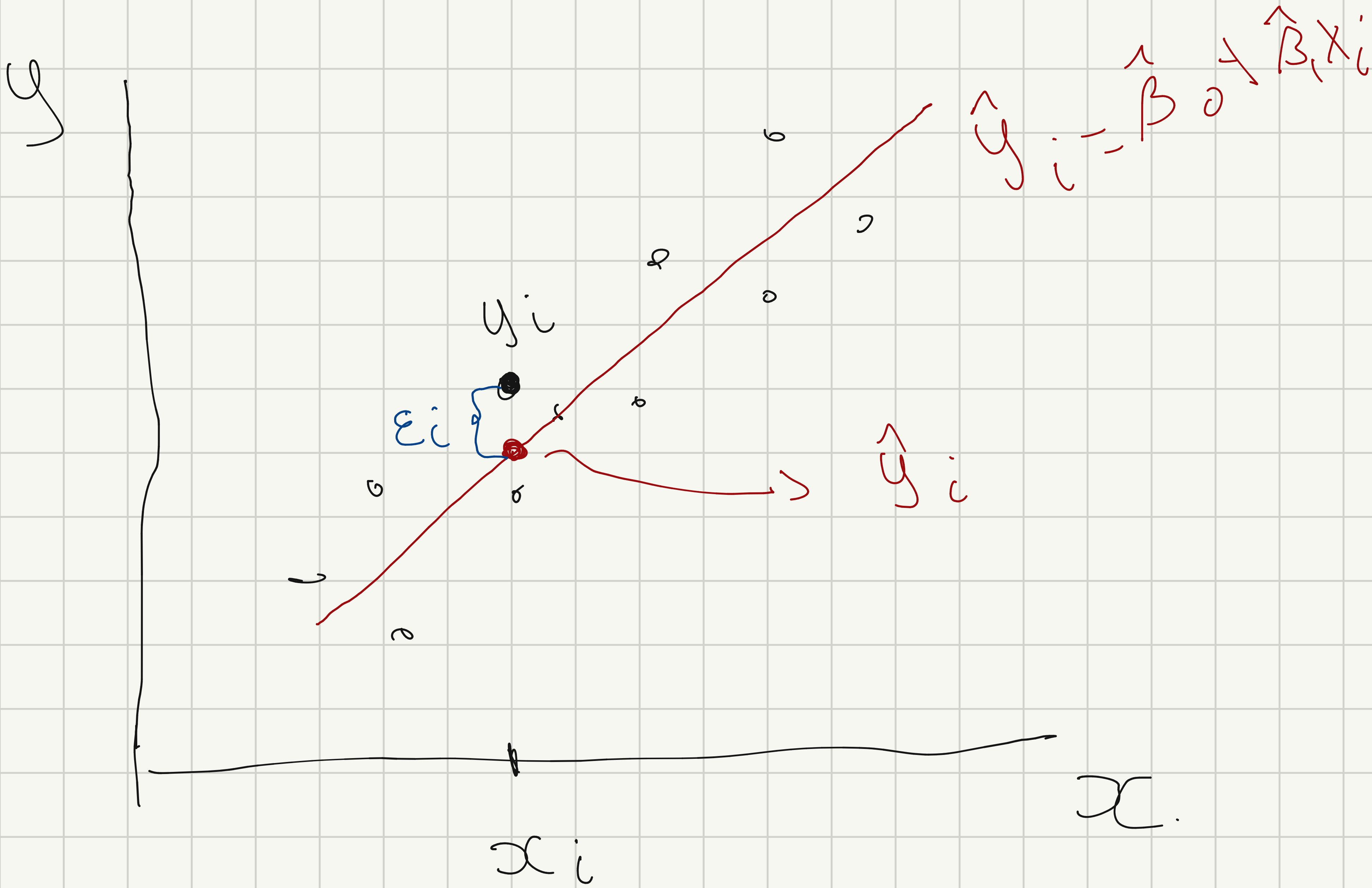

$$S(\beta_0,\beta_1) = \sum_{i=1}^{n} \left[Y_i - (\beta_0 + \beta_1 X_i) \right]^2$$

(1) $$\frac{\partial S(\beta_0,\beta_1)}{\partial \beta_0} = -2 \sum_{i=1}^{n} \left[Y_i - (\beta_0 + \beta_1 X_i) \right]$$

(2) $$\frac{\partial S(\beta_0,\beta_1)}{\partial \beta_1} = -2 \sum_{i=1}^{n} X_i \left[Y_i - (\beta_0 + \beta_1 X_i) \right]$$

De (1) $$\sum_{i=1}^{n} \left[Y_i - (\beta_0 + \beta_1 X_i) \right] = 0$$

$$\sum_{i=1}^{n} Y_i - \underline{n\beta_0} - \beta_1 \sum X_i = 0$$

$$\sum_{i=1}^{n} Y_i - \beta_1 \sum X_i = n\beta_0$$

$$\bar{Y} - \beta_1 \bar{X} = \beta_0$$

$$\therefore \boxed{\bar{y} - \hat{\beta}_1 \bar{x} = \hat{\beta}_0} \leftarrow (1)$$

(1) en (2)

$$\sum_{i=1}^{n} x_i \left[Y_i - \left[(\bar{Y} - \beta_1 \bar{X}) + \beta_1 X_i \right] \right] = 0$$

$$\Rightarrow \sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \bar{Y} + \beta_1 \bar{X} \sum X_i - \beta_1 \sum X_i^2 = 0$$

$$\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \bar{Y} = \beta_1 \sum X_i^2 - \beta_1 \bar{X} \sum X_i$$

$$\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \bar{Y} = \beta_1 \left(\sum_{i=1}^{n} X_i^2 - \bar{X} \sum_{i=1}^{n} X_i \right)$$

$$\frac{\sum_{i=1}^{n} X_i Y_i - \sum X_i \bar{Y}}{\sum_{i=1}^{n} X_i^2 - \bar{X} \sum_{i=1}^{n} X_i} = \beta_1$$

$$\frac{\sum X_i Y_i - \sum X_i \sum Y_i / n}{\sum X_i^2 - \bar{X} \sum X_i} = \beta_1$$

$$\frac{\sum Y_i (X_i - \bar{X})}{\underbrace{\sum X_i^2 - \bar{X} \sum X_i}} = \beta_1$$

En la presentación el denominador es $\sum_{i=1}^{n} (X_i - \bar{X})^2$

$$\sum_{i=1}^{n} (X_i - \bar{X})^2 = \sum_{i=1}^{n} (X_i^2 - 2 X_i \bar{X} + \bar{X}^2)$$

$$= \sum_{i=1}^{n} X_i^2 - \underbrace{n 2 \bar{X} \frac{\sum X_i}{n}}_{2n\bar{X}^2} + n\bar{X}^2$$

$$= \sum_{i=1}^{n} X_i^2 - n\bar{X}^2 = \sum_{i=1}^{n} X_i^2 - \cancel{n} \frac{\sum X_i}{\cancel{n}} \bar{X}$$

$$= \sum_{i=1}^{n} X_i^2 - \sum_{i=1}^{n} X_i \bar{X}$$

$$\therefore \boxed{\hat{\beta}_1 = \frac{\sum_{i=1}^{n} y_i (x_i - \bar{x})}{\sum x_i^2 - \bar{x} \sum x_i}}$$