UNIVERSIDAD NACIONAL DE COLOMBIA

Introducción a la Criptografía y a la Seguridad de la Información

Sesión 3
Data Encryption Standard

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Session 3

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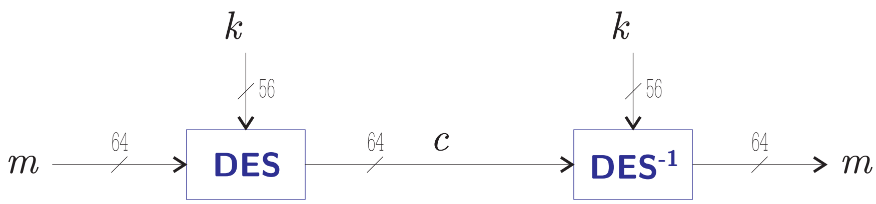
Data Encryption Standard (DES)

The Data Encryption Standard (DES) is a **block cipher** invented in the early 1970s by IBM and the U.S. government (US patent 3,962,539).

It operates on blocks of 64 bits using a secret key that is 56 bits long. $\mathcal{M} = \mathcal{C} = \{0,1\}^{64}$, $\mathcal{K} = \{0,1\}^{56}$.



At the time it was believed that trying out all 72,057,594,037,927,936 (72 quadrillion) possible keys (a seven with 16 zeros) would be impossible because computers could not possibly ever become fast enough.

DES has been replaced by AES as a standard. We will use DES to illustrate the principles of modern symmetric ciphers

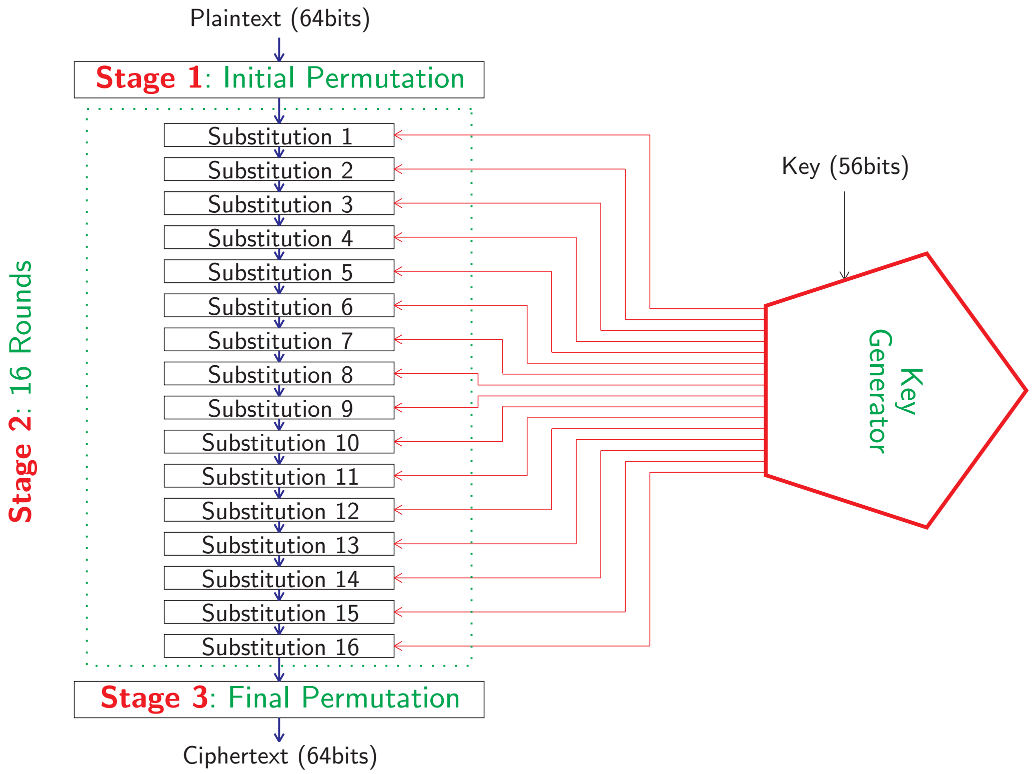
Steps of DES Algorithm

The algorithm has 3 stages (18 steps):

- **Stage 1**: Initial Permutation
- **Stage 2**: 16 Operations (rounds)
- **Stage 3**: Final Permutation

In 1998, the Electronic Frontier Foundation (EFF) built a machine that could crack the DES algorithm by brute-force; called DES Deep Crack, it could find a 56-bit DES key in an average of 4.5 days.

Steps of DES Algorithm (cont.)



