Comparison of Results on the 2019 CEC Competition on Constrained Real Parameter

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The 28 benchmarked constrained optimization problems can be transformed into the following format:

$$\text{Minimize: } f(X), \quad X = (x_1, x_2, ..., x_n) \text{ and } X \in S \tag{1}$$

$$\text{Subject to: } \begin{cases} g_i(X) \le 0, & i = 1, ..., p \\ h_j(X) = 0, & j = p+1, ..., m \end{cases} \tag{2}$$

Equality constraints are transformed into inequalities of the form

$$|h_j(X)| - \varepsilon \le 0, \text{ for } \quad j = p+1, ..., m \tag{3}$$

Solution X is regarded as feasible if $g_i(X) \le 0, i = 1, ..., p$ and

$|h_j(X)| - \varepsilon \le 0$, for $j = p+1, ..., m$ (ε=0.0001).

Reference:

Guohua Wu, R. Mallipeddi, P. N. Suganthan, "Problem Definitions and Evaluation Criteria for the CEC 2017 Competition and Special Session on Constrained Single Objective Real-Parameter Optimization", Technical Report, Nanyang Technological University, Singapore, December 2016.

Problem Properties

Problem/Search Range	Type of Objective	Number of Constraints	
		E	*I*
C01 $[-100,100]^D$	Non Separable	0	1 Separable
C02 $[-100,100]^D$	Non Separable, Rotated	0	1 Non Separable, Rotated
C03 $[-100,100]^D$	Non Separable	1 Separable	1 Separable
C04 $[-10,10]^D$	Separable	0	2 Separable
C05 $[-10,10]^D$	Non Separable	0	2 Non Separable, Rotated
C06 $[-20,20]^D$	Separable	6	0 Separable
C07 $[-50,50]^D$	Separable	2 Separable	0
C08 $[-100,100]^D$	Separable	2 Non Separable	0
C09 $[-10,10]^D$	Separable	2 Non Separable	0

Problem Properties

Problem/Search Range	Type of Objective	Number of Constraints	
		E	*I*
C10 $[-100,100]^D$	Separable	2 Non Separable	0
C11 $[-100,100]^D$	Separable	1 Non Separable	1 Non Separable
C12 $[-100,100]^D$	Separable	0	2 Separable
C13 $[-100,100]^D$	Non Separable	0	3 Separable
C14 $[-100,100]^D$	Non Separable	1 Separable	1 Separable
C15 $[-100,100]^D$	Separable	1	1
C16 $[-100,100]^D$	Separable	1 Non Separable	1 Separable
C17 $[-100,100]^D$	Non Separable	1 Non Separable	1 Separable
C18 $[-100,100]^D$	Separable	1	2 Non Separable

Problem Properties

Problem/Search Range	Type of Objective	Number of Constraints	
		E	*I*
C19 $[-50,50]^D$	Separable	0	2 Non Separable
C20 $[-100,100]^D$	Non Separable	0	2
C21 $[-100,100]^D$	Rotated	0	2 Rotated
C22 $[-100,100]^D$	Rotated	0	3 Rotated
C23 $[-100,100]^D$	Rotated	1 Rotated	1 Rotated
C24 $[-100,100]^D$	Rotated	1 Rotated	1 Rotated
C25 $[-100,100]^D$	Rotated	1 Rotated	1 Rotated
C26 $[-100,100]^D$	Rotated	1 Rotated	1 Rotated
C27 $[-100,100]^D$	Rotated	1 Rotated	2 Rotated
C28 $[-50,50]^D$	Rotated	0	2 Rotated

Evaluation Criteria

(1) Rank all algorithms on one problem with multiple runs

- The procedure for ranking algorithms based on mean values:
  - ① Rank the algorithms based on feasibility rate;
  - ② Then rank the algorithms according to the mean violation amounts;
  - ③ At last, rank the algorithms in terms of mean objective function value.
- The procedure for ranking the algorithms based on the median solutions:
  - ① A feasible solution is better than an infeasible solution;
  - ② Rank feasible solutions based on their objective function values;
  - ③ Rank infeasible solutions according to their constraint violation amounts.

