

University of São Paulo
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Sistemas de Controle

List 3

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1 Question 1

A control system with state \mathbf{x} is described by matrices:

$$\mathbf{A} = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix}$$

(a) The transfer function ($G_{(s)}$) can be computed as

$$\begin{aligned} G_{(s)} &= \mathbf{C}(\mathbf{I}s - \mathbf{A})^{-1}\mathbf{B}, \\ G_{(s)} &= \frac{s + 3}{s^2 + 2s + 2}, \end{aligned} \tag{1}$$

(b) Canonical form of system using (1) is given by

$$\mathbf{A}_c = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}, \mathbf{B}_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{C}_c = \begin{bmatrix} 1 & 3 \end{bmatrix}, \mathbf{D}_c = \begin{bmatrix} 0 \end{bmatrix}.$$

Considering \mathbf{z} as states of canonical form and $\mathbf{z} = \mathbf{T}\mathbf{x}$. Then, matrix \mathbf{T} can be computed solving $\mathbf{AT} = \mathbf{TA}_c$ and $\mathbf{b} = \mathbf{Tb}_c$:

$$\mathbf{T} = \begin{bmatrix} -0.08 & 0.36 \\ 0.36 & 0.88 \end{bmatrix}.$$

2 Question 2

(a) State-space matrices

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{CR_1} & 0 \\ 0 & -\frac{R_2}{L} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \frac{1}{CR_1} \\ \frac{1}{L} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \frac{-1}{R_1} & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} \frac{1}{R_1} \end{bmatrix}$$

and state-space equations

$$\begin{aligned} \begin{bmatrix} \dot{v}_c \\ \dot{i}_l \end{bmatrix} &= \mathbf{A} \begin{bmatrix} v_c \\ i_l \end{bmatrix} + \mathbf{B}\mathbf{u}, \\ \mathbf{y} &= \mathbf{C} \begin{bmatrix} v_c \\ i_l \end{bmatrix} + \mathbf{D}\mathbf{u}, \end{aligned}$$

(b) Controllability matrix can be computed as

$$\begin{aligned} \mathbf{C}_{ctb} &= \begin{bmatrix} B & AB \end{bmatrix}, \\ \mathbf{C}_{ctb} &= \begin{bmatrix} \frac{1}{CR_1} & \frac{-1}{C^2R_1^2} \\ \frac{1}{L} & -\frac{R_2}{L^2} \end{bmatrix}. \end{aligned}$$

System is controllable if $|\mathbf{C}_{ctb}| \neq 0$. Thus, system is not controllable when relation between R_1, R_2, C, L is

$$CR_1 = \frac{L}{R_2}.$$

(c) Constant time of a resistant-capacitor (RC) circuit is $\tau_{RC} = CR_1$ and resistant-inductor (RL) is $\tau_{RL} = \frac{L}{R_2}$. On one hand, when $t = 5\tau_{RC}$ RC circuit will set $v_c = u(t)$ v and $i_c = 0$ mA. On the other hand, when $t = 5\tau_{RL}$ RL circuit will set $v_c = 0$ v and $i_c = i_y$ mA. For this reason, both circuits must not have the same constant time because system will become unstable.

(d) Transfer function ($G_{(s)}$) can be computed as

$$\begin{aligned} G_{(s)} &= \mathbf{C}(\mathbf{I}s - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}, \\ G_{(s)} &= \frac{(LCR_1^2)s^2 + (CR_1^3 + R_2CR_1^2)s + R_1^2}{(LCR_1^3)s^2 + (R_2CR_1^3 + LR_1^2)s + R_2R_1^2}, \end{aligned} \quad (2)$$

when $CR_1 = \frac{L}{R_2}$ the poles and zeros are

$$\begin{aligned} \text{poles} &: -\frac{R_2}{L}, \\ \text{zeros} &: -\frac{R_1}{L}. \end{aligned}$$

3 Question 3

(a) Control law can be computed as

$$u = -\mathbf{K}_x\mathbf{x} + K_r r \quad (3)$$

where \mathbf{K}_x is state gain, K_r is reference gain and r reference signal.

Then, state-space close loop is given by

$$\begin{aligned} \dot{\mathbf{x}} &= (\mathbf{A} - \mathbf{BK}_x)\mathbf{x} + \mathbf{BK}_r u(t), \\ y &= \mathbf{C}\mathbf{x} + \mathbf{D}u(t). \end{aligned}$$

Time-domain requirements are settling time lower than 4.6 seconds and %PO lower than 5%. Thus, desired poles are: $\lambda_1 = -5, \lambda_2 = -1, \lambda_3 = -1$. Finally, control gains are

$$\begin{aligned} \mathbf{K}_x &= \begin{bmatrix} 1.8 & 0.8 & 0.4 \end{bmatrix}, \\ \mathbf{K}_r &= \begin{bmatrix} 0.3333 \end{bmatrix}. \end{aligned}$$

Figure 1 shows step response of state-space close loop system. The settling time is close to 3.8 seconds and %PO is %0.