Key Generator

DES must first create 16 subkeys as follows:

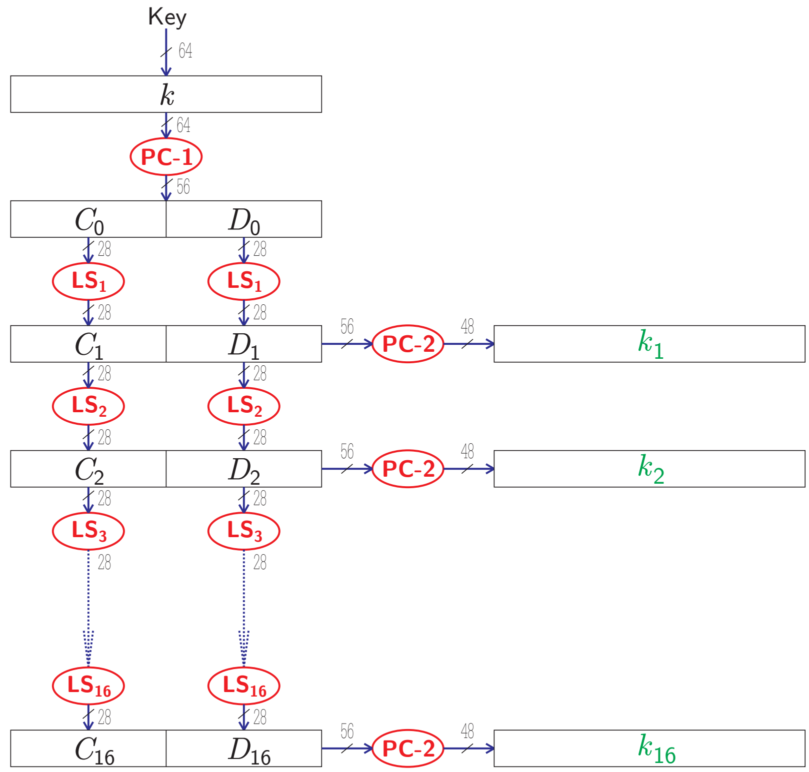
1. From a given bitstring key k of length 64, of which 56 bits comprise the key and 8 bits are parity-check bits(for error-detection), compute $k_0 = PC\text{-}1(k) = C_0D_0$, where C_0 comprise the first 28 bits of $PC\text{-}1(k)$ and D_0 the last 28 bits.

2. For i ranging 1 to 16, compute:
$$C_i = LS_i(C_{i-1}), D_i = LS_i(D_{i-1}), k_i = PC\text{-}2(C_iD_i).$$

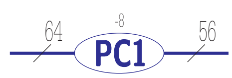
LS_i represents a cyclic shift (to the left) of either one or two positions, depending on the value of i: by 1 if $i = 1, 2, 9, 16$, by 2 otherwise. PC-2 is another fixed permutation.

The bits in positions 8,16,24,32,40,48,56 and 64 of k are defined so that each byte contains an odd number of 1's. Hence, a single error can be detected within each group of 8 bits. The parity-check bits are ignored in the computation of the key.

Computation of the 16 DES-Subkeys

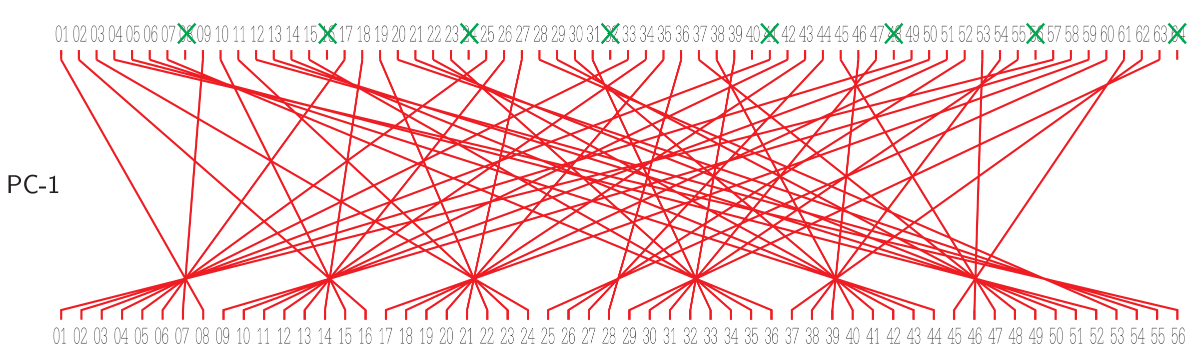


Permuted Choice 1 (PC-1)



bit	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
PC-1	57	49	41	33	25	17	09	01	58	50	42	34	26	18	10	02	59	51	43	35	27	19	11	03	60	52	44	36

bit	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
PC-1	63	55	47	39	31	23	15	07	62	54	46	38	30	22	14	06	61	53	45	37	29	21	13	05	28	20	12	04

