$$Y_{ijk} = \overbrace{\mu + \alpha_i + \beta_j}^{\mu_{ij}} + \varepsilon_{ijk} \quad , \quad \varepsilon_{ijk} \overset{iid}{\sim} N(0,\sigma^2)$$

$$\sum_{i=1}^{a} \alpha_i = \sum_{j=1}^{b} \beta_j = 0$$

Los EMCO $\hat{\mu} = \overline{Y}_{\bullet\bullet\bullet}$ $\quad \hat{\alpha}_i = \overline{Y}_{i\bullet\bullet} - \overline{Y}_{\bullet\bullet\bullet}$

$$\hat{\beta}_j = \overline{Y}_{\bullet j \bullet} - \overline{Y}_{\bullet\bullet\bullet}$$

$$\Rightarrow \quad E(Y_{ijk}) = \mu_{ij} \Rightarrow \hat{\mu}_{ij} = \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j$$

$$\Rightarrow \hat{\mu}_{ij} = \cancel{\overline{Y}_{\bullet\bullet\bullet}} + \overline{Y}_{i\bullet\bullet} - \cancel{\overline{Y}_{\bullet\bullet\bullet}} + \overline{Y}_{\bullet j \bullet} - \overline{Y}_{\bullet\bullet\bullet}$$

$$= \underbrace{\overline{Y}_{i\bullet\bullet} + \overline{Y}_{\bullet j \bullet}} - \overline{Y}_{\bullet\bullet\bullet}$$

↳ estas medias no son estadística/ indep/.

$$\mathrm{Var}(\hat{\mu}_{ij}) = \mathrm{Var}(\bar{Y}_{i..}) + \mathrm{Var}(\bar{Y}_{.j.}) + \mathrm{Var}(\bar{Y}_{...})$$

$$+ 2\,\mathrm{cov}(\bar{Y}_{i..}, \bar{Y}_{.j.}) - 2\,\mathrm{cov}(\bar{Y}_{i..}, \bar{Y}_{...})$$

$$- 2\,\mathrm{cov}(\bar{Y}_{.j.}, \bar{Y}_{...})$$

$$= ?? = \frac{\sigma (a+b-1)}{abn}$$

$$\hat{\mu}_{ij} \sim N\left(\mu + \alpha_i + \beta_j, \ \sigma^2 \frac{(a+b-1)}{abn}\right)$$

ANOVA en un DCA balanceado, con dos factores de efectos aleatorios con interacción

Fuente	g.l	SC	CM	CME	F_0	Valor P
A	$a-1$	SSA	$MSA = \frac{SSA}{a-1}$	$\underline{\sigma^2 + n\sigma^2_{\alpha\beta}} + nb\sigma^2_{\alpha}$	$\frac{MSA}{MS(AB)}$	$P(f_{a-1,\text{dfi}} > F_0)$
B	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$\sigma^2 + n\sigma^2_{\alpha\beta} + na\sigma^2_{\beta}$	$\frac{MSB}{MS(AB)}$	$P(f_{b-1,\text{dfi}} > F_0)$
AB	dfi	$SS(AB)$	$MS(AB) = \frac{SS(AB)}{\text{dfi}}$	$\sigma^2 + n\sigma^2_{\alpha\beta}$	$\frac{MS(AB)}{MSE}$	$P(f_{\text{dfi},\text{dfe}} > F_0)$
Error	dfe	SSE	$MSE = \frac{SSE}{\text{dfe}}$	σ^2		
Total	$abn-1$	SST	Con dfi $= (a-1)(b-1)$, dfe $= ab(n-1)$			

bajo $H_0: \sigma^2_\alpha = 0$

$$E(MSA) = E(MS(AB)) = \sigma^2 + n\sigma^2_{\alpha\beta}$$

$$\rightarrow F_0 = \frac{MSA}{MS(AB)}$$

Observe que,

bajo: $H_0: \sigma^2_{\alpha\beta} = 0 \longrightarrow E(MS(AB)) = E(MSE) = \sigma^2$

$$\Rightarrow F_0 = \frac{MS(AB)}{MSE}$$

bajo H_0: $H_0: \sigma^2_\beta = 0 \longrightarrow E(MSB) = E(MS(AB)) = \sigma^2 + n\sigma^2_{\alpha\beta}$

$$\Rightarrow F_0 = \frac{MSB}{MS(AB)}$$