Toward Robust Vehicle Platooning With Bounded Spacing Error

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***Abstract*—Intelligent transportation has become an essential field of cyber-physical systems. Among various intelligent transportation technologies, the automated highway system (AHS) has its unique advantage of being able to coordinate a platoon of vehicles as a whole unit. The major challenge of building a robust AHS is the nonlinear and (potentially) fast time-varying uncertainty induced by parameter variations and external disturbances. Finally, reflected as the spacing between neighboring vehicles, such uncertainties can be a serious concern for maintaining safety. This paper addresses the problem by proposing a mathematical transformation scheme to bound the spacing error and build a distributed control algorithm on such a basis. The propose algorithm achieves a spacing error satisfying both uniformly boundedness and uniformly ultimate boundedness. Our decentralized algorithm is communication efficient in the sense that it only requires the state information of the preceding car and the acceleration feedback and does not need to communicate with all other cars.**

***Index Terms*—Collision avoidance, strict spacing bound, string stability, vehicle platooning.**

I. Introduction

INTELLIGENT transportation system (ITS) has become one of the important applications of cyber-physical systems (CPSs) [1]–[3]. ITS applications are enabled by the advances in both vehicle-to-vehicle (V2V) and vehicle-to-infrastructure communications. Vehicles and the supporting infrastructure constitute a cooperative system in which the users share information and collaborate with each other to improve characteristics such as safety, fuel economy, traffic efficiency and riding comfortableness. Among various ITS technologies, automated highway system (AHS) is a promising technology by organizing vehicles into a platoon with a closer intervehicle spacing and then coordinating the platoon as a single unit [4]–[6]. Vehicle platooning can be defined as a collection of vehicles that travel together and actively coordinated in formation [7]. It has been shown that AHS significantly reduces the fuel consumption of heavy duty vehicles [8] and increases safety by over 90% in accidents due to human errors [9], [10].

The vehicle platooning technology has gained tremendous attention in the ITS research community. In the past 30 years, well-known projects of vehicle platooning, such as SARTRE (a European platooning project) [11], PATH (a California traffic automation program that includes platooning) [12], [13], Energy ITS (a Japanese truck platooning project) [14], SCANIA-platooning (Heavy Duty Vehicle Platoon by Scania and KTH) [15], and KONVOI (German truck platooning project) [16], have received significant progress. In a vehicle platoon, vehicles automatically follow the lead vehicle both laterally and longitudinally. Vehicles may dynamically join or leave the platoon (e.g., leave on arriving at the desired destination). V2V communication allows the sharing of vehicle states such as speed and sensor data among vehicles in the platoon. The shared data are used by the control algorithm of vehicles. The platoon actually forms a cooperative system where sensing, control and actuation are distributed throughout the vehicles in the platoon.

One key component for vehicle platooning is the control algorithm that coordinates the vehicles in a line. Much work has been done on the control algorithm. An excellent overview of research achievements is given in [17]. Solving the robust control problem using sliding mode controllers was also discussed in [18]. Recently, a Lp string stability of cascaded systems was applied to vehicle platooning in [19]. A distributed framework for coordinated heavy-duty vehicle platooning is introduced in [20]. Among these, the string stability in leader-follower platoons has been a topic of great concern. It has proved to be an important tool in analyzing the stability of platoons of vehicles. Intuitively, string stability implies uniform boundedness of all states of an interconnected system at all times, given that the initial states of the interconnected system are uniformly bounded [21]. A vehicle platoon is an interconnected system in which the stability of any single vehicle is not sufficient to guarantee the boundedness of the spacing errors for all the vehicles [22]. Under this setting, the concept of string stability is introduced to describe the performance of interconnected system [23]. A sufficient condition for string stability is that under a zero initial condition the spacing error should not amplify downstream from one vehicle to another [24]. A large body of research work (see [25]–[27])

Manuscript received July 5, 2015; revised October 31, 2015 and February 25, 2016; accepted April 29, 2016. Date of publication May 18, 2016; date of current version March 17, 2017. This work was supported by the National Natural Science Foundation of China under Grant 11302074. This paper was recommended by Associate Editor A. Zomaya. *(Corresponding author: Yangdong Deng.)*

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Digital Object Identifier 10.1109/TCAD.2016.2570431

has been dedicated to improving string stability by utilizing platoon information (e.g., acceleration feedback function). Nevertheless, the string stability does not formally guarantee collision avoidance of the platoon (i.e., no collision at any time). In addition, the requirement of zero initial condition, which suggest that the initial condition of a platoon is collision free (i.e., be able to avoid collision in operation) [28], [29], is not practical in real applications. Besides, a shorter spacing between vehicles in a platoon increases the risk of collision under systematic and external disturbances [30]–[32]. Such disturbances come from parameter variations (e.g., the mass variation and the movement of the passengers and/or the stowage), aerodynamics (e.g., the unmeasurable side wind), and environmental factors (e.g., the rolling resistance and the slope of road), which tend to be nonlinear and (potentially) fast time-varying. Besides the collision of a single vehicle, it is desirable for the controller to bound the spacing error of a platoon under the presence of uncertainty from the perspective of robustness.

In this paper, we propose a decentralized control algorithm for the vehicle platoon. With the state of the preceding vehicles and the bounds of the uncertainties as the input, the algorithm is guaranteed to maintain collision avoidance and string stability under practical initial conditions. State transformations for the error dynamics are devised so that the bounded error intervals of vehicles can be mapped into a single state in the transformed space. A back-stepping based scheme is developed to make the error dynamics practically stable and suppress the nonlinearity of uncertainties. We use two classes of transformations to illustrate the proposed approach.

The main contribution of this paper is trifold. First, a novel decentralized control algorithm as well its formal framework is proposed for vehicle platooning. With the algorithm, a vehicle only needs to know the state information of its preceding vehicle. Such a relaxation significantly lowers the demand for communication bandwidth. In addition, the proposed algorithm relaxes the initial condition for each vehicle in the platoon and only requires that there is no vehicle collision at the beginning. Second, a complete worst case analysis shows that the string stability can be guaranteed under the uniform boundedness. Third, the proposed algorithm is proven to be able to tolerate nonlinear uncertainties.

II. Problem Formulation

A vehicle platoon is a group of vehicles that travels in close proximity to one another, nose-to-tail, at highway speeds. Fig. 1 demonstrates the idea of a vehicle platoon [33]. Except the lead vehicle, every vehicle in a platoon automatically tries to maintain the specified gap with its preceding vehicle and follow the trajectory specified by the lead vehicle. Another aim is to maintain a sufficient gap to discourage interference from other vehicles. The goal of longitudinal control, e.g., the speed of the following vehicles, is to make coordinated movements accurate and safe.

