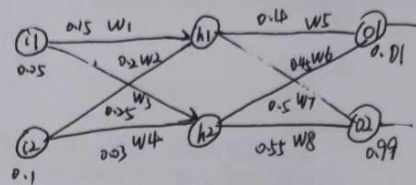
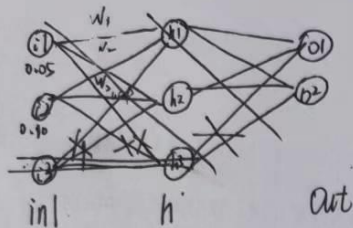


推理: 输入数据, 对输入神经元乘以相应权重, 经过隐藏神经元, 输出层神经元, 最后得到值与期望值相乘。

训练: 从结果向前推算计算输入神经元的权重(反向传播)



前向传播:

神经元  $h_1$  加权输入和  $z_{h_1}$ ,  $z_{h_2}$

$$z_{h_1} = i_1 * w_1 + i_2 * w_2 + i_3 * w_3 = 0.05 * 0.5 + 0.1 * 0.2 + 0.1 * 0.05 = 0.0275$$

$$z_{h_2} = i_1 * w_3 + i_2 * w_4 = 0.05 * 0.05 + 0.1 * 0.3 = 0.0155$$

神经元  $h_1$  的输出  $a_{h_1}$ ,  $a_{h_2}$

$$a_{h_1} = \frac{1}{1 + e^{-z_{h_1}}} = \frac{1}{1 + e^{-0.0275}} = 0.506874568$$

$$a_{h_2} = \frac{1}{1 + e^{-z_{h_2}}} = \frac{1}{1 + e^{-0.0155}} = 0.5038749224$$

输出层神经元  $o_1$  和  $o_2$  的值.

$$z_{o_1} = a_{h_1} * w_5 + a_{h_2} * w_6 = 0.506874568 * 0.4 + 0.5038749224 * 0.01 = 0.20275$$

$$z_{o_2} = a_{h_1} * w_7 + a_{h_2} * w_8 = 0.506874568 * 0.05 + 0.5038749224 * 0.3 = 0.15155$$

$$a_{o_1} = \frac{1}{1 + e^{-z_{o_1}}} = \frac{1}{1 + e^{-0.20275}} = 0.5605752244$$

$$a_{o_2} = \frac{1}{1 + e^{-z_{o_2}}} = \frac{1}{1 + e^{-0.15155}} = 0.6296156936$$

前向传播结果  $[0.5605752244, 0.6296156936]$

实际值  $[0.01, 0.99]$

> 相差较大, 进行反向传播, 调整权重值.

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反向传播

1. 计算损失函数

$$E_{total} = \sum \frac{1}{2} (target - output)^2$$

$$E_{o1} = \frac{1}{2} (0.01 - 0.751365)^2 = 0.274811$$

$$E_{o2} = \frac{1}{2} (0.99 - 0.772728485)^2 = 0.02356026$$

$$E_{total} = E_{o1} + E_{o2} = 0.274811 + 0.02356026 = 0.29837136$$

隐藏层 → 输出层的权值更新

$$2. \frac{\partial E_{total}}{\partial W_5} = \frac{\partial E_{total}}{\partial a_{o1}} \cdot \frac{\partial a_{o1}}{\partial z_{h1}} \cdot \frac{\partial z_{h1}}{\partial W_5}$$

$$\frac{\partial E_{total}}{\partial a_{o1}} = 2 \cdot \frac{1}{2} (target - a_{o1}) * (-1)$$

$$= (0.01 - 0.751365) * (-1) = 0.741365$$

$$\frac{\partial a_{o1}}{\partial z_{h1}} = a_{o1} \cdot (1 - a_{o1}) = 0.2388163613$$

$$\frac{\partial z_{h1}}{\partial W_5} = a_{h1} = 0.593201998 \cdot 0.5058745668$$

