University of São Paulo

São Carlos School of engineering

Sistemas de Controle

List 4

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2021 - 2

1 Question 1

Considering the transfer function (G)

$$G = \frac{{w_n}^2}{s^2 + 2w_n\zeta s + {w_n}^2},$$

where ζ is damping factor and w_n is natural frequency.

The modulus of G at resonance frequency (w_r) is given by

$$|G(jw_r)| = \frac{{w_n}^2}{\sqrt{({w_n}^2 - {w_r}^2)^2 + (2w_n\zeta w_r)^2}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}},$$

Then resonance frequency can be computed in function of w_n and ζ as

$$\begin{aligned} 0 &= {w_n}^4 + {w_r}^2(-2{w_n}^2 + 4{w_n}^2\zeta^2) + {w_n}^4(1 - 4\zeta^2(1-\zeta^2)) \\ {w_r}^2 &= {w_n}^2 - 2{w_n}^2\zeta^2, \\ w_r &= w_n\sqrt{(1-2\zeta^2)} \end{aligned}$$

2 Question 2

(a) The bode diagram indicates that close-loop system attenuates signals with frequency higher than 4.5 Hz and delay output signal with respect to input. Thus, response of close-loop system will present lower oscillations. Figure 1 describes a sketch of response of close-loop system.

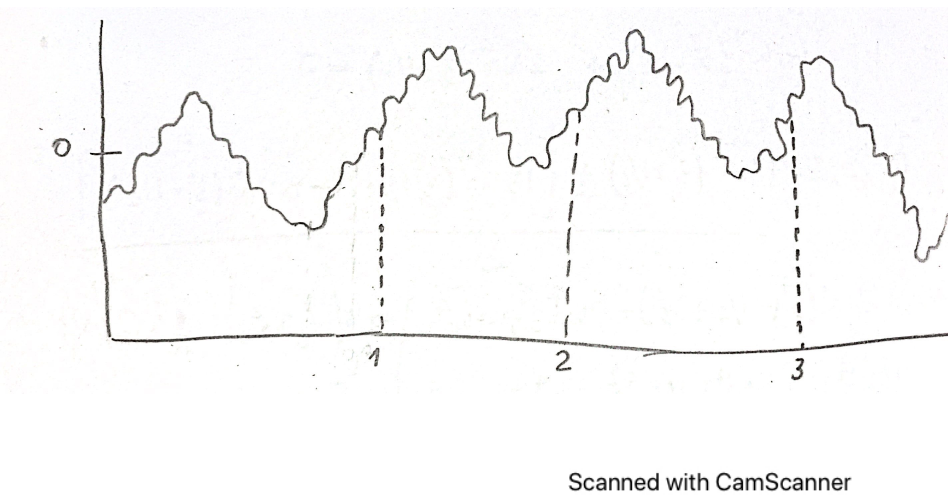

Figure 1: Sketch of response of close-loop system.

(b) Considering input signal

$$r(t) = 1\sin(2\pi 0.1t) + 0.5\sin(2\pi t) + 0.2\sin(2\pi 10t),$$

the output signal will be

$$y(t) = 1\sin(0.628t + 0 \text{ rad}) + 0.48\sin(6.28t - 0.262 \text{ rad}) + 0.0796\sin(62.8t - 1.08 \text{ rad}),$$

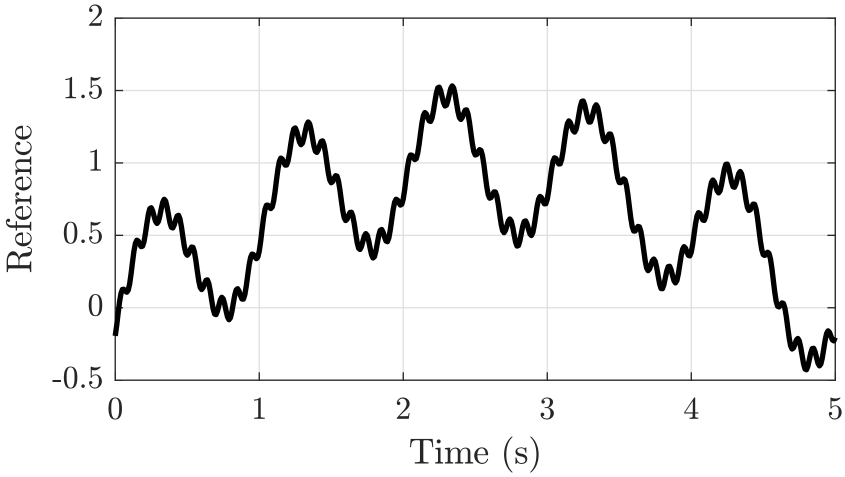

Figure 2: Response of the close-loop system with input (2).