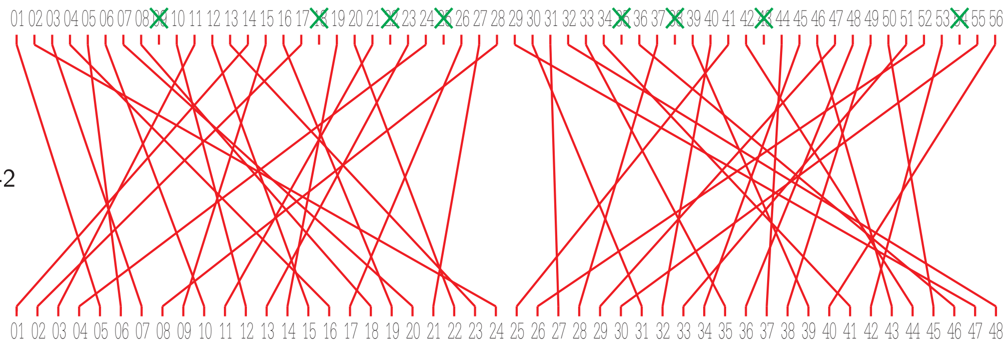


Permuted Choice 2 (PC-2)

bit	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
PC-2	14	17	11	24	01	05	03	28	15	06	21	10	23	19	12	04	26	08	16	07	27	20	13	02

bit	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
PC-2	41	52	31	37	47	55	30	40	51	45	33	48	44	49	39	56	34	53	46	42	50	36	29	32

PC-2



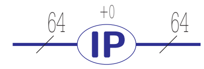
Stage 1: Initial Permutation

Given a plaintext m, a bitstring m_0 is constructed by permuting the bits of m according to a (fixed) **initial permutation IP**.

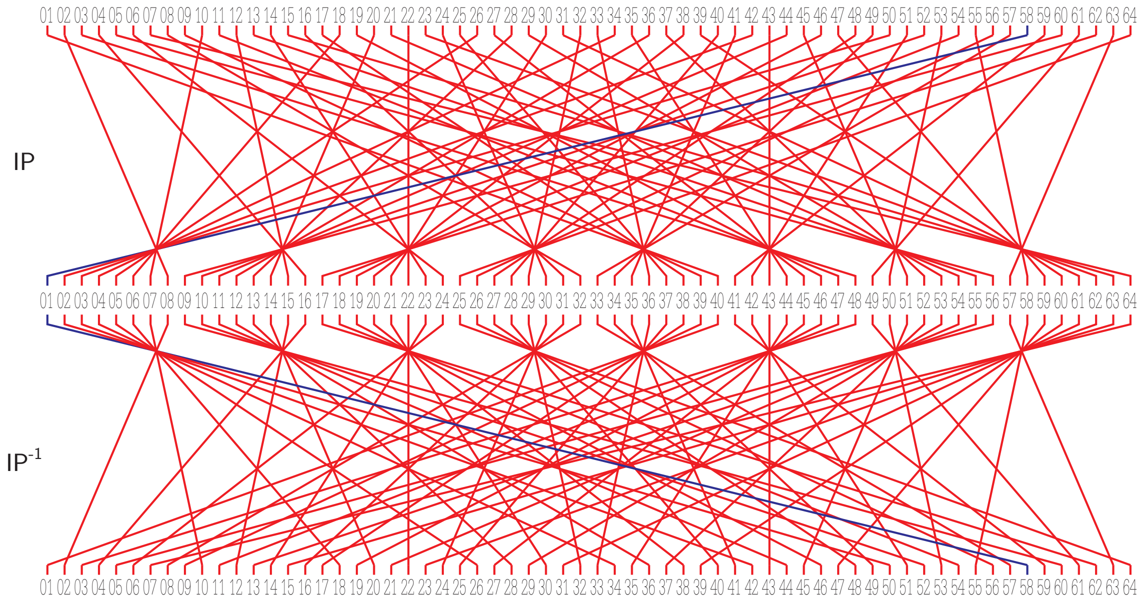
We write $m_0 = IP(x) = L_0 R_0$, where L_0 comprises the first 32 bits of m_0 and R_0 the last 32 bits.

If the block is shorter than 64 bits, it should be padded with zeros.