Consider a platoon of automated vehicles traveling in the same lane on a highway, as shown in Fig. 2. The position of the leading vehicle and the ith following vehicle in the platoon

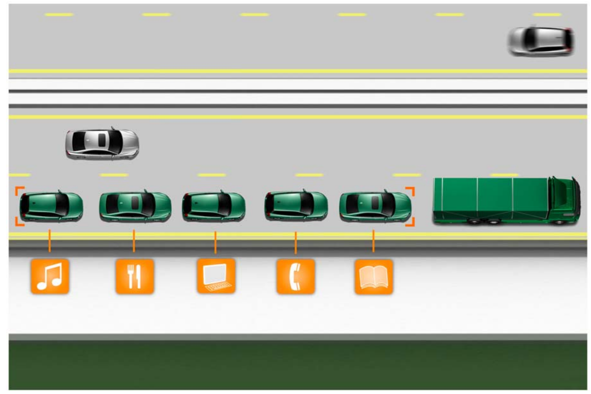

Fig. 1. Vehicle platooning as shown by the SARTRE demo [33].

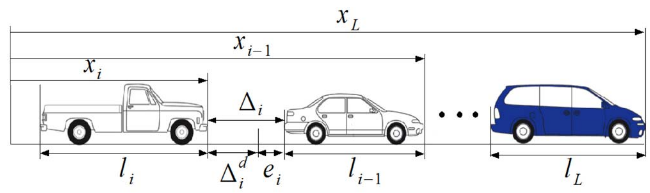

Fig. 2. Definitions of intervehicle spacing for a platoon of vehicles.

are denoted by x_L and x_i with respect to an inertial frame, respectively. The actual space Δ_i between the ith vehicle and its preceding one is given by

$$\Delta_i(t) = x_{i-1}(t) - x_i(t) - l_{i-1}, \quad i = 2, 3, \ldots, n \tag{1}$$

where l_{i-1} is the length of the of the $i-1$th vehicle in the platoon, n is the total vehicle number in the platoon. Henceforth, subscripts L and i are used to label the leader and the ith follower in the platoon, respectively.

Suppose the desired spacing between the ith and $i-1$th vehicle is denoted by a constant scalar Δ_i^d. Then, the spacing error is given by

$$e_i(t) = \Delta_i^d - \Delta_i(t) = \Delta_i^d + x_i(t) - x_{i-1}(t) + l_{i-1}. \tag{2}$$

Notice that the critical point for a collision to happen at time t_s is $\Delta_i(t_s) = 0$, i.e., $e_i(t_s) = \Delta_i^d$. The following definition related to the platoon performance is needed for later development.

A platoon of controlled vehicles are string stable if given any $\gamma > 0$, there exists a $\delta > 0$ such that [23]

$$\max\left\{\sup_i |e_i(t_0)|, \sup_i |\dot{e}_i(t_0)|\right\} < \delta \tag{3}$$

implies

$$\sup_i \|e_i(t)\|_\infty < \gamma \tag{4}$$

for all $t > t_0$. A sufficient condition for the platoon to be string stable is that with zero initial errors [24]

$$\|e_i(t)\|_\infty \leq \|e_{i-1}(t)\|_\infty \tag{5}$$

for all i. This requires the spacing errors in the platoon attenuate or at least should not amplify as they propagated down to the platoon.

It is worth pointing out here that collision avoidance (16) is the first and foremost thing which should be guaranteed for a platoon of vehicles. However, as it was demonstrated in [34] that string stability alone does not provide a formal warranty of collision avoidance among vehicles. From the practical design point of view, one should first guarantee that the platoon is collision avoidable. Based on this premise, a further consideration is that the spacing error should not amplify or could attenuate as they propagated down to the platoon.

The longitudinal equation of motion for the ith vehicle is determined by Newton's second law as

$$M_i(x_i(t), \sigma_i(t), t)\ddot{x}_i(t) = u_i(t) - c_i(x(t), \dot{x}_i(t), \sigma_i(t), t) \times \dot{x}_i(t)|\dot{x}_i(t)| - F_i(x_i(t), \dot{x}_i(t), \sigma_i(t), t) \tag{6}$$

where t is the time variable, σ_i, which may be a vector in form, represents all the uncertainty in the vehicle, u_i is the control input which is from the vehicle propulsive/braking effort. Moreover, $M_i(x_i, \sigma_i, t)$ represents the vehicle mass, $-c_i(x, \dot{x}_i, \sigma_i, t)\dot{x}_i|\dot{x}_i|$ represents the aerodynamic drag force, $-F_i(x_i, \dot{x}_i, \sigma_i, t)$ represents the rolling resistance force and other external disturbances acting on the vehicle.

By differentiating (2) with respect to time, we get the spacing error velocity and spacing error dynamics as follows:

$$\dot{e}_i(t) = \dot{x}_i(t) - \dot{x}_{i-1}(t) \tag{7}$$

$$\ddot{e}_i(t) = \ddot{x}_i(t) - \ddot{x}_{i-1}(t). \tag{8}$$

Substitute (6) into (8), we can get (henceforth, arguments of functions are sometimes omitted when no confusion is likely to arise)

$$\ddot{e}_i = M_i^{-1}(u_i - c_i\dot{x}_i|\dot{x}_i| - F_i) - M_{i-1}^{-1}(u_{i-1} - c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - F_{i-1}). \tag{9}$$

There are (possible) time-varying uncertainties in the vehicle systems and we suppose the scalars M_i, c_i, and F_i in (6) can be decomposed as follows:

$$\begin{aligned} M_i(x_i, \sigma_i, t) &= \bar{M}_i(x_i, t) + \Delta M_i(x_i, \sigma_i, t) \\ c_i(x_i, \dot{x}_i, \sigma_i, t) &= \bar{c}_i(x_i, \dot{x}_i, t) + \Delta c_i(x_i, \dot{x}_i, \sigma_i, t) \\ F_i(x_i, \dot{x}_i, \sigma_i, t) &= \bar{F}_i(x_i, \dot{x}_i, t) + \Delta F_i(x_i, \dot{x}_i, \sigma_i, t) \end{aligned} \tag{10}$$

where $\bar{M}_i$, $\bar{c}_i$, $\bar{F}_i$, denote the "nominal" portions with $\bar{M}_i > 0$ (this is always feasible since the nominal portion is the designer's discretion), while ΔM_i, Δc_i and Δg_i are the uncertain portions. The functions $\bar{M}_i(\cdot)$, $\Delta M_i(\cdot)$, $\bar{c}_i(\cdot)$, $\Delta c_i(\cdot)$, $\bar{F}_i(\cdot)$, and $\Delta F_i(\cdot)$ are all continuous.