3 Question 3

(a) Dynamic model is given by

$$I_1\ddot{\theta}_1 = \tau - K(\theta_1 - \theta_2) - D(\dot{\theta}_1 - \dot{\theta}_2),$$
$$I_2\ddot{\theta}_2 = K(\theta_1 - \theta_2) + D(\dot{\theta}_1 - \dot{\theta}_2),$$

where K is stiffness and D is damping.

(b) Transfer functions are:

$$G_1(s) = \frac{\theta_1(s)}{\tau(s)} = \frac{I_2 s^2 + Ds + k}{I_1 I_2 s^4 + s^3(I_1 D + I_2 D) + s^2(I_1 K + I_2 K)},$$
$$G_2(s) = \frac{\theta_2(s)}{\tau(s)} = \frac{Ds + k}{I_1 I_2 s^4 + s^3(I_1 D + I_2 D) + s^2(I_1 K + I_2 K)}.$$

(c) Figure 3 describes time-response of open-loop systems ($G_1(s)$ and $G_2(s)$) for different values of stiffness ($K = 10[1, 1.4, 1.8, 2.2, 2.6, 3]\ \frac{\text{N.m}}{\text{rad}}$) and damping ($D = 1000[1, 1.8, 2.6, 3.4, 4.2, 5]\ \frac{\text{N.m.s}}{\text{rad}}$).

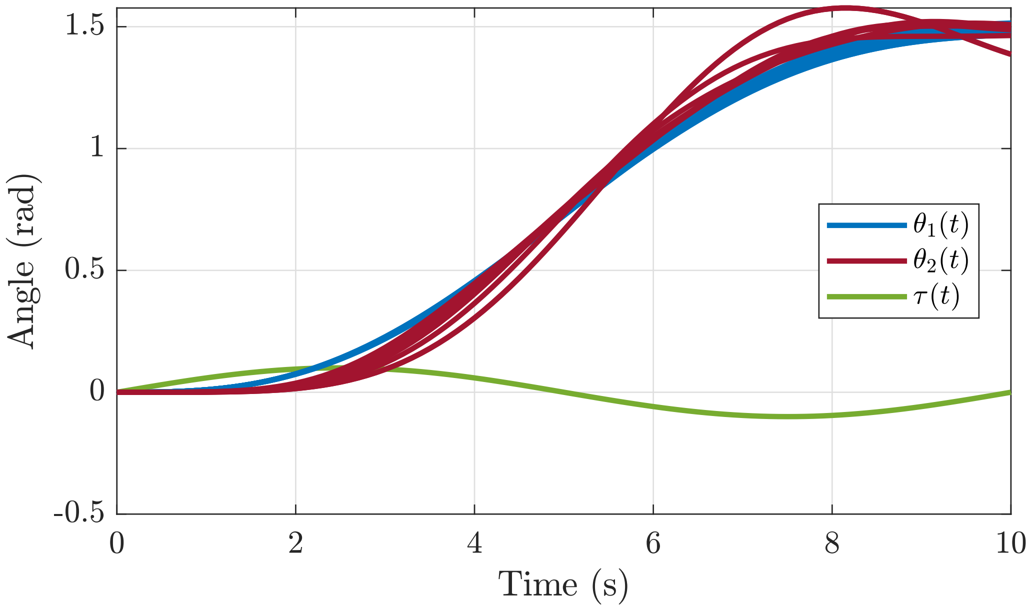

Figure 3: Time-response of open-loop systems ($G_1(s)$ and $G_2(s)$) considering sinusoidal input ($\tau = 0.1 \sin 0.2\pi t$) and different values of stiffness and damping ($K = 10[1, 1.4, 1.8, 2.2, 2.6, 3]\ \frac{\text{N.m}}{\text{rad}}$ and $D = 1000[1, 1.8, 2.6, 3.4, 4.2, 5]\ \frac{\text{N.m.s}}{\text{rad}}$).

(d) On one hand, Figure 4 describes root-locus of close-loop system with proportional gain. In this figure, third and fourth poles (p_3, p_4) are located in the right half-plane for any proportional gain value. On the other hand, Figure 5 describes bode diagram of open-loop system ($G_2(s)$). In this figure, gain margin is $G_m = -13$ dB and phase margin is $P_m = 179$. Likewise, phase diagram is below $-180°$, thus gain margin will be negative for any proportional gain value. In conclusion, both root-locus and frequency response method indicates that close-loop system cannot be controlled with just a proportional gain.