Initial Permutation (IP)



IP
(01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64
58 50 42 34 26 18 10 02 60 52 44 36 28 20 12 04 62 54 46 38 30 22 14 06 64 56 48 40 32 24 16 08 57 49 41 33 25 17 09 01 59 51 43 35 27 19 11 03 61 53 45 37 29 21 13 05 63 55 47 39 31 23 15 07)



IP-1
(01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64
40 08 48 16 56 24 64 32 39 07 47 15 55 23 63 31 38 06 46 14 54 22 62 30 37 05 45 13 53 21 61 29 36 04 44 12 52 20 60 28 35 03 43 11 51 19 59 27 34 02 42 10 50 18 58 26 33 01 41 09 49 17 57 25)

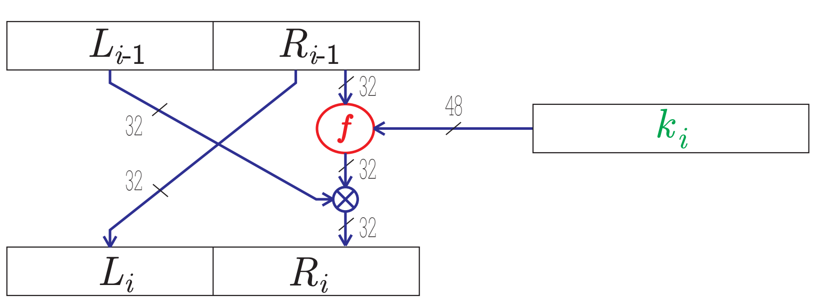
Stage 2: 16 Rounds

16 iterations of a certain function are computed.

We compute L_iR_i, for $1 \leq i \leq 16$, according to the following rule:

$$L_i = R_{i-1}, R_i = L_{i-1} \otimes f(R_{i-1}, k_i),$$

where $\otimes$ denotes the exclusive-or, f is the inner function of DES and it's described later, and $k_1, k_2, \ldots, k_{16}$ are the subkeys we already computed.

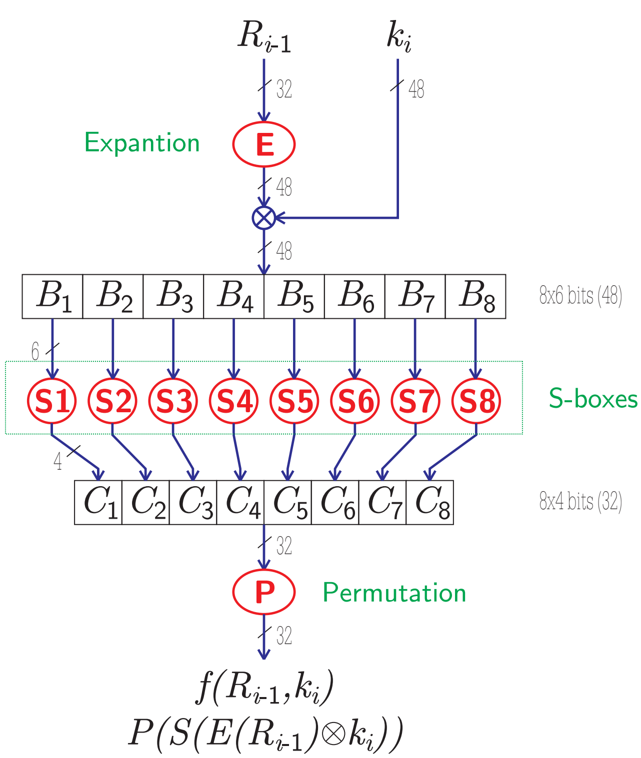


Inner Function f

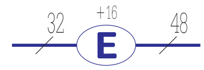
The function f takes as input a first argument R_{i-1} of length 32, and a second argument k_i of length 48, and produces as output a bitstring of length 32. The following steps are executed:

1. The first argument R_{i-1} is expanded to a bitstring of length 48 according to a fixed expansion function E. $E(R_{i-1})$ consists of 32 bits from R_{i-1}, permuted in a certain way, with 16 of the bits appearing twice.
2. Compute $R_{i-1} \otimes k_i$ and write the result as the concatenation of eight 6-bit strings $B = B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8$.
3. Apply $S_j(B_j)$ to every block B_j. S_j is a fixed 4×16 array whose entries come from the integers 0-15. For each $B_j = b_1 b_2 b_3 b_4 b_5 b_6$, the two bits $b_1 b_6$ determine the binary representation of the row r of $S_j(r, c)$ and the four bits $b_2 b_3 b_4 b_5$ determines the binary representation of the column. $C_j = S_j(B_j)$ is defined to be the entry $S_j(r, c)$, written in binary as a bitstring of length four.
4. The bitstring $C = C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8$ of length 32 is permuted according to a fixed permutation P. $f(R_{i-1}, k_i) = P(C)$.

