Part 1. Solve for the following questions. Use the appropriate relationship in probability in answering.

1. A coin is flipped once. What is the probability of getting heads?
2. If $P(A) = 0.45$, what is $P(A^c)$?
3. If $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \cap B) = 0.1$, find $P(A \cup B)$.
4. Two events are disjoint. If $P(A) = 0.2$ and $P(B) = 0.25$, find $P(A \cup B)$.
5. A bag has 12 balls: 5 red, 4 blue, and 3 green. If one ball is drawn at random, what is $P(\text{blue})$?

Part 2. Using Bayes' Theorem, solve for the following.

1. A factory has two machines. Machine A produces 60% of the items with a defect rate of 2%. Machine B produces 40% of the items with a defect rate of 4%. If a randomly chosen item is defective, what is the probability it came from Machine B?
2. In a certain city, 1% of people have a particular disease. A test detects the disease correctly 95% of the time, but gives a false positive 2% of the time. If a person tests positive, what is the probability they actually have the disease?

Part 3. Using the appropriate probability distribution (mean, variance, and binomial distribution, look for the following.

1. A fair die is rolled. Let X be the outcome. Construct the probability distribution of X and find $P(X \leq 3)$.
2. The probability distribution of a discrete random variable X is:

$$X : 1 \quad 2 \quad 3 \quad 4$$

$$P(X) : 0.2 \quad 0.3 \quad 0.4 \quad 0.1$$

Find the **mean** and **variance** of X.

Part 4. Give what is asked in each item. Show your solution.

1. A license plate consists of two letters, followed by three digits, and then one more letter. If the first letter must be a vowel (A, E, I, O, U) and repetition is allowed, how many unique license plates are possible?
2. How many ways can a president, vice president, and treasurer be chosen from a group of 15 people?
3. How many distinct arrangements can be made using the letters of the word 'BOOKKEEPER'?
4. A student must answer 5 questions out of 8 on an exam. How many different selections of questions can the student make?
5. In a survey of 100 students, 60 like math, 45 like science, and 20 like both. How many students like neither math nor science?