Evaluation Criteria

(2) Rank all algorithms on multiple problems

For each problem, algorithm ranks are determined in terms of the mean values and median solutions at the maximum allowed number of evaluations, respectively. The total rank value of each algorithm is calculated as below.

$$\text{Rank value} = \sum_{i=1}^{28} rank_i(\text{using mean value}) + \sum_{i=1}^{28} rank_i(\text{using median solution})$$

Algorithms

CAL-SHADE	Ales Zamuda. Adaptive Constraint Handling and Success History Differential Evolution for CEC 2017 Constrained Real-Parameter Optimization, CEC 2017
LSHADE44 + IDE	Josef Tvrdik and Radka Polakova. A Simple Framework for Constrained Problems with Application of L-SHADE44 and IDE, CEC 2017
LSHADE44	Radka Polakova. L-SHADE with Competing Strategies Applied to Constrained Optimization, CEC 2017
UDE	Anupam Trivedi, Krishnendu Sanyal, Pranjal Verma and Dipti Srinivasan. A Unified Differential Evolution Algorithm for Constrained Optimization Problems, CEC 2017
MA-ES	Michael Hellwig and Hans-Georg Beyer. A Matrix Adaptation Evolution Strategy for Constrained Real-parameter Optimization(2018)
IUDE	Anupam Trivedi, Dipti Srinivasan and Nimagna Biswas. An Improved Unified Differential Evolution Algorithm for Constrained Optimization Problems(2018)
LSHADE-IEpsilon	Zhun Fan et. AL. LSHADE with an Improved ε Constraint-handling Method for Solving Constrained Single-objective Optimization Problems(2018)
L-SHADE+BOC	Takeshi Kawachi, Jun-ichi Kushida, Akira Hara, Tetsuyuki Takahama, L-SHADE with an Adaptive Penalty Method of Balancing the Objective Value and the Constraint Violation(GECCO 2019)
HECO-DE	Tao Xu, Jun He and Changjing Shang. Helper and Equivalent Objective Different Evolution for Constrained Optimisation(GECCO 2019)

Ranks of the algorithms

Table 1 Ranks based on mean values on the 28 functions of 10 dimensions

Problems	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
CAL-SHADE	1	8	6	7	9	8	9	1	7	9	9	6	9	9	9	8	1	1	8	6	7	9	8	9	1	7	9	9	**182**
LSHADE44 + IDE	3	1	8	4	7	5	8	4	4	2	3	8	7	7	5	9	7	3	1	8	4	7	5	8	4	4	2	3	**128**
LSHADE44	8	1	7	8	8	6	7	3	2	3	5	7	6	8	6	7	8	8	1	7	8	8	6	7	3	2	3	5	**142**
UDE	1	7	4	5	5	7	6	4	9	6	8	5	1	5	8	6	5	1	7	4	5	5	7	6	4	9	6	8	**137**
MA-ES	9	6	9	9	1	9	1	9	8	7	4	9	5	1	7	2	6	9	6	9	9	1	9	1	9	8	7	4	**127**
IUDE	6	1	1	3	1	4	5	4	6	1	7	1	1	1	3	5	4	6	1	1	3	1	4	5	4	6	1	7	**81**
LSHADE-IEpsilon	4	9	3	6	1	2	2	4	5	4	6	4	8	4	4	1	9	4	9	3	6	1	2	2	4	5	4	6	**104**
L-SHADE+BOC	5	1	5	1	6	3	4	4	1	8	1	2	1	6	2	4	3	5	1	5	1	6	3	4	4	1	8	1	**82**
HECO-DE	7	1	2	2	1	1	3	2	3	5	1	3	1	1	1	3	2	7	1	2	2	1	1	3	2	3	5	1	**65**

Ranks of the algorithms

Table 2 Ranks based on median solution on the 28 functions of 10 dimensions

Problems	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
CAL-SHADE	1	1	9	9	1	7	5	9	9	1	1	2	1	6	7	9	9	9	1	7	4	1	8	9	9	9	9	1	**154**
LSHADE44 + IDE	1	1	8	5	1	9	6	1	1	2	1	3	1	9	6	8	5	8	5	4	5	1	9	8	8	7	8	3	**134**
LSHADE44	1	1	7	6	1	8	7	1	1	2	1	9	1	8	8	7	6	6	3	2	1	1	7	7	7	5	7	8	**129**
UDE	1	1	4	7	1	6	4	1	1	2	9	1	1	1	1	5	8	7	5	9	1	1	1	1	5	8	6	3	**101**
MA-ES	1	1	1	8	1	1	1	1	1	2	1	4	1	3	9	1	7	1	5	8	6	1	3	6	1	6	1	7	**89**
IUDE	1	1	1	1	1	1	8	1	1	2	1	7	1	1	1	1	1	5	5	6	1	1	1	1	1	1	5	3	**61**
LSHADE-IEpsilon	1	1	5	4	1	1	3	1	1	2	7	5	1	3	1	1	4	3	4	5	8	1	5	1	1	4	2	9	**85**
L-SHADE+BOC	1	1	6	1	1	1	2	1	1	2	1	6	1	7	1	6	2	2	5	1	9	1	4	1	6	3	4	6	**83**
HECO-DE	1	1	1	1	1	1	9	1	1	2	8	8	1	3	1	1	3	4	2	3	7	1	6	1	1	2	3	2	**76**