Inner Function f

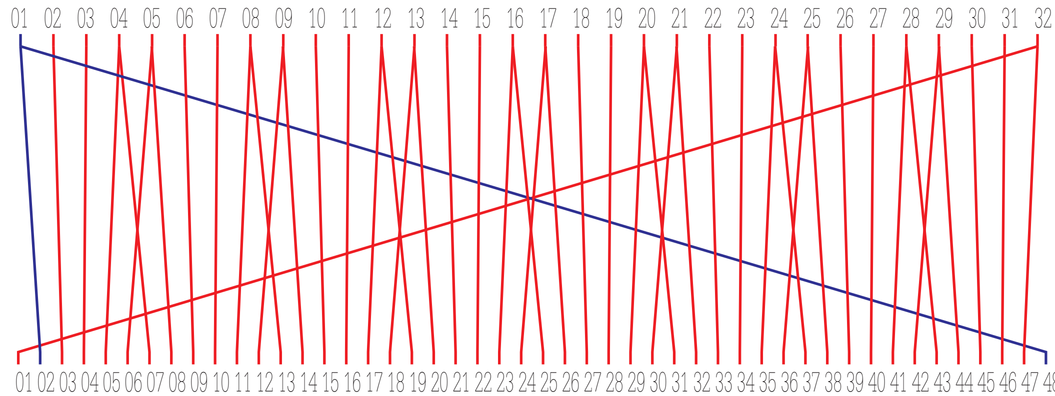


Expansion (E)

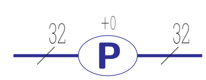


bit	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
E	32	01	02	03	04	05	04	05	06	07	08	09	08	09	10	11	12	13	12	13	14	15	16	17	16	17	18	19	20	21	20	21	22	23	24	25	24	25	26	27	28	29	28	29	30	31	32	01

E

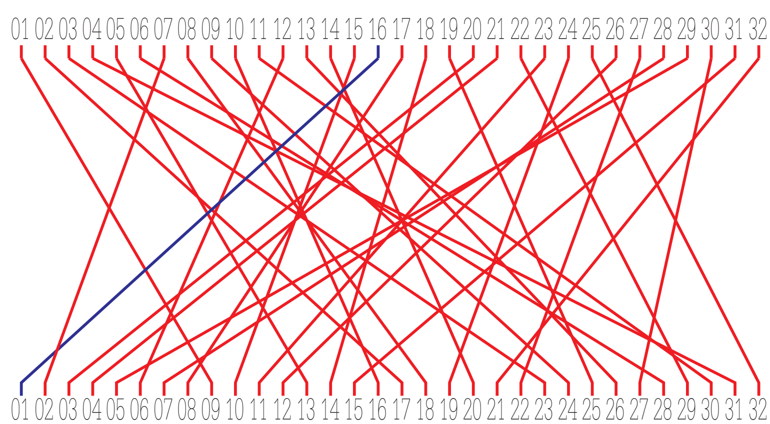


Permutation (P)

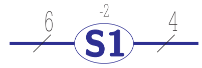


$$P \begin{pmatrix} 01 & 02 & 03 & 04 & 05 & 06 & 07 & 08 & 09 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 \\ 16 & 07 & 20 & 21 & 29 & 12 & 28 & 17 & 01 & 15 & 23 & 26 & 05 & 18 & 31 & 10 & 02 & 08 & 24 & 14 & 32 & 27 & 03 & 09 & 19 & 13 & 30 & 06 & 22 & 11 & 04 & 25 \end{pmatrix}$$

P



S-Box 1 (S1)



		c															
	S1	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
	00	14	04	13	01	02	15	11	08	03	10	06	12	05	09	00	07
r	01	00	15	07	04	14	02	13	01	10	06	12	11	09	05	03	08
	02	04	01	14	08	13	06	02	11	15	12	09	07	03	10	05	00
	03	15	12	08	02	04	09	01	07	05	11	03	14	10	00	06	13

$B_j = b_1 b_2 b_3 b_4 b_5 b_6$

$r = b_1 b_6$ [0..3]

$c = b_2 b_3 b_4 b_5$ [0..15]

Example: Compute S1(011000)

S1(011000) = S1(00, 1100) = S1(0, 12) = 5 = 0101

S1(011000) = 0101

S-Boxes 1–8 (S1,S2,S3,S4,S5,S6,S7,S8)

c

	S1	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
	00	14	04	13	01	02	15	11	08	03	10	06	12	05	09	00	07
r	01	00	15	07	04	14	02	13	01	10	06	12	11	09	05	03	08
	02	04	01	14	08	13	06	02	11	15	12	09	07	03	10	05	00
	03	15	12	08	02	04	09	01	07	05	11	03	14	10	00	06	13
	S2	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
	00	15	01	08	14	06	11	03	04	09	07	02	13	12	00	05	10
r	01	03	13	04	07	15	02	08	14	12	00	01	10	06	09	11	05
	02	00	14	07	11	10	04	13	01	05	08	12	06	09	03	02	15
	03	13	08	10	01	03	15	04	02	11	06	07	12	00	05	14	09
	S3	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
	00	10	00	09	14	06	03	15	05	01	13	12	07	11	04	02	08
r	01	13	07	00	09	03	04	06	10	02	08	05	14	12	11	15	01
	02	13	06	04	09	08	15	03	00	11	01	02	12	05	10	14	07
	03	01	10	13	00	06	09	08	07	04	15	14	03	11	05	02	12
	S4	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
	00	07	13	14	03	00	06	09	10	01	02	08	05	11	12	04	15
r	01	13	08	11	05	06	15	00	03	04	07	02	12	01	10	14	09
	02	10	06	09	00	12	11	07	13	15	01	03	14	05	02	08	04
	03	03	15	00	06	10	01	13	08	09	04	05	11	12	07	02	14