$$\frac{\partial E_{total}}{\partial W_5} = 0.07211583135$$

$$W_5^+ = W_5 - \frac{\partial E_{total}}{\partial W_5} * \eta = 0.3639420843$$

$$\frac{\partial E_{total}}{\partial W_b} = \frac{\partial E_{total}}{\partial a_{o1}} \cdot \frac{\partial a_{o1}}{\partial z_{h2}} \cdot \frac{\partial z_{h2}}{\partial W_b}$$

$$\frac{\partial E_{total}}{\partial a_{o1}} = \frac{1}{2} \cdot 2 \cdot (\text{target} - a_{o1}) \cdot (-1)$$

$$= 0.6057527244 - 0.01$$

$$= 0.5957527244$$

$$\frac{\partial a_{o1}}{\partial z_{h2}} = \frac{\cancel{0.6057527244}}{a_{o1} \cdot (1 - a_{o1})}$$

$$= 0.6057527244 \cdot (1 - 0.6057527244)$$

$$= 0.2391055791$$

$$\frac{\partial z_{h2}}{\partial W_b} = a_{h2} = 0.503849224 = 0.07177221358$$

$$\frac{\partial E_{total}}{\partial W_b} = 0.07177221358$$

$$W_b^+ = W_b - \eta \cdot \frac{\partial E_{total}}{\partial W_b} = 0.4141138932$$



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$$\frac{\partial E_{total}}{\partial W_7} = \frac{\partial E_{total}}{\partial a_{o2}} \cdot \frac{\partial a_{o2}}{\partial z_{h1}} \cdot \frac{\partial z_{h1}}{\partial W_7}$$

$$\frac{\partial E_{total}}{\partial a_{o2}} = \frac{1}{2} \cdot 2 \cdot (\text{target} - \text{output}) \cdot a_{o2} = -0.3603843064$$

$$\frac{\partial a_{o2}}{\partial z_{h1}} = a_{o2} \cdot (1 - a_{o2}) = 0.233199772$$

$$\frac{\partial a_{o2}}{\partial W_7} = a_{h1} = 0.5068745668$$

$$\frac{\partial E_{total}}{\partial W_7} = -0.0425985182$$

$$W_7^+ = W_7 - \eta \cdot \frac{\partial E_{total}}{\partial W_7} = 0.5 - 0.5 \times (-0.0425985182) = 0.5212992591$$

 $W_8^+$ 

$$\frac{\partial E_{total}}{\partial W_8} = \frac{\partial E_{total}}{\partial a_{o2}} \cdot \frac{\partial a_{o2}}{\partial z_{h2}} \cdot \frac{\partial z_{h2}}{\partial W_8}$$

$$\frac{\partial E_{total}}{\partial a_{o2}} = \frac{1}{2} \cdot 2 \cdot (\text{target} - a_{o2}) \cdot (-1) = 0.6296156936 - 0.99 = -0.3603843064$$

$$\frac{\partial a_{o2}}{\partial z_{h2}} = 0.233199772$$

$$\frac{\partial z_{h2}}{\partial W_8} = a_{h2} = 0.5038749224$$

$$\frac{\partial E_{total}}{\partial W_8} = -0.0423464394$$

$$W_8^+ = W_8 - \eta \times \frac{\partial E_{total}}{\partial W_8} = 0.5711732117$$

隐藏层 → 输入层权重值  $W_1^+$

$$\frac{\partial E_{total}}{\partial W_1} = \left( \frac{\partial E_{01}}{\partial a_{h1}} + \frac{\partial E_{02}}{\partial a_{h1}} \right) \cdot \frac{\partial a_{h1}}{\partial W_1}$$

$$\frac{\partial E_{01}}{\partial W_1} = \frac{\partial E_{01}}{\partial a_{h1}} \cdot \frac{\partial a_{h1}}{\partial z_{01}} \cdot \frac{\partial z_{01}}{\partial W_1}$$

$$\frac{\partial E_{total}}{\partial W_1} = \frac{\partial E_{total}}{\partial a_{h1}} \cdot \frac{\partial a_{h1}}{\partial z_{01}} \cdot \frac{\partial z_{01}}{\partial W_1}$$