Ranks of the algorithms

Table 3 Ranks of the methods on problems of 10 dimensions based on mean value and median solution

	CAL-SHADE	LSHADE44 + IDE	LSHADE44	UDE	MA-ES	IUDE	LSHADE-IEpsilon	L-SHADE+BOC	HECO-DE
Total rank values	336	262	271	238	216	142	189	165	141
Rank	**9**	**7**	**8**	**6**	**5**	**2**	**4**	**3**	**1**

Ranks of the algorithms

Table 4 Ranks based on mean values on the 28 functions of 30 dimensions

Problems	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
CAL-SHADE	1	1	9	9	9	4	8	1	9	1	4	7	9	4	8	9	9	9	1	4	4	9	9	9	9	9	8	1	**174**
LSHADE44 + IDE	1	1	8	6	1	9	6	9	1	9	3	6	6	9	6	7	6	8	5	5	8	5	8	8	7	6	9	7	**170**
LSHADE44	1	1	7	3	1	8	5	2	1	2	2	4	4	8	7	8	5	6	3	2	7	6	7	7	8	5	7	8	**135**
UDE	1	1	3	8	7	3	2	8	1	8	8	8	7	5	5	5	7	7	8	8	5	4	5	5	4	7	6	5	**151**
MA-ES	1	1	1	7	1	2	1	2	1	2	1	9	1	6	9	1	8	2	9	9	9	1	6	6	1	8	1	4	**110**
IUDE	1	1	4	5	1	6	7	2	1	2	5	1	3	1	4	3	2	5	7	7	6	3	4	3	3	2	5	6	**100**
LSHADE-IEpsilon	1	1	6	2	1	7	4	2	1	2	9	5	8	3	3	4	4	1	4	3	3	7	2	4	5	4	2	9	**107**
L-SHADE+BOC	1	1	5	1	1	5	3	2	1	2	6	2	5	7	1	6	3	4	6	1	2	8	1	1	6	1	4	3	**89**
HECO-DE	1	1	1	3	8	1	9	2	1	2	7	3	1	2	2	1	1	3	2	6	1	2	3	2	1	3	3	2	**74**

Ranks of the algorithms

Table 5 Ranks based on median solution on the 28 functions of 30 dimensions

Problems	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
CAL-SHADE	1	1	9	9	1	5	8	1	1	1	4	7	9	5	8	9	9	8	1	4	1	9	6	9	9	9	7	1	**152**
LSHADE44 + IDE	1	1	8	3	1	9	7	9	2	9	3	1	1	9	6	7	7	9	4	5	8	6	9	8	7	5	9	7	**161**
LSHADE44	1	1	7	4	1	8	6	2	2	2	2	6	7	8	7	8	6	5	3	2	7	7	8	6	8	7	8	8	**147**
UDE	1	1	3	8	1	4	2	8	2	8	8	8	6	4	5	4	8	7	4	8	1	5	1	5	4	8	6	5	**135**
MA-ES	1	1	1	7	1	3	1	2	2	2	1	9	1	6	9	1	5	1	9	9	9	1	7	7	1	6	1	4	**108**
IUDE	1	1	4	6	1	7	5	2	2	2	5	2	1	1	3	3	2	6	4	7	1	1	1	2	3	1	5	6	**85**
LSHADE-IEpsilon	1	1	6	2	1	6	4	2	2	2	9	3	8	2	4	5	4	2	4	3	6	8	4	4	5	2	2	9	**111**
L-SHADE+BOC	1	1	5	1	1	1	3	2	2	2	6	4	1	7	1	6	3	4	4	1	1	1	3	1	6	3	4	3	**78**
HECO-DE	1	1	1	4	1	1	9	2	2	2	7	5	1	3	2	1	1	3	2	6	1	4	5	3	1	4	3	2	**78**

