

* Dynamic model

$$\bullet M \ddot{q} + b = \tau$$

* sliding variable

$$\bullet s = \lambda^2 e + 2\lambda \dot{e}$$

$$\bullet \dot{s} = 2\lambda \dot{\lambda} e + \lambda^2 \ddot{e} + 2\lambda \ddot{e} + 2\lambda \ddot{e}$$

* PD control signal

$$\bullet u = M(s) + b$$

* SMC control signal

$$\bullet u = M(s + \tanh(s)) + b$$

* Cost function

$$\bullet J = \underbrace{\frac{\lambda^2}{2} s^2}_{\text{reduce } \Delta \ddot{q}} + \underbrace{\frac{(1-\lambda)^2}{2} \dot{s}^2}_{\text{reduce } \Delta \ddot{\ddot{q}}}$$

* Gradient descent

$$\bullet \frac{\partial J}{\partial \lambda} = \frac{\partial J}{\partial s} \cdot \frac{\partial s}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial u} \cdot \frac{\partial u}{\partial \lambda} + \frac{\partial J}{\partial \dot{s}} \cdot \frac{\partial \dot{s}}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial u} \cdot \frac{\partial u}{\partial \lambda}$$

* $\frac{\partial u}{\partial \lambda}$ for PD control signal

$$\bullet \frac{\partial u}{\partial \lambda} = M \cdot [2\lambda e + 2\dot{e}]$$

* $\frac{\partial u}{\partial \lambda}$ for SM control signal

$$\bullet \frac{\partial u}{\partial \lambda} = M \cdot (2 - \tanh^2(s)) \cdot (2\lambda e + 2\dot{e})$$

$$\bullet \frac{\partial s}{\partial \lambda} = -\lambda^2$$

$$\bullet \frac{\partial J}{\partial s} = \lambda s$$

$$\bullet \lambda_{(k+1)} = \lambda_{(k)} - \alpha \frac{\partial J}{\partial \lambda}$$

$$\bullet \frac{\partial \dot{s}}{\partial \lambda} = -2\lambda \dot{\lambda}$$

$$\bullet \frac{\partial J}{\partial \dot{s}} = (1-\lambda) \dot{s}$$

$$\bullet \frac{\lambda_{(k+1)} - \lambda_{(k)}}{\Delta t} = \frac{-\alpha}{\Delta t} \cdot \frac{\partial J}{\partial \lambda}$$

$$\bullet \dot{\lambda} = \frac{-\alpha}{\Delta t} \cdot \frac{\partial J}{\partial \lambda}$$