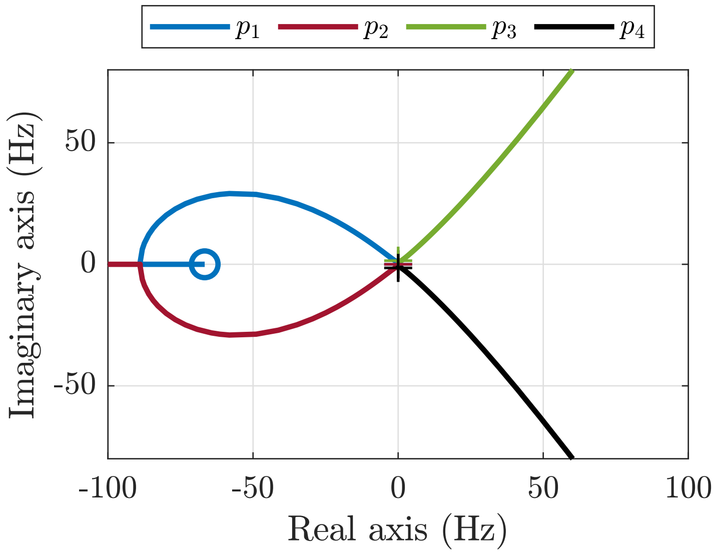

Figure 4: Root-Locus of close-loop system with proportional gain.

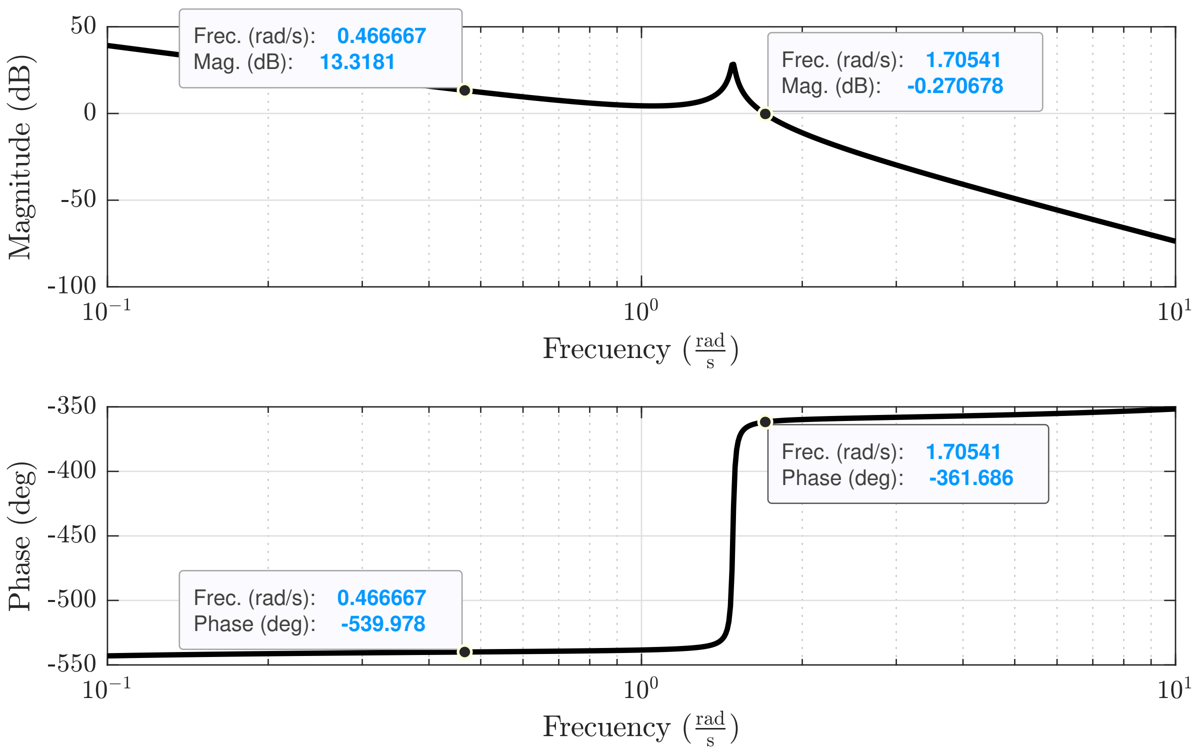

Figure 5: Frequency response of open-loop system $G_2(s) = \frac{\theta_2(s)}{\tau(s)}$.