$$= \left( \frac{\partial E_{01}}{\partial a_{h1}} + \frac{\partial E_{02}}{\partial a_{h1}} \right) \cdot \frac{\partial a_{h1}}{\partial z_{01}} \cdot \frac{\partial z_{01}}{\partial W_1}$$

$$= \left( \frac{\partial E_{01}}{\partial a_{01}} \cdot \frac{\partial a_{01}}{\partial z_{01}} \cdot \frac{\partial z_{01}}{\partial a_{h1}} + \frac{\partial E_{02}}{\partial a_{02}} \cdot \frac{\partial a_{02}}{\partial z_{02}} \cdot \frac{\partial z_{02}}{\partial a_{h1}} \right) \cdot \frac{\partial a_{h1}}{\partial z_{01}} \cdot \frac{\partial z_{01}}{\partial W_1}$$

$$= (0.575752744 \cdot 0.2388163613 \cdot 0.4 + (-0.360841064) \cdot 0.233389772 \cdot 0.45) \cdot 0.5068745668 \cdot (-0.5068745668) \cdot 0.05$$

$$= 0.01909150701$$

$$W_1^+ = W_1 - \eta \cdot \frac{\partial E_{total}}{\partial W_1} = 0.15 - 0.5 \times 0.01909150701 = 0.149880706$$

$$= 0.149880706$$

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$$\frac{\partial \bar{E}_{total}}{\partial W_2} = \frac{\partial \bar{E}_{total}}{\partial a_{h1}} \cdot \frac{\partial a_{h1}}{\partial z_{h1}} \cdot \frac{\partial z_{h1}}{\partial W_2}$$

$$= \left( \frac{\partial \bar{E}_{01}}{\partial a_{h1}} + \frac{\partial \bar{E}_{02}}{\partial a_{h1}} \right) \cdot \frac{\partial a_{h1}}{\partial z_{h1}} \cdot \frac{\partial z_{h1}}{\partial W_2}$$

$$= \left( \frac{\partial \bar{E}_{01}}{\partial a_{01}} \cdot \frac{\partial a_{01}}{\partial z_{01}} \cdot \frac{\partial z_{01}}{\partial a_{h1}} + \frac{\partial \bar{E}_{02}}{\partial a_{02}} \cdot \frac{\partial a_{02}}{\partial z_{02}} \cdot \frac{\partial z_{02}}{\partial a_{h1}} \right) \cdot \frac{\partial a_{h1}}{\partial z_{h1}} \cdot \frac{\partial z_{h1}}{\partial W_2}$$

$$= 0.1997614013$$

 $W_3^+$ 

$$\frac{\partial \bar{E}_{total}}{\partial W_3} = \frac{\partial \bar{E}_{total}}{\partial a_{h2}} \cdot \frac{\partial a_{h2}}{\partial z_{h2}} \cdot \frac{\partial z_{h2}}{\partial W_3}$$

$$= \left( \frac{\partial \bar{E}_{01}}{\partial a_{h2}} + \frac{\partial \bar{E}_{02}}{\partial a_{h2}} \right) \cdot \frac{\partial a_{h2}}{\partial z_{h2}} \cdot \frac{\partial z_{h2}}{\partial W_3}$$

$$= \left( \frac{\partial \bar{E}_{01}}{\partial a_{01}} \cdot \frac{\partial a_{01}}{\partial z_{01}} \cdot \frac{\partial z_{01}}{\partial a_{h2}} + \frac{\partial \bar{E}_{02}}{\partial a_{02}} \cdot \frac{\partial a_{02}}{\partial z_{02}} \cdot \frac{\partial z_{02}}{\partial a_{h2}} \right) \cdot \frac{\partial a_{h2}}{\partial z_{h2}} \cdot \frac{\partial z_{h2}}{\partial W_3}$$

$$= (0.0711377489) +$$

$$= 0.02491490299 \cdot (0.503874924 \cdot (1 - 0.503874924)) \cdot 0.05$$

$$= 5.96071525 \times 10^{-4}$$

$$W_3^+ = W_3 - \eta \cdot \frac{\partial \bar{E}_{total}}{\partial W_3} = 0.2497019642$$