Denote

$$D_i(x_i, t) := \bar{M}_i^{-1}(x_i, t) \tag{11}$$

$$\Delta D_i(x_i, \sigma_i, t) := M_i^{-1}(x_i, \sigma_i, t) - \bar{M}_i^{-1}(x_i, t) \tag{12}$$

$$E_i(x_i, \sigma_i, t) := \bar{M}_i(x_i, t)M_i^{-1}(x_i, \sigma_i, t) - 1. \tag{13}$$

It follows that $\Delta D_i(x_i, \sigma_i, t) = D_i(x_i, t)E_i(x_i, \sigma_i, t)$.

The following standard assumptions are introduced to restrict the structure of the uncertainty of system (6).

Assumption 1: There exists a known function $\rho_{E_i}(\cdot) : \mathbf{R} \times \mathbf{R} \to (-1, \infty)$ such that

$$E_i(x_i, \sigma_i, t) \geq \rho_{E_i}(x_i, t) \tag{14}$$

for all $(x_i, t) \in \mathbf{R} \times \mathbf{R}$, $\sigma_i \in \Sigma_i$.

In the special case that $M_i = \bar{M}_i$ (i.e., no uncertainty in the ith vehicle mass), we get $E_i = 0$. Hence one can choose $\rho_{E_i} = 0$. The assumption imposes the effect of uncertainty on the possible deviation of M_i from $\bar{M}_i$ to be within a unidirectional threshold.

Assumption 2: There exists a known function $\Pi_i(\cdot) : \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \to \mathbf{R}_+$ such that for all $(x_i, x_{i-1}, \dot{x}_i, \dot{x}_{i-1}, u_{i-1}, t) \in \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R}$, $\sigma_i \in \Sigma_i$

$$\begin{aligned} &\Pi_i(x_i, x_{i-1}, \dot{x}_i, \dot{x}_{i-1}, u_{i-1}, t) \\ &\quad \geq D_i \max_{\sigma_i \in \Sigma_i} \big\| \Delta D_i(-c_i\dot{x}_i|\dot{x}_i| - F_i + p_{i1} + p_{i2}) \\ &\qquad - \Delta D_{i-1}(u_{i-1} - C_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - F_{i-1}) \\ &\qquad + D_i(-\Delta c_i\dot{x}_i|\dot{x}_i| - \Delta F_i) \\ &\qquad - D_{i-1}(-\Delta c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - \Delta F_{i-1}) \big\|. \end{aligned} \tag{15}$$

The vehicle platooning problem is to design a control algorithm for vehicle i in (6) such that the resulting platoon system possesses the following performance: given any $\underline{\Delta}_i > 0$, $\bar{\Delta}_i > 0$, if $-\underline{\Delta}_i < e_i(t_0) < \bar{\Delta}_i$, then:

$$-\underline{\Delta}_i < e_i(t) < \bar{\Delta}_i \tag{16}$$

for all $t > t_0$.

III. Formal Framework for Designing Robust Platooning Control Algorithms

To resolve the problem formulated in the previous section, we develop a formal framework to transform the error space in this section. The mathematical framework enables us to develop a robust control algorithm that matches the requirement of the platoon system performance.

Step 1 (Bijective Function Selection): We first introduce a bijective function for later state transforming use.

Bijective Function: We propose a bijective function $g(\cdot) : \mathbf{R} \to \mathbf{R}$ which can be used to transform the spacing error from state e_i to z_i as follows:

$$z_i = g_i(e_i) \tag{17}$$

where

$$g_i(0) = 0, \quad \lim_{e_i \to -\underline{\Delta}_i} g_i(e_i) = -\infty, \quad \lim_{e_i \to \bar{\Delta}_i} g_i(e_i) = +\infty \tag{18}$$

and

$$\frac{\partial g_i}{\partial e_i} \neq 0 \tag{19}$$

for all $\underline{\Delta}_i < e_i < \bar{\Delta}_i$.

The existence of $g(\cdot)$ can be demonstrated by examples (20) followed and (54) in the experiments. The Bijective function also means that e_i with $e_i = g_i^{-1}(z_i)$ is one-to-one correspondence with z_i.

From (17), we have for all $-\underline{\Delta}_i < e_i < \bar{\Delta}_i$, the corresponding z_i will be $-\infty < z_i < \infty$. Furthermore, $z_i = 0 \Leftrightarrow e_i = 0$. This is the primary motivation for us to transform the system from state $e_i, \dot{e}_i$ to state z_{i1}, z_{i2}. If both $z_{i1}(t)$ and $z_{i2}(t)$ are bounded, then spacing error $e_i(t) \in (-\underline{\Delta}_i, \bar{\Delta}_i)$, hence strictly bounded. If the origin $z_{i1} = z_{i2} = 0$ of the transformed system is uniformly bounded, the spacing error e_i of the system will be located in $(-\underline{\Delta}_i, \bar{\Delta}_i)$. Hence collision avoidance guaranteed.

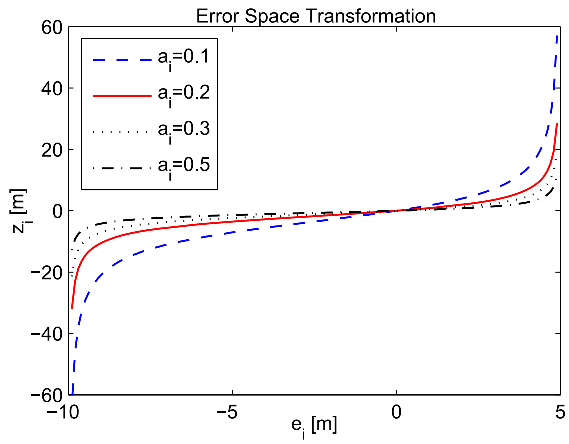

Fig. 3. Error space under algebraic transformation.

