - This allows $O(\log n)$ average time complexity for insertion/deletion while maintaining the order

  - # of nodes is roughly N + N/2 + N/4 + … = 2N
  - Therefore, **the space complexity** of the skip list is $O(n)$



Time & Space Complexities

Time Complexity

Operation	(standard) Array	(standard) Linked List	Skip List (average case)	Skip List (worst case)
Insertion	$O(n)$	$O(1)$	$O(\log n)$	$O(n)$
Deletion	$O(n)$	$O(1)$	$O(\log n)$	$O(n)$
Search by Index	$O(1)$	$O(n)$	$O(\log n)$	$O(n)$
Search by Value	$O(n)$	$O(n)$	$O(\log n)$	$O(n)$
Reversion	$O(n)$	$O(n)$	-	-

Space Complexity

(standard) Array	(standard) Linked List	Skip List (average case)	Skip List (worst case)
$O(n)$	$O(n)$	$O(n)$	$O(n \log n)$

- **Note.** The array operations can be improved to $O(\log n)$ with advanced structures

Any Questions?