c

S5	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
00	02	12	04	01	07	10	11	06	08	05	03	15	13	00	14	09
01	14	11	02	12	04	07	13	01	05	00	15	10	03	09	08	06
02	04	02	01	11	10	13	07	08	15	09	12	05	06	03	00	14
03	11	08	12	07	01	14	02	13	06	15	00	09	10	04	05	03
S6	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
00	12	01	10	15	09	02	06	08	00	13	03	04	14	07	05	11
01	10	15	04	02	07	12	09	05	06	01	13	14	00	11	03	08
02	09	14	15	05	02	08	12	03	07	00	04	10	01	13	11	06
03	04	03	02	12	09	05	15	10	11	14	01	07	06	00	08	13
S7	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
00	04	11	02	14	15	00	08	13	03	12	09	07	05	10	06	01
01	13	00	11	07	04	09	01	10	14	03	05	12	02	15	08	06
02	01	04	11	13	12	03	07	14	10	15	06	08	00	05	09	02
03	06	11	13	08	01	04	10	07	09	05	00	15	14	02	03	12
S8	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
00	13	02	08	04	06	15	11	01	10	09	03	14	05	00	12	07
01	01	15	13	08	10	03	07	04	12	05	06	11	00	14	09	02
02	07	11	04	01	09	12	14	02	00	06	10	13	15	03	05	08
03	02	01	14	07	04	10	08	13	15	12	09	00	03	05	06	11

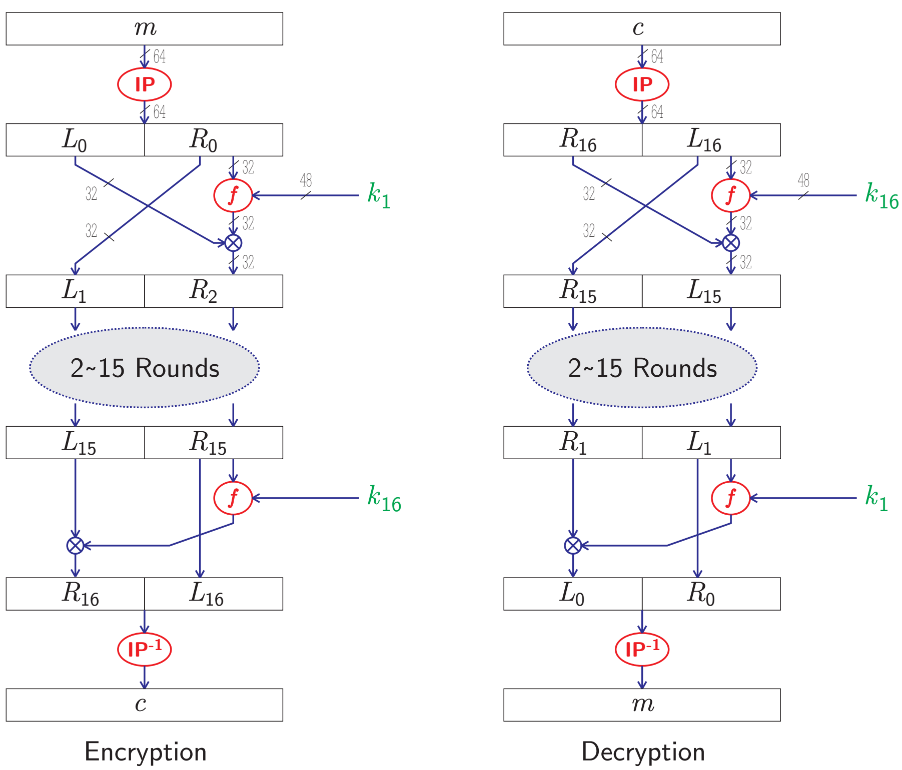
Stage 1: Final Permutation

Apply the inverse permutation IP^{-1} to the bitstring $R_{16}L_{16}$.

Obtain the ciphertext $c = IP^{-1}(R_{16}L_{16})$.

Note the inverted order of L_{16} and R_{16}.

