Scientific Computing for DPhil Students I
Assignment 1

Due at lecture at 9:00 on Tuesday, 23 October 2018.
This is the first of four assignments this term.
It will count for only 5% of the overall mark for the term.

In this as in all assignments in this course, the answers you turn in should be at-tractive and brief but complete, and should include program listings and plots where appropriate, as well as appropriate discussion. It is ok to discuss problems in a general way with other students, but your programs and your writeups must be your own.

1. Consider the 4×4 matrix
    $$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \\ 1 & 4 & 2 & 3 \\ 4 & 1 & 2 & 3 \end{pmatrix}.$$
    Use Matlab to compute A^2, A^{-1}, and $\det(A)$. Also compute the eigenvalues of A and verify that their product equals $\det(A)$.

2. Compute the exponential e^A of the same matrix A by two methods: (a) using Matlab's `expm` command, and (b) by writing a `for` loop that sums the Taylor series $e^A = I + A + A^2/2! + \cdots$ to sufficiently many terms. How closely do the two results agree?

3. Write a program that creates the following anonymous functions to generate matrices for a given positive integer N:

    ```
    D = @(N) sparse(toeplitz([2 -1 zeros(1,N-2)]));
    I = @(N) speye(N);
    A = @(N) kron(I(N),kron(I(N),D(N))) + kron(I(N),kron(D(N),I(N))) ...
                + kron(D(N),kron(I(N),I(N)));
    ```

    What is the dimension of $A(N)$ as a function of N? List $A(2)$ explicitly. Draw a spy plot of $A(4)$. Now make another anonymous function `b = @(N) ...` to create a column vector with the same number of rows as $A(N)$, with all entries equal to 1. Apply these functions and Matlab's `pcg` code to solve $Ax = b$ for $N = 40$ to high accuracy. How many iterations does it take? How long does it take on your computer? What are the first five entries of the solution vector x?