We take the a bijective function as follows for the purpose of illustration:

$$z_i = \frac{1}{a_i}\frac{e_i + \Delta_2}{\sqrt{\Delta_1^2 - (e_i+\Delta_2)^2}} - \frac{1}{a_i}\Delta_3 \tag{20}$$

where

$$\Delta_1 := \frac{1}{2}\left(\underline{\Delta}_i + \bar{\Delta}_i\right),\ \ \Delta_2 := \frac{1}{2}\left(\underline{\Delta}_i - \bar{\Delta}_i\right),\ \ \Delta_3 := \frac{\underline{\Delta}_i - \bar{\Delta}_i}{2\sqrt{\underline{\Delta}_i\bar{\Delta}_i}}.$$

Here $a_i > 0$ is a design parameter for the algebraic function. A numerical demonstration with different a_i is shown in Fig. 3.

For the bijective function proposed in (20), one can check that

$$z_i\big|_{e_i=0} = \frac{1}{a_i}\frac{\Delta_2}{\sqrt{\Delta_1^2-\Delta_2^2}} - \frac{1}{a_i}\Delta_3 = 0 \tag{21}$$

$$\lim_{e_i\to-\underline{\Delta}_i}\frac{1}{a_i}\frac{-\underline{\Delta}_i+\Delta_2}{\sqrt{\Delta_1^2-\left(-\underline{\Delta}_i+\Delta_2\right)^2}} - \frac{1}{a_i}\Delta_3 = -\infty \tag{22}$$

$$\lim_{e_i\to\bar{\Delta}_i}\frac{1}{a_i}\frac{\bar{\Delta}_i+\Delta_2}{\sqrt{\Delta_1^2-\left(\bar{\Delta}_i+\Delta_2\right)^2}} - \frac{1}{a_i}\Delta_3 = +\infty. \tag{23}$$

Hence (20) is a legitimate transformation. Moreover, the corresponding inverse transformation from z_i to e_i is given by

$$e_i = \Delta_1\frac{a_i z_i + \Delta_3}{\sqrt{1+(a_i z_i+\Delta_3)^2}} - \Delta_2 \tag{24}$$

where for all $-\infty < z_i < +\infty$, the spacing error $-\underline{\Delta}_i < e_i < \bar{\Delta}_i$.

Step 2 (State Transformation): Differentiating (17) with respect to time, we get

$$\dot{z}_i = \frac{\partial g_i}{\partial e_i}\dot{e}_i \tag{25}$$

$$\ddot{z}_i = \frac{\partial^2 g_i}{\partial e_i^2}\dot{e}_i^2 + \frac{\partial g_i}{\partial e_i}\ddot{e}_i. \tag{26}$$

Let

$$z_{i1} := z_i \tag{27}$$

$$z_{i2} := z_{i1} + \frac{\partial g_i}{\partial e_i}\dot{e}_i \tag{28}$$

yields

$$\begin{aligned}\dot{z}_{i1} &= -z_{i1} + z_{i2}\\ \dot{z}_{i2} &= \dot{z}_{i1} + \frac{\partial^2 g_i}{\partial e_i^2}\dot{e}_i^2 + \frac{\partial g_i}{\partial e_i}\ddot{e}_i.\end{aligned} \tag{29}$$

With (9) in (29), the transformed equation of motion is obtained

$$\dot{z}_{i1} = -z_{i1} + z_{i2} \tag{30}$$

$$\begin{aligned}\dot{z}_{i2} = &-z_{i1} + z_{i2} + \frac{\partial^2 g_i}{\partial e_i^2}\dot{e}_i^2\\ &+ \frac{\partial g_i}{\partial e_i}\Big[M_i^{-1}(u_i - c_i\dot{x}_i|\dot{x}_i| - F_i)\\ &\qquad - M_{i-1}^{-1}(u_{i-1} - c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - F_{i-1})\Big].\end{aligned} \tag{31}$$

The state z_{i2} given by (28) is selected to make the transformed equation of motion in a lower triangular form so that the backstepping method [35] can be adopted, which will be more convenient for programming in implementation.

Take the bijective function in (20) for example. Differentiate (20) with respect to time, and denote

$$z_{i1} = z_i \tag{32}$$

$$\begin{aligned}z_{i2} = &\, z_{i1} + \frac{1}{a_i}\Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{1}{2}}\dot{e}_i\\ &+ \frac{1}{a_i}\Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{3}{2}}(e_i+\Delta_2)^2\dot{e}_i\end{aligned} \tag{33}$$

we get

$$\dot{z}_{i1} = -z_{i1} + z_{i2} \tag{34}$$

$$\begin{aligned}\dot{z}_{i2} = &-z_{i1} + z_{i2} + \frac{3}{a_i}\Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{3}{2}}(e_i+\Delta_2)\dot{e}_i^2\\ &+ \frac{3}{a_i}\Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{5}{2}}(e_i+\Delta_2)^3\dot{e}_i^2\\ &+ \frac{1}{a_i}\Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{1}{2}}\ddot{e}_i\\ &+ \frac{1}{a_i}\Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{3}{2}}(e_i+\Delta_2)^2\ddot{e}_i.\end{aligned} \tag{35}$$

Plugging (9) into (35), the equation of motion with state space $\delta_i = [z_{i1}\ z_{i2}]^T$ is

$$\dot{z}_{i1} = -z_{i1} + z_{i2} \tag{36}$$

$$\begin{aligned}\dot{z}_{i2} = &-z_{i1} + z_{i2} + \frac{3}{a_i}\Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{3}{2}}(e_i+\Delta_2)\dot{e}_i^2\\ &+ \frac{3}{a_i}\Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{5}{2}}(e_i+\Delta_2)^3\dot{e}_i^2\\ &+ \frac{1}{a_i}\Bigg\{\Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{1}{2}}\\ &\qquad + \Big[\Delta_1^2 - (e_i+\Delta_2)^2\Big]^{-\frac{3}{2}}(e_i+\Delta_2)^2\Bigg\}\\ &\times\Big[M_i^{-1}(u_i - c_i\dot{x}_i|\dot{x}_i| - F_i)\\ &\qquad - M_{i-1}^{-1}(u_{i-1} - c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - F_{i-1})\Big].\end{aligned} \tag{37}$$

Algorithm 1: Robust Control Algorithm

Input: The initial state $x_i(o)$'s of the platooning, platooning parameters n, the bijective function g_i's, and the nominal parameters $\bar{M}_i$'s, $\bar{c}_i$'s, $\bar{F}_i$'s
Output: The controller output sequence