Ranks of the algorithms

Table 6 Ranks of the methods on problems of 30 dimensions based on mean value and median solution

	CAL-SHADE	LSHADE44 + IDE	LSHADE44	UDE	MA-ES	IUDE	LSHADE-IEpsilon	L-SHADE+BOC	HECO-DE
Total rank values	326	331	282	286	218	185	218	167	152
Rank	**8**	**9**	**6**	**7**	**4**	**3**	**4**	**2**	**1**

Ranks of the algorithms

Table 7 Ranks based on mean values on the 28 functions of 50 dimensions

Problems	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
CAL-SHADE	9	9	9	9	8	3	8	9	9	7	5	6	9	4	9	9	9	8	1	4	5	8	9	9	9	9	7	1	**201**
LSHADE44 + IDE	8	1	8	5	1	8	6	8	7	9	1	4	6	9	6	7	7	7	4	5	7	4	8	8	8	7	8	8	**175**
LSHADE44	1	1	7	2	1	7	5	1	3	1	2	8	5	8	7	8	6	6	3	2	8	5	6	7	7	6	6	7	**136**
UDE	1	1	3	8	9	2	2	7	1	8	6	5	8	5	5	4	8	9	7	7	2	7	5	5	4	8	9	5	**151**
MA-ES	1	1	1	7	1	1	1	2	8	1	3	9	7	7	8	1	5	1	9	9	6	3	7	6	1	4	1	4	**115**
IUDE	1	1	4	6	1	9	7	4	1	1	7	3	4	1	3	3	2	5	6	8	3	2	4	3	3	1	5	6	**104**
LSHADE-IEpsilon	1	1	6	4	7	6	4	6	6	6	8	7	2	2	4	5	4	2	5	3	4	9	2	4	6	5	2	9	**130**
L-SHADE+BOC	1	1	5	1	1	4	3	2	3	1	4	2	3	6	1	6	3	4	8	1	9	1	1	1	5	3	4	2	**86**
HECO-DE	1	1	1	2	1	5	9	5	3	5	9	1	1	3	2	1	1	3	2	6	1	6	3	2	1	2	3	3	**83**

Ranks of the algorithms

Table 8 Ranks based on median solution on the 28 functions of 50 dimensions

Problems	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
CAL-SHADE	1	1	8	9	1	5	7	9	7	7	5	6	9	4	9	9	9	9	1	4	5	8	6	9	9	9	7	1	**174**
LSHADE44 + IDE	1	1	9	2	1	8	6	8	9	9	3	3	7	9	6	8	7	8	5	5	8	6	9	8	8	7	9	8	**178**
LSHADE44	1	1	7	3	1	7	5	1	3	1	1	9	6	8	8	7	6	6	4	2	9	7	8	7	7	6	6	7	**144**
UDE	1	1	3	8	9	4	2	7	1	8	6	5	8	5	5	4	8	7	9	7	1	5	1	6	4	8	8	5	**146**
MA-ES	1	1	1	7	1	3	1	2	8	1	2	8	1	6	7	1	4	1	8	9	7	4	7	5	1	4	1	4	**106**
IUDE	1	1	4	6	1	9	8	4	1	1	7	4	1	2	2	3	1	5	2	8	3	3	1	2	3	1	5	6	**95**
LSHADE-IEpsilon	1	1	6	1	1	6	4	6	3	6	8	7	5	1	4	5	5	2	6	3	4	9	5	4	6	5	2	9	**125**
L-SHADE+BOC	1	1	5	5	1	1	3	2	3	1	4	2	1	7	1	6	3	4	7	1	6	2	1	1	5	3	4	3	**84**
HECO-DE	1	1	1	3	1	2	9	5	3	5	9	1	1	3	3	1	2	3	3	6	2	1	4	3	1	2	3	2	**81**

Ranks of the algorithms

Table 9 Ranks of the methods on problems of 50 dimensions based on mean value and median solution

	CAL-SHADE	LSHADE44 + IDE	LSHADE44	UDE	MA-ES	IUDE	LSHADE-IEpsilon	L-SHADE+BOC	HECO-DE
Total rank values	375	353	280	297	221	199	255	170	164
Rank	**9**	**8**	**6**	**7**	**4**	**3**	**5**	**2**	**1**

