Scientific Computing for DPhil Students I
Assignment 4

Due at the Andrew Wiles reception before 11:00 on Tuesday morning, 4 Dec. 2018. No papers will be accepted after this hour, and solutions will be posted at this time. This is the last of four assignments this term. See you again next term! — Tuesdays Weeks 1–7 and Thursdays Weeks 1–3 and 5–6 at 11:00 in L3.

(If you will be out of town, you can submit your assignment as a pdf file to our TA Abi Gopal at `gopal@maths.ox.ac.uk` before the due hour. Please do not do this unless it is really necessary.)

Warning: this assignment is harder than the previous ones. Use whatever tools you like, which might include `fminsearch` or `fminunc` in Matlab, for example. You may find it helpful to make exploratory plots along the way to get a sense of where the solution may lie. If you prefer to use a language other than Matlab, that is ok.

1. *A degenerate eigenvalue problem.* Consider the 10×10 matrices $D = \mathrm{diag}\,(1,\ldots,10)$, $T = \mathrm{tridiag}\,(1,2,1)$, and $G = S^T S$, where $s_{i,j} = \sin(i\cdot j)$, and define

$$A(g,t) = D + gG + tT.$$

What is the smallest λ which is a <u>double</u> eigenvalue of $A(g,t)$ for some $g > 0$ and $t > 0$? Try to get the right answer to a number of digits of precision, and turn in plots and discussion as appropriate to persuade us that you've got it right. *(Hint: One approach is to look for minima of the function* `sep(g,t) = min(diff(sort(eig(A(g,t)))))`*.*

This problem exemplifies the effect of "level repulsion" in eigenvalue problems, which goes back to von Neumann and Wigner in 1929. A beautiful paper on this subject is by Berry and Wilkinson in *Proc. Roy. Soc. A,* 1984. These references won't help you do the problem; I just mention them for interest.

2. *A maximization problem.* Consider the function

$$f(x,y) = x\,e^{-r^2}\sin(5(\theta + r))$$

where $r = \sqrt{x^2+y^2}$ and $\theta = \tan^{-1}(y/x)$. If we throw a needle of length 1 on the x-y plane, what is the maximum difference in values that f can take at the two ends of the needle? Be sure to explain clearly what you have done and why you think you've found the maximum. Aim for at least six digits of accuracy.