$x \leftarrow x(o)$
$t \leftarrow 0$
while *true* **do**
  if $n >= 2$ **then**
    get $x_i(t)$'s
    for $i \leftarrow 1$ ***to*** n **do**
      $e_i(t) \leftarrow \Delta_i^d + x_i(t) - x_{i-1}(t) + l_{i-1}$
      $\mu_i \leftarrow z_{i2} \frac{\partial g_i}{\partial e_i} \Pi_i$
      $p_{i1} \leftarrow \bar{c}_i \dot{x}_i |\dot{x}_i| + \bar{F}_i + D_i^{-1} D_{i-1}(u_{i-1}$
        $-\bar{c}_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - \bar{F}_{i-1})$
      $p_{i2} \leftarrow D_i^{-1} \left(\frac{\partial g_i}{\partial e_i}\right)^{-1} \left(-2z_{i2} - \frac{\partial^2 g_i}{\partial e_i^2} \dot{e}_i^2\right)$
      $p_{i3} \leftarrow -D_i^{-1} \frac{2(1+\rho_{E_i})^{-1}\mu_i \Pi_i}{\|\mu_i\| + \epsilon_i}$
      $u_i \leftarrow p_{i1} + p_{i2} + p_{i3}$

      if *the i'th vehicle quit the platooning* **then**
        for i ***to*** n **do**
          $i \leftarrow i - 1$
    if *one more vehicle join the platooning* **then**
      $n \leftarrow n + 1$
  if *the platooning stops* **then**
    break
return

We thus get the state transformation for the platoon system with collision avoidance.

Step 3 (Robust Control Algorithm for Vehicle Platooning): With the above formal framework, we are able to develop a robust control algorithm listed as in Algorithm 1.

Algorithm 1 is a generalized form of the control algorithm. Specifically, if we take the bijective function as in the example in (20), the respective control algorithm will have the form as listed in Algorithm 2.

Algorithm 2: Robust Control Algorithm Under Algebraic Transformation

Input: The initial state $x_i(o)$'s of the platooning, platooning parameters n, and the nominal parameters $\bar{M}_i$'s, $\bar{c}_i$'s, $\bar{F}_i$'s
Output: The controller output sequence

$x \leftarrow x(o)$
$t \leftarrow 0$
while *true* **do**
  if $n >= 2$ **then**
    get $x_i(t)$'s
    for $i \leftarrow 1$ ***to*** n **do**
      $e_i(t) \leftarrow \Delta_i^d + x_i(t) - x_{i-1}(t) + l_{i-1}$
      $\mu_i \leftarrow z_{i2} a_i^{-1} \{[\Delta_1^2 - (e_i + \Delta_2)^2]^{-\frac{1}{2}}$
        $+[\Delta_1^2 - (e_i + \Delta_2)^2]^{-\frac{3}{2}} (e_i + \Delta_2)^2\} \Pi_i$
      $p_{i1} \leftarrow \bar{c}_i \dot{x}_i |\dot{x}_i| + \bar{F}_i + D_i^{-1} D_{i-1}(u_{i-1}$
        $-\bar{c}_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - \bar{F}_{i-1})$
      $p_{i2} \leftarrow D_i^{-1} a_i \{[\Delta_1^2 - (e_i + \Delta_2)^2]^{-\frac{1}{2}}$
        $+[\Delta_1^2 - (e_i + \Delta_2)^2]^{-\frac{3}{2}} (e_i + \Delta_2)^2\}^{-1}$
        $\times\{-2z_{i2} - \frac{3}{a_i}[\Delta_1^2 - (e_i + \Delta_2)^2]^{-\frac{3}{2}}$
        $(e_i + \Delta_2)\dot{e}_i^2 - \frac{3}{a_i}[\Delta_1^2 - (e_i + \Delta_2)^2]^{-\frac{5}{2}}$
        $(e_i + \Delta_2)^3 \dot{e}_i^2\}$
      $p_{i3} \leftarrow -D_i^{-1} \frac{2(1+\rho_{E_i})^{-1}\mu_i \Pi_i}{\|\mu_i\| + \epsilon_i}$
      $u_i \leftarrow p_{i1} + p_{i2} + p_{i3}$

      if *the i'th vehicle quit the platooning* **then**
        for i ***to*** n **do**
          $i \leftarrow i - 1$
    if *one more vehicle join the platooning* **then**
      $n \leftarrow n + 1$
  if *the platooning stops* **then**
    break
return

IV. Theoretical Analysis on the Platoon System Performance

In this section, we perform a theoretical analysis on the performance of the platoon system under the control algorithm devised in the previous section to illustrate the effectiveness of the proposed techniques.

Let $\delta_i := [z_{i1} \;\; z_{i2}]^T \in \mathbf{R}^2$. Algorithm 1 renders the transformed system (31) the following performance.

1) *Uniform Boundedness:* For any $r_i > 0$, there is a $d_i(r_i) < \infty$ such that if $\|\delta_i(t_0)\| \leq r_i$, then $\|\delta_i(t)\| \leq d_i(r_i)$ for all $t > t_0$.
2) *Uniform Ultimate Boundedness:* For any $r_i > 0$ with $\|\delta_i(t_0)\| \leq r_i$, there exists a $\underline{d}_i > 0$ such that $\|\delta_i(t)\| \leq \bar{d}_i$ for any $\bar{d}_i > \underline{d}_i$ as $t \geq t_0 + T_i(\bar{d}_i, r_i)$, where $T_i(\bar{d}_i, r_i) < \infty$.

The performance can be analyzed by the following Lyapunov minimax framework [36], [37] as follows:

$$V_i = \frac{1}{2}\left(z_{i1}^2 + z_{i2}^2\right). \tag{38}$$

For a given uncertainty $\sigma(\cdot)$ and the corresponding trajectory of the controlled system, the derivative of V_i is evaluated as

$$\dot{V}_i = z_{i1}\dot{z}_{i1} + z_{i2}\dot{z}_{i2}. \tag{39}$$

In view of (30), the first term on the right-hand side (RHS) of (39) is given by

$$z_{i1}\dot{z}_{i1} = -z_{i1}^2 + z_{i1}z_{i2} \tag{40}$$

and the second term on the RHS of (39) is

$$z_{i2}\dot{z}_{i2} = z_{i2}(-z_{i1}+z_{i2}) + z_{i2}\frac{\partial^2 g_i}{\partial e_i^2}\dot{e}_i^2 + z_{i2}\frac{\partial g_i}{\partial e_i}\Big[M_i^{-1}(u_i - c_i\dot{x}_i|\dot{x}_i| - F_i) - M_{i-1}^{-1}(u_{i-1} - c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - F_{i-1})\Big]. \tag{41}$$