Encryption/Decryption



Example

Let $m =$ `0123456789ABCDEF` and $k =$ `133457799BBCDFF1`, where m and k are in hexadecimal (base 16) format.

Part 1: Create 16 subkeys:

$k =$ `133457799BBCDFF1`.

This gives us as the binary key (setting 1 = 0001, 3 = 0011, etc., and grouping together every eight bits, of which the last one in each group will be unused):

$k =$ 00010011 00110100 01010111 01111001 10011011 10111100 11011111 11110001

Compute $k' = PC\text{-}1(k)$

$k' =$ 1111000 0110011 0010101 0101111 0101010 1011001 1001111 0001111

Next, split this key into left and right halves, C_0 and D_0, where each half has 28 bits.

$C_0 =$ 1111000 0110011 0010101 0101111 $D_0 =$ 0101010 1011001 1001111 0001111

Now, compute $C_i = LS_i(C_{i-1}), D_i = LS_i(D_{i-1})$ for $i = 1, 2, \ldots, 16$

$C_0 =$	1111000011001100101010101111	$D_0 =$	0101010101100110011110001111
$C_1 =$	1110000110011001010101011111	$D_1 =$	1010101011001100111100011110
$C_2 =$	1100001100110010101010111111	$D_2 =$	0101010110011001111000111101
$C_3 =$	0000110011001010101011111111	$D_3 =$	0101011001100111100011110101
$C_4 =$	0011001100101010101111111100	$D_4 =$	0101100110011110001111010101
$C_5 =$	1100110010101010111111110000	$D_5 =$	0110011001111000111101010101
$C_6 =$	0011001010101011111111000011	$D_6 =$	1001100111100011110101010101
$C_7 =$	1100101010101111111100001100	$D_7 =$	0110011110001111010101010110
$C_8 =$	0010101010111111110000110011	$D_8 =$	1001111000111101010101011001
$C_9 =$	0101010101111111100001100110	$D_9 =$	0011110001111010101010110011
$C_{10} =$	0101010111111110000110011001	$D_{10} =$	1111000111101010101011001100
$C_{11} =$	0101011111111000011001100101	$D_{11} =$	1100011110101010101100110011
$C_{12} =$	0101111111100001100110010101	$D_{12} =$	0001111010101010110011001111
$C_{13} =$	0111111110000110011001010101	$D_{13} =$	0111101010101011001100111100
$C_{14} =$	1111111000011001100101010101	$D_{14} =$	1110101010101100110011110001
$C_{15} =$	1111100001100110010101010111	$D_{15} =$	1010101010110011001111000111
$C_{16} =$	1111000011001100101010101111	$D_{16} =$	0101010101100110011110001111

We now form the keys $k_i = PC\text{-}2(C_iD_i)$ for $i = 1, 2, 3, \ldots, 16$.

$k_1 =$ 000110 110000 001011 101111 111111 000111 000001 110010
$k_2 =$ 011110 011010 111011 011001 110110 111100 100111 100101
$k_3 =$ 010101 011111 110010 001010 010000 101100 111110 011001
$k_4 =$ 011100 101010 110111 010110 110110 110011 010100 011101
$k_5 =$ 011111 001110 110000 000111 111010 110101 001110 101000
$k_6 =$ 011000 111010 010100 111110 010100 000111 101100 101111
$k_7 =$ 111011 001000 010010 110111 111101 100001 100010 111100
$k_8 =$ 111101 111000 101000 111010 110000 010011 101111 111011
$k_9 =$ 111000 001101 101111 101011 111011 011110 011110 000001
$k_{10} =$ 101100 011111 001101 000111 101110 100100 011001 001111
$k_{11} =$ 001000 010101 111111 010011 110111 101101 001110 000110
$k_{12} =$ 011101 010111 000111 110101 100101 000110 011111 101001
$k_{13} =$ 100101 111100 010111 010001 111110 101011 101001 000001
$k_{14} =$ 010111 110100 001110 110111 111100 101110 011100 111010
$k_{15} =$ 101111 111001 000110 001101 001111 010011 111100 001010
$k_{16} =$ 110010 110011 110110 001011 000011 100001 011111 110101

Part 2: Encode each 64-bit block of data.

Rewriting m=`0123456789ABCDEF` in binary format, we get the 64-bit block of text:

$$m = 0000000100100011010001010110011110001001101010111100110111101111$$

Compute m'=$IP(m)$

$$m' = 1100110000000000110011001111111111110000101010101111000010101010$$

Here the 58th bit of m is "1", which becomes the first bit of m'. The 50th bit of m is "1", which becomes the second bit of m'. The 7th bit of m is "0", which becomes the last bit of m'.

Next divide the permuted block IP into a left half L0 of 32 bits, and a right half R0 of 32 bits.