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$W_4^+$

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$$\frac{\partial E_{total}}{\partial W_4} = \frac{\partial E_{01}}{\partial W_4} + \frac{\partial E_{02}}{\partial W_4}$$

$$= \frac{\partial E_{total}}{\partial a_{n2}} \cdot \frac{\partial a_{n2}}{\partial z_{n2}} \cdot \frac{\partial z_{n2}}{\partial W_4}$$

$$= \left( \frac{\partial E_{01}}{\partial a_{n2}} + \frac{\partial E_{02}}{\partial a_{n2}} \right) \cdot \frac{\partial a_{n2}}{\partial z_{n2}} \cdot \frac{\partial z_{n2}}{\partial W_4}$$

$$= \left( \frac{\partial E_{01}}{\partial a_{01}} \cdot \frac{\partial a_{01}}{\partial z_{01}} \cdot \frac{\partial z_{01}}{\partial a_{n2}} + \frac{\partial E_{02}}{\partial a_{02}} \cdot \frac{\partial a_{02}}{\partial z_{02}} \cdot \frac{\partial z_{02}}{\partial a_{n2}} \right) \cdot \frac{\partial a_{n2}}{\partial z_{n2}} \cdot \frac{\partial z_{n2}}{\partial W_4}$$

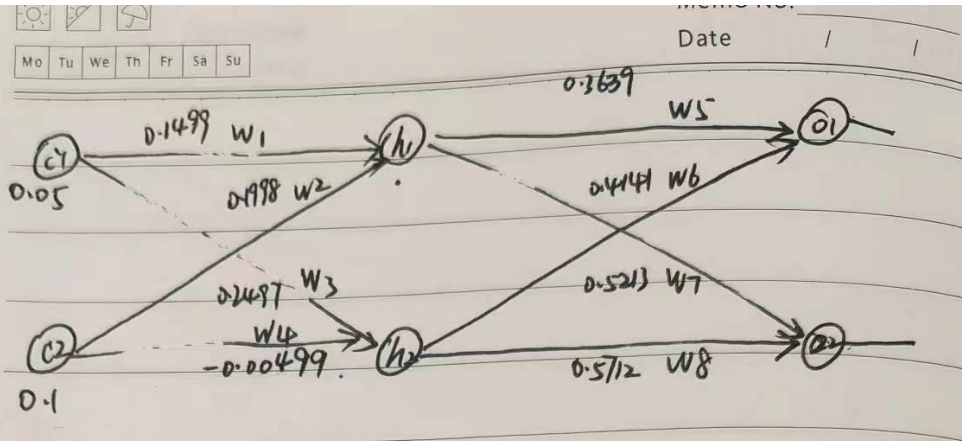
$$= (0.595752744 \cdot 0.2588163613 \cdot 0.5 - 0.3603842064 \cdot 0.221177172 \cdot 0.55)$$

$$= 0.5038749224 \cdot (-0.5038749224) \cdot 0.1$$

$$= 0.00998224719$$

$$W_4^+ = W_4 - \eta \cdot \frac{\partial E_{total}}{\partial W_4} = 0.03 - 0.5 \times 0.00998224719$$

$$= -0.04991123595$$



$$Z_{h1} = 0.05 \times 0.1499 + 0.1 \times 0.1998 = 0.009493$$

$$A_{h1} = \frac{1}{1 + e^{-0.009493}} = 0.5023724822$$

$$Z_{h2} = 0.05 \times 0.2497 + 0.1 \times (-0.00499) = 0.011986$$

$$A_{h2} = \frac{1}{1 + e^{-0.011986}} = 0.5029964641$$

$$Z_{o1} = 0.5023724822 \times 0.3639 + 0.5029964641 \times 0.4141$$

$$= 0.3911041821$$

$$A_{o1} = \frac{1}{1 + e^{-0.3911041821}} = 0.5965484804$$

$$Z_{o2} = 0.5023724822 \times 0.5213 + 0.5029964641 \times 0.5712$$

$$= 0.5491983553$$

$$A_{o2} = \frac{1}{1 + e^{-0.5491983553}} = 0.6339495833$$