The third term on the RHS of (40), after decomposing M_i^{-1}, c_i, and F_i by (13) and recalling that $M_i^{-1} = D_i + \Delta D_i$, we get

$$\begin{aligned}
z_{i2}&\frac{\partial g_i}{\partial e_i}\Big[M_i^{-1}(u_i - c_i\dot{x}_i|\dot{x}_i| - F_i) - M_{i-1}^{-1}(u_{i-1} - c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - F_{i-1})\Big]\\
&= z_{i2}\frac{\partial g_i}{\partial e_i}\big\{(D_i+\Delta D_i)\big[p_{i1}+p_{i2}+p_{i3} - (\bar{c}_i+\Delta c_i)\dot{x}_i|\dot{x}_i| - \big(\bar{F}_i - \Delta F_i\big)\big] - (D_{i-1}+\Delta D_{i-1})\big[u_{i-1} - (\bar{c}_{i-1} + \Delta c_{i-1})\dot{x}_{i-1}|\dot{x}_{i-1}| - \big(\bar{F}_{i-1} - \Delta F_{i-1}\big)\big]\big\}\\
&= z_{i2}\frac{\partial g_i}{\partial e_i}\big\{\big[D_i\big(p_{i1} - \bar{c}_i\dot{x}_i|\dot{x}_i| - \bar{F}_i\big) - D_{i-1}\big(u_{i-1} - \bar{c}_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - \bar{F}_{i-1}\big)\big]\\
&\quad + D_i p_{i2} + \big[\Delta D_i(p_{i1}+p_{i2} - c_i\dot{x}_i|\dot{x}_i| - F_i) - \Delta D_{i-1}(u_{i-1} - c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - F_{i-1})\\
&\quad + D_i(-\Delta c_i\dot{x}_i|\dot{x}_i| - \Delta F_i) - D_{i-1}(-\Delta c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - \Delta F_{i-1})\big] + (D_i+\Delta D_i)p_{i3}\big\}.
\end{aligned} \tag{42}$$

When the system is in the absence of uncertainty (hence $\Delta M_i = \Delta c_i = \Delta F_i = 0$), we have

$$z_{i2}\frac{\partial g_i}{\partial e_i}\big\{D_i\big(p_{i1} - \bar{c}_i\dot{x}_i|\dot{x}_i| - \bar{F}_i\big) - D_{i-1}\big(u_{i-1} - \bar{c}_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - \bar{F}_{i-1}\big)\big\} = 0. \tag{43}$$

Next

$$\begin{aligned}
z_{i2}\frac{\partial g_i}{\partial e_i}D_i p_{i2} &= z_{i2}\frac{\partial g_i}{\partial e_i}D_i D_i^{-1}\left(\frac{\partial g_i}{\partial e_i}\right)^{-1}\left(-2z_{i2} - \frac{\partial^2 g_i}{\partial e_i^2}\dot{e}_i^2\right)\\
&= -2z_{i2}^2 - z_{i2}\frac{\partial^2 g_i}{\partial e_i^2}\dot{e}_i^2.
\end{aligned} \tag{44}$$

By (15), we get

$$\begin{aligned}
z_{i2}&\frac{\partial g_i}{\partial e_i}D_i\big\{\big[\Delta D_i(p_{i1}+p_{i2} - c_i\dot{x}_i|\dot{x}_i| - F_i) - \Delta D_{i-1}(u_{i-1} - c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - F_{i-1}) + D_i(-\Delta c_i\dot{x}_i|\dot{x}_i| - \Delta F_i) - D_{i-1}(-\Delta c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}| - \Delta F_{i-1})\big]\big\}\\
&\le |z_{i2}|\frac{\partial g_i}{\partial e_i}\Pi_i.
\end{aligned} \tag{45}$$

By (14) and (40) (noting that $\Delta D_i = D_i E_i$ and $\mu_i := z_{i2}((\partial g_i)/(\partial e_i))\Pi_i$)

$$\begin{aligned}
z_{i2}\frac{\partial g_i}{\partial e_i}(D_i+\Delta D_i)p_{i3} &\le -z_{i2}\frac{\partial g_i}{\partial e_i}(D_i + D_iE_i)D_i^{-1}\frac{2(1+\rho_{Ei})^{-1}\mu_i\Pi_i}{\|\mu_i\|+\epsilon_i}\\
&\le -D_iD_i^{-1}(1+\rho_{Ei})\frac{2(1+\rho_{Ei})^{-1}\|\mu_i\|^2}{\|\mu_i\|+\epsilon_i}\\
&= -\frac{2\|\mu_i\|^2}{\|\mu_i\|+\epsilon_i}.
\end{aligned} \tag{46}$$

Substituting equations (42)–(46) into (41), we have

$$\begin{aligned}
z_{i2}\dot{z}_{i2} &\le z_{i2}(-z_{i1}+z_{i2}) + z_{i2}\frac{\partial^2 g_i}{\partial e_i^2}\dot{e}_i^2 - 2z_{i2}^2 - z_{i2}\frac{\partial^2 g_i}{\partial e_i^2}\dot{e}_i^2 + |z_{i2}|\frac{\partial g_i}{\partial e_i}\Pi_i - \frac{2\|\mu_i\|^2}{\|\mu_i\|+\epsilon_i}\\
&= -z_{i2}^2 - z_{i1}z_{i2} + \|\mu_i\| - \frac{2\|\mu_i\|^2}{\|\mu_i\|+\epsilon_i}.
\end{aligned} \tag{47}$$

If $\|\mu_i\| > \epsilon_i$

$$\|\mu_i\| - \frac{2\|\mu_i\|^2}{\|\mu_i\|+\epsilon_i} \le \|\mu_i\| - \frac{2\|\mu_i\|^2}{\|\mu_i\|+\|\mu_i\|} = 0 \tag{48}$$

else if $\|\mu_i\| \le \epsilon_i$

$$\|\mu_i\| - \frac{2\|\mu_i\|^2}{\|\mu_i\|+\epsilon_i} \le \|\mu_i\| - \frac{\|\mu_i\|^2}{\epsilon_i} \le \frac{\epsilon_i}{4}. \tag{49}$$

Consequently, for all $\|\mu_i\|$, we have

$$z_{i2}\dot{z}_{i2} \le -z_{i2}^2 - z_{i1}z_{i2} + \frac{\epsilon_i}{4}. \tag{50}$$

