Daniela Pico

1. considere el vector

$$\underline{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ -1 \\ -3 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} \underline{X}^{(1)} \\ \hline \underline{X}^{(2)} \end{bmatrix} \qquad \text{donde } X^{(1)} = [X_1] \quad \underline{X}^{(2)} = \begin{bmatrix} X_4 \\ X_2 \end{bmatrix}$$

$$\underline{u}^{(1)} = -1 \qquad \Sigma = \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{14} \\ \theta_{12} & \theta_{22} & \theta_{24} \\ \theta_{14} & \theta_{42} & \theta_{44} \end{bmatrix}$$

$$\underline{u}^{(2)} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & \big| & 0 & 2 \\ \hline 0 & \big| & 1 & 0 \\ 2 & \big| & 0 & 4 \end{bmatrix} \qquad \boxed{\Sigma_{11} = 3 \qquad \Sigma_{12} = [0 \ 2]}$$
$$\boxed{\Sigma_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \quad \Sigma_{21} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}}$$

b) Hallar la distribución z

$$\underline{z} = \begin{bmatrix} -4X_4 & -2X_2 \\ 4X_4 & +2X_2 \end{bmatrix} \qquad \overline{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -4X_4 & -2X_2 \\ 4X_4 & +2X_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 0 & -4 \\ 0 & 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

donde $\overline{z} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \sim N(AM, A\Sigma A^t)$

$$\overline{A}u = \begin{bmatrix} 0 & -2 & 0 & -4 \\ 0 & 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 18 \\ -18 \end{bmatrix}$$

$$A\Sigma A^T = \begin{bmatrix} 0 & -2 & 0 & -4 \\ 0 & 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -2 & 2 \\ 0 & 0 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 68 & -68 \\ -68 & 68 \end{bmatrix}$$

$$\boxed{\overline{z} \sim N_2 \left(\begin{bmatrix} 18 \\ -18 \end{bmatrix}, \begin{bmatrix} 68 & -68 \\ -68 & 68 \end{bmatrix} \right)}$$

c) Distribución condicional $\underline{X}^{(1)} \mid \underline{X}^{(2)} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$$\underline{X}^{(1)} \mid \underline{X}^{(2)} = \underline{X}^{(1)} = N_2\left[\mu^{(1)} + \Sigma_{12}\,\Sigma_{22}^{-1}\,(\underline{X}^2 - \underline{\mu}^2),\ \Sigma_{11} - \Sigma_{12}\,\Sigma_{22}^{-1}\,\Sigma_{21} \right]$$

$$= \underline{X}^{(1)} \mid \underline{X}^{(2)} = N_2\left(-1 + \underbrace{[0\ 2]}_{A}\ \underbrace{\begin{bmatrix} 1.0 & 0 \\ 0 & 0.25 \end{bmatrix}}_{B}\ \underbrace{\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} -3 \\ 3 \end{bmatrix}\right)}_{c},\ \underbrace{3}_{d} - \underbrace{[0\ 2]}_{A}\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0.25 \end{bmatrix}}_{B}\underbrace{\begin{pmatrix} 0 \\ 2 \end{pmatrix}}_{c} \right)$$

$$\boxed{= \underline{X}^{(1)} \mid \underline{X}^{(2)} \sim N_2\left(\tfrac{1}{2},\ 2 \right)}$$

d) Distribución condicional $\underline{X}^{(2)} \mid \underline{X}^{(1)} = 3$

$$X^{(2)} \mid X^{(1)} \sim N_1\left(\mu^{(2)} + \Sigma_{21}\,\Sigma_{11}^{-1}\,(X^{(1)} - \mu^{(1)}),\ \Sigma_{22} - \Sigma_{21}\,\Sigma_{11}^{-1}\,\Sigma_{12} \right)$$

$$X^{(2)} \mid X^{(1)} \sim N_1\left(\underbrace{\begin{bmatrix} -3 \\ -3 \end{bmatrix}}_{A} + \underbrace{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}_{b}\, 0.3333\,\underbrace{(3 - (-1))}_{c},\ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}}_{d} - \underbrace{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}_{b}\, 0.3333\,\underbrace{[0\ 2]}_{c\ \ B'} \right)$$

$$\boxed{X^{(2)} \mid X^{(1)} \sim N_2\left(\begin{bmatrix} -3 \\ -0.336 \end{bmatrix},\ \begin{bmatrix} 1 & 0 \\ 0 & 2.6668 \end{bmatrix} \right)}$$