Ranks of the algorithms

Table 10 Ranks based on mean values on the 28 functions of 100 dimensions

Problems	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
CAL-SHADE	8	8	9	9	6	2	9	9	7	8	5	6	8	2	9	9	9	9	1	2	4	6	9	9	9	9	7	1	**189**
LSHADE44 + IDE	9	9	7	3	4	8	5	6	2	7	1	2	4	9	4	7	7	7	4	5	5	5	6	6	8	7	8	6	**161**
LSHADE44	1	1	6	2	3	7	6	2	1	1	4	9	5	8	7	8	5	6	3	4	8	4	8	5	7	5	6	8	**140**
UDE	7	7	3	8	9	3	2	8	9	9	6	3	9	6	6	3	8	8	8	8	3	9	2	7	3	8	9	5	**176**
MA-ES	1	1	1	6	8	1	1	3	8	4	2	8	2	7	8	1	4	1	9	9	7	1	7	3	1	4	2	4	**114**
IUDE	1	1	8	7	5	9	7	4	5	5	3	4	7	1	3	4	2	5	6	7	1	7	5	4	5	2	5	7	**130**
LSHADE-IEpsilon	6	6	5	5	7	5	4	7	6	6	8	5	3	4	5	5	6	2	5	3	2	8	4	8	6	6	3	9	**149**
L-SHADE+BOC	1	1	4	4	1	6	3	1	4	2	7	1	6	5	1	6	3	4	7	1	9	3	1	1	4	3	4	2	**95**
HECO-DE	1	1	1	1	1	4	8	5	3	3	9	7	1	3	2	1	1	3	2	6	6	2	3	2	2	1	1	3	**83**

Ranks of the algorithms

Table 11 Ranks based on median solution on the 28 functions of 100 dimensions

Problems	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
CAL-SHADE	8	8	9	9	6	3	9	9	5	8	5	5	8	1	9	9	9	9	1	2	2	6	6	9	9	9	9	1	**183**
LSHADE44 + IDE	9	9	8	1	3	8	7	6	2	7	4	3	5	9	5	7	7	6	4	5	6	5	7	7	8	7	6	7	**168**
LSHADE44	1	1	7	2	4	7	8	2	1	1	2	9	6	8	8	8	5	7	3	4	7	4	9	8	7	4	7	8	**148**
UDE	7	7	3	8	9	2	3	8	7	9	6	2	9	6	7	3	8	8	8	8	4	9	3	5	3	8	8	5	**173**
MA-ES	1	1	1	6	8	1	2	3	9	5	1	7	3	7	1	1	4	1	9	9	5	2	8	3	1	5	1	4	**109**
IUDE	1	1	6	7	5	9	6	4	8	4	3	4	7	2	6	4	2	5	7	7	1	7	5	4	5	2	5	6	**133**
LSHADE-IEpsilon	6	6	5	4	7	4	5	7	6	6	8	6	4	5	1	5	6	2	6	3	3	8	4	6	6	6	4	9	**148**
L-SHADE+BOC	1	1	4	5	1	5	4	1	4	2	7	1	2	3	1	6	3	4	5	1	9	3	1	1	4	3	3	2	**87**
HECO-DE	1	1	1	2	1	6	1	5	3	3	9	8	1	4	1	1	1	3	2	6	8	1	2	2	2	1	2	3	**81**

Ranks of the algorithms

Table 12 Ranks of the methods on problems of 100 dimensions based on mean value and median solution

	CAL-SHADE	LSHADE44 + IDE	LSHADE44	UDE	MA-ES	IUDE	LSHADE-IEpsilon	L-SHADE+BOC	HECO-DE
Total rank values	372	329	288	349	223	263	297	182	164
Rank	**9**	**7**	**5**	**8**	**3**	**4**	**6**	**2**	**1**

Ranks of the algorithms

Table 13 Ranks of the methods on problems of all considered dimensions

	CAL-SHADE	LSHADE44 + IDE	LSHADE44	UDE	MA-ES	IUDE	LSHADE-IEpsilon	L-SHADE+BOC	HECO-DE
Total rank values	1409	1275	1121	1170	878	789	959	684	621
Rank	**9th**	**8th**	**6th**	**7th**	**4th**	**3rd**	**5th**	**2nd**	**1st**