With (40) and (50) in (39), we get (noting that $\|\delta_i\|^2 = \|z_{i1}\|^2 + \|z_{i2}\|^2$)

$$\begin{aligned}
\dot{V}_i &\le -z_{i1}^2 - z_{i2}^2 + \frac{\epsilon_i}{4}\\
&= -\|\delta_i\|^2 + \frac{\epsilon_i}{4}.
\end{aligned} \tag{51}$$

Since the design parameter ϵ_i is a constant scalar. We conclude that $\dot{V}_i$ is negative definite for sufficiently large $\|\delta_i\|$. Therefore, upon invoking the standard arguments as in [38], we conclude the solution to the transformed error dynamics of the ith vehicle is uniform boundedness with

$$\begin{aligned}
d_i(r_i) &= \begin{cases} R_i & \text{if} \quad r_i \le R_i \\ r_i & \text{if} \quad r_i > R_i \end{cases}\\
R_i &= \frac{\sqrt{\epsilon_i}}{2}.
\end{aligned} \tag{52}$$

Furthermore, uniform ultimate boundedness also follows with:

$$\begin{aligned}
\underline{d}_i &= R_i\\
T_i\big(\bar{d}_i, r_i\big) &= \begin{cases} 0 & \text{if} \quad r_i \le \bar{d}_i \\ \frac{r_i^2 - \bar{d}_i^2}{\bar{d}_i^2 - R_i^2} & \text{otherwise.} \end{cases}
\end{aligned} \tag{53}$$

Based on (52) and (53), we conclude that the uniform boundedness and uniform ultimate boundedness regions can be made arbitrary small with suitable choice of the design parameter ϵ_i, that is, $\bar{d}_i \to 0$ as $\epsilon_i \to 0$. This can be used to discuss the string stability given in Section II.

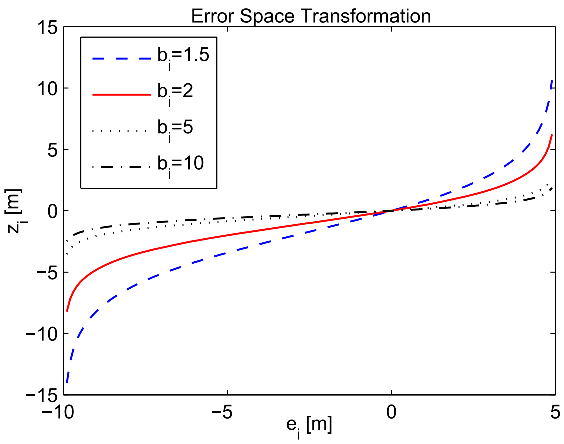

Fig. 4. Error space under the logarithm transformation.

V. Simulation Experiments

In this section, we conduct numerical simulations to examine the behavior of a vehicle platoon under the control algorithm generated by the framework proposed earlier. Consider a platoon of four vehicles traveling on a highway.

We illustrate the three-step framework by proposing another bijective function

$$z_i = -\log_{b_i}\left[\Lambda_1(e_i+\Lambda_2)^{-1}-\Lambda_3\right] \tag{54}$$

where

$$\Lambda_1 := \frac{\underline{\Delta}_i}{\bar{\Delta}_i}(\underline{\Delta}_i+\bar{\Delta}_i),\ \ \Lambda_2 := \underline{\Delta}_i,\ \ \Lambda_3 := \frac{\underline{\Delta}_i}{\bar{\Delta}_i}. \tag{55}$$

Here $b_i > 1$ is a design parameter for the logarithmic function and parameters $\Lambda_{1,2,3} > 0$. A numerical demonstration with different b_i is shown in Fig. 4.

Similarly, one can check that

$$z_i\big|_{e_i=0} = -\log_{b_i}\left[\Lambda_1\Lambda_2^{-1}-\Lambda_3\right] = 0 \tag{56}$$

$$\lim_{e_i\to-\underline{\Delta}_i} -\log_{b_i}\left[\Lambda_1\left(-\underline{\Delta}_i+\Lambda_2\right)^{-1}-\Lambda_3\right] = -\infty \tag{57}$$

$$\lim_{e_i\to\bar{\Delta}_i} -\log_{b_i}\left[\Lambda_1\left(\bar{\Delta}_i+\Lambda_2\right)^{-1}-\Lambda_3\right] = +\infty. \tag{58}$$

Hence, (54) is a legitimate transformation by the property of the bijective function. For clarity, we denote $\lambda_i := \ln b_i$, here $\lambda_i > 0$ since $b_i > 1$. Thus, (54) can be rewritten as

$$z_i = -\frac{\ln\left[\Lambda_1(e_i+\Lambda_2)^{-1}-\Lambda_3\right]}{\lambda_i}. \tag{59}$$

Differentiate (59) with respect to time, and with similar process as in step 2, the equation of motion with state space $\delta_i = [z_{i1}\ z_{i2}]^T$ is

$$\dot{z}_{i1} = -z_{i1}+z_{i2} \tag{60}$$

$$\begin{aligned}\dot{z}_{i2} &= -z_{i1}+z_{i2}+\lambda_i z_{i1}^2 - 2(e_i+\Lambda_2)^{-1}\dot{e}_i\dot{z}_{i1}\\ &\quad + \lambda_i^{-1}\Lambda_1\exp(\lambda_i z_i)(e_i+\Lambda_2)^{-2}\Big[M_i^{-1}(u_i - c_i\dot{x}_i|\dot{x}_i| - F_i)\\ &\quad - M_{i-1}^{-1}(u_{i-1}-c_{i-1}\dot{x}_{i-1}|\dot{x}_{i-1}|-F_{i-1})\Big].\end{aligned} \tag{61}$$

Algorithm 3: Robust Control Algorithm

Input: The initial state $x_i(o)$'s of the platooning, platooning parameters n, and the nominal parameters $\bar{M}_i$'s, $\bar{c}_i$'s, $\bar{F}_i$'s