$L_0 =$ 1100 1100 0000 0000 1100 1100 1111 1111
$R_0 =$ 1111 0000 1010 1010 1111 0000 1010 1010

We now proceed through 16 iterations, for $1 \leq i \leq 16$, using function f which operates on two blocks — a data block of 32 bits and a key k_i of 48 bits — to produce a block of 32 bits. Then for i going from 1 to 16 we calculate:

$$L_i = R_{i-1}, R_i = L_{i-1} \otimes f(R_{i-1}, k_i),$$

For $i = 1$, we have

$k_1 =$ 000110 110000 001011 101111 111111 000111 000001 110010
$L_1 = R_0 =$ 1111 0000 1010 1010 1111 0000 1010 1010
$R_1 = L_0 \otimes f(R_0, K_1)$

We calculate $E(R_0)$ from R_0 as follows:

$R_0 =$ 1111 0000 1010 1010 1111 0000 1010 1010
$E(R_0) =$ 011110 100001 010101 010101 011110 100001 010101 010101

Note that each block of 4 original bits has been expanded to a block of 6 output bits.

Next in the f calculation, we XOR the output $E(R_{i-1})$ with the key k_i:

$k_1 =$ 000110 110000 001011 101111 111111 000111 000001 110010
$E(R_0) =$ 011110 100001 010101 010101 011110 100001 010101 010101
$k_1 \otimes E(R_0) =$ 011000 010001 011110 111010 100001 100110 010100 100111

Write the previous result, which is 48 bits in the form $k_i \otimes E(R_{i-1}) = B_1B_2B_3B_4B_5B_6B_7B_8$, where each B_i is a group of six bits.

$k_1 \otimes E(R_0) =$ 011000 010001 011110 111010 100001 100110 010100 100111

We now calculate

$S_1(B_1)S_2(B_2)S_3(B_3)S_4(B_4)S_5(B_5)S_6(B_6)S_7(B_7)S_8(B_8)$

where $S_i(B_i)$ referres to the output of the i-th S-box.

$S_1(B_1)S_2(B_2)S_3(B_3)S_4(B_4)S_5(B_5)S_6(B_6)S_7(B_7)S_8(B_8) =$ 0101 1100 1000 0010 1011 0101 1001 0111

The final stage in the calculation of f is to do permutation P, we get

$f =$ 0010 0011 0100 1010 1010 1001 1011 1011

So $R_1 = L_0 \otimes f(R_0, k_1)$

$=$ 1100 1100 0000 0000 1100 1100 1111 1111 $\otimes$ 0010 0011 0100 1010 1010 1001 1011 1011
$=$ 1110 1111 0100 1010 0110 0101 0100 0100

After the first round we get

$L_1 =$ 11110000101010101111000010101010
$R_1 =$ 11101111010010100110010101000100

Now we simply repeat this simple process 15 more times.

$L_2 =$	11101111010010100110010101000100	$R_2 =$	11001100000000010111011100001001
$L_3 =$	11001100000000010111011100001001	$R_3 =$	10100010010111000000101111110100
$L_4 =$	10100010010111000000101111110100	$R_4 =$	01110111001000100000000001000101
$L_5 =$	01110111001000100000000001000101	$R_5 =$	10001010010011111010011000110111
$L_6 =$	10001010010011111010011000110111	$R_6 =$	11101001011001111100110101101001
$L_7 =$	11101001011001111100110101101001	$R_7 =$	?00001?00100?0101?1110?0000?000?
$L_8 =$	?00001?00100?0101?11?0?0000?000?	$R_8 =$	11010101011010010100101110010000
$L_9 =$	11010101011010010100101110010000	$R_9 =$	00100100011111001100011001111010
$L_{10} =$	00100100011111001100011001111010	$R_{10} =$	10110111110101011101011110110010
$L_{11} =$	10110111110101011101011110110010	$R_{11} =$	11000101011110000011110001111000
$L_{12} =$	11000101011110000011110001111000	$R_{12} =$	01110101101111010001100001011000
$L_{13} =$	01110101101111010001100001011000	$R_{13} =$	00011000110000011000101010101010
$L_{14} =$	00011000110000011000101010101010	$R_{14} =$	11000010100011001001011000001101
$L_{15} =$	11000010100011001001011000001101	$R_{15} =$	01000011010000100011001000110100
$L_{16} =$	01000011010000100011001000110100	$R_{16} =$	00001010010011001101100110010101

We then reverse the order of the two blocks $R_{16}L_{16}$ and apply IP^{-1} with the following result:

$c =$10000101 11101000 00010011 01010100 00001111 00001010 10110100 00000101

which in hexadecimal format is 85E813540F0AB405.

Therefore, the encrypted form of $m =$ 0123456789ABCDEF is $c =$ 85E813540F0AB405.