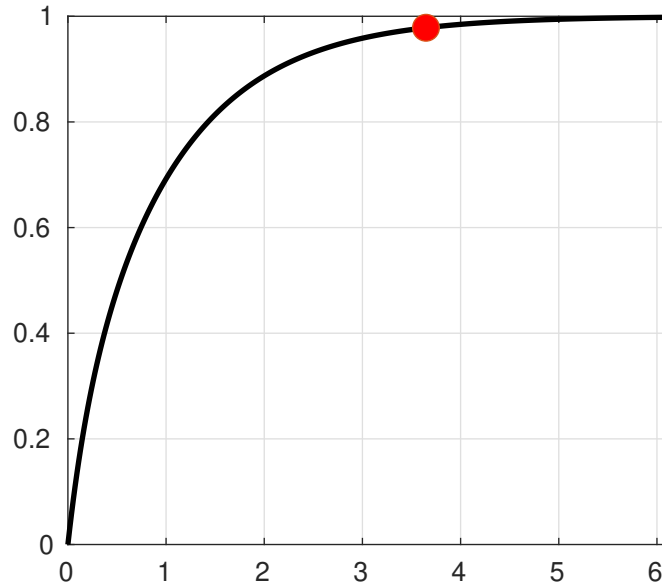


Figure 1: Step response. Settling time is represented with red dot.

4 Question 4

(a) Considering transfer function

$$G = \frac{10}{s^3 + 10s^2 + 16s}, \quad (4)$$

then, state-state representation in canonical form is

$$A_c = \begin{bmatrix} -10 & -16 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, B_c = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, C_c = \begin{bmatrix} 0 & 0 & 10 \end{bmatrix}, D_c = \begin{bmatrix} 0 \end{bmatrix},$$

and state-state representation in observable form is

$$A_o = \begin{bmatrix} -10 & 1 & 0 \\ -16 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B_o = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}, C_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, D_o = \begin{bmatrix} 0 \end{bmatrix}.$$

(b) The controller gains can be $k_c = \begin{bmatrix} -47.1 & 5.84 & -0.66 \end{bmatrix}$ and observer gains are $k_o = \begin{bmatrix} 0.25 & 59.5 & 212 \end{bmatrix}$.

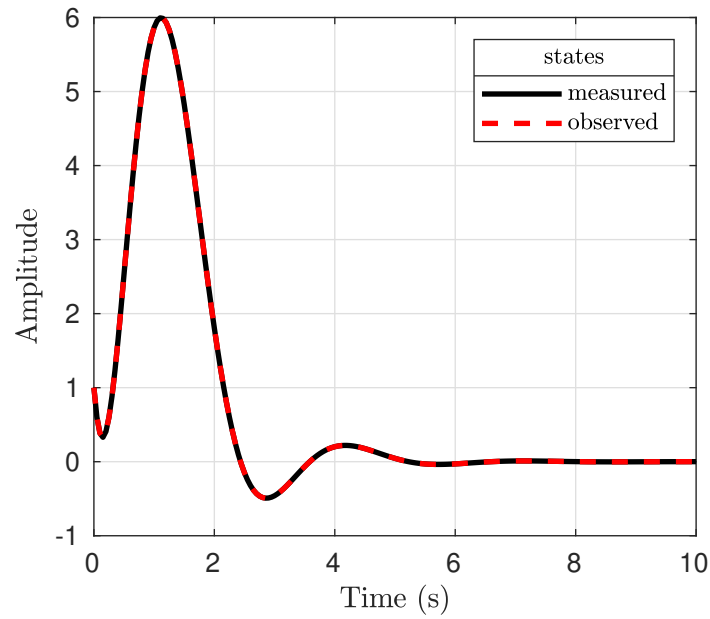


Figure 2: Close loop system with controller poles $s = -1.4, s = -1 \pm 2.15i$ and observer poles $s = -4.25, s = -3 \pm 6.4i$.

(c) Figure 2 shows the performance of close loop system with controller and observer when initial condition is $\mathbf{x} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$

5 Question 5

(a) Transfer function is given by

$$T(s) = \frac{k_p 65}{s^2 + (37 + 65k_v)s + k_p 65} \quad (5)$$

(b) Natural frequency $w_n = \sqrt{k_p 65}$, damping ratio $\zeta = \frac{37 + 65k_v}{2\sqrt{k_p 65}}$