Output: The controlled system performance measured by the state $x_i(t)$'s

```
x ← x(o)
t ← 0
while true do
    if n >= 2 then
        get x_i(t)'s
        for i ← 1 to n do
            e_i(t) ← Δ_i^d + x_i(t) − x_{i−1}(t) + l_{i−1}
            μ_i ← z_{i2} λ_i^{−1} Λ_1 exp(λ_i z_{i1})(e_i + Λ_2)^{−2} Π_i
            p_{i1} ← c̄_i ẋ_i|ẋ_i| + F̄_i + D_i^{−1} D_{i−1}(u_{i−1}
                    −c̄_{i−1} ẋ_{i−1}|ẋ_{i−1}| − F̄_{i−1})
            p_{i2} ← D_i^{−1} λ_i Λ_1^{−1} exp(−λ_i z_{i1})(e_i + Λ_2)^2
                    [ − 2z_{i2} − λ_i(z_{i2} − z_{i1})^2 + (e_i + Λ_2)^{−1}
                    ė_i(z_{i2} − z_{i1})]
            p_{i3} ← −D_i^{−1} 2(1+ρ_{E_i})^{−1} μ_i Π_i / (‖μ_i‖+ε_i)
            u_i ← p_{i1} + p_{i2} + p_{i3}

            if the i'th vehicle quit the platooning then
                for i to n do
                    i ← i − 1

        if one more vehicle join the platooning then
            n ← n + 1

    if the platooning stops then
        break

return
```

Followed step 3, we can get a control algorithm listed in Algorithm 3.

Since the algorithm generated by the mathematical framework proposed in this paper matches the property examined in Section III, the performance of a respective platoon system shall be guaranteed.

Next we perform numerical simulation to validate the performance of the control algorithm. The following parameters are chosen for numerical simulation. The nominal vehicle masses (kg): $\bar{M}_L = 1000$, $\bar{M}_1 = 950$, $\bar{M}_2 = 850$, $\bar{M}_3 = 750$, the nominal aerodynamic coefficients: $\bar{c}_L = \bar{c}_1 = \bar{c}_2 = \bar{c}_3 = 0.3$, the nominal resistance forces (N): $\bar{F}_L = 200$, $\bar{F}_1 = 180$, $\bar{F}_2 = 160$, $\bar{F}_3 = 150$. The vehicle lengths: $l_L = l_1 = l_2 = l_3 = 5$m. The uncertainties in each vehicle which are (possibly) time-varying are given by: $\Delta M_L = 50\sin 0.1t$, $\Delta M_1 = 50\cos 0.5t$, $\Delta M_2 = 50\cos t$, $\Delta M_3 = 50\cos 0.1t$, $\Delta c_L = 0.02$, $\Delta c_1 = 0.01$, $\Delta c_2 = -0.03$, $\Delta c_3 = -0.02$, $\Delta F_L = 180\sin 0.5t$, $\Delta F_1 = 160\sin t$, $\Delta F_2 = 140\sin(t-\pi/6)$, $\Delta F_3 = 120\sin(t-\pi/6)$. The modeling uncertainties, which may come from the tire property, complex road surface situations, the movement of the passengers/stowage, the side wind force, etc., are very hard

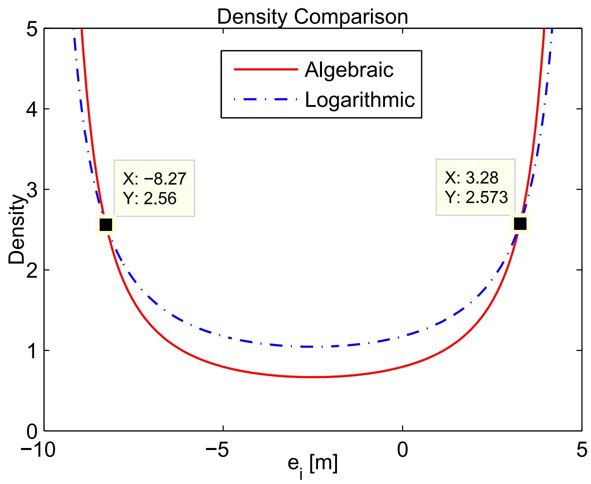

Fig. 5. Comparison of the density under the algebraic and logarithmic transformations.

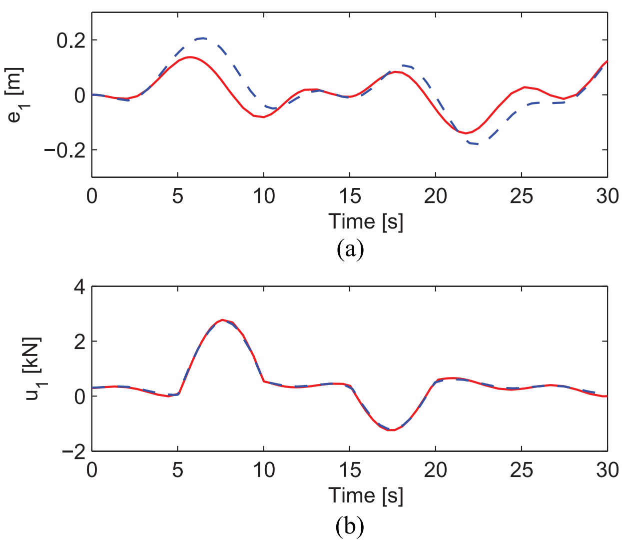

Fig. 6. Spacing error and control force histories of the first follower under algebraic (solid line) and logarithmic (dashed line) controllers, respectively. (a) Space error history. (b) Control force history.

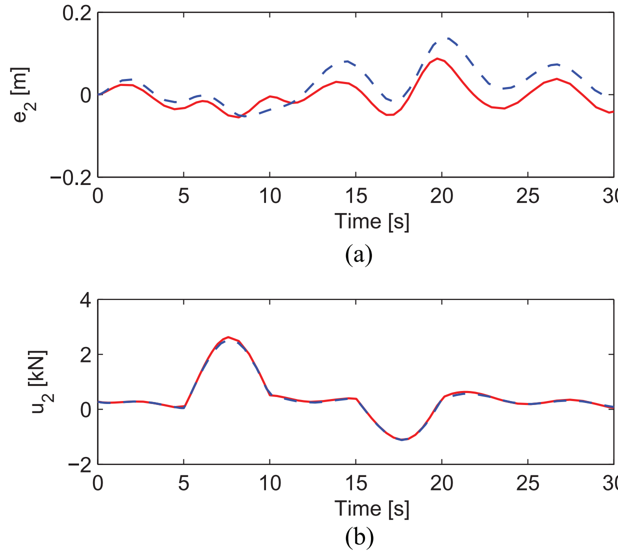

Fig. 7. Spacing error and control force histories of the second follower under algebraic (solid line) and logarithmic (dashed line) controllers, respectively. (a) Space error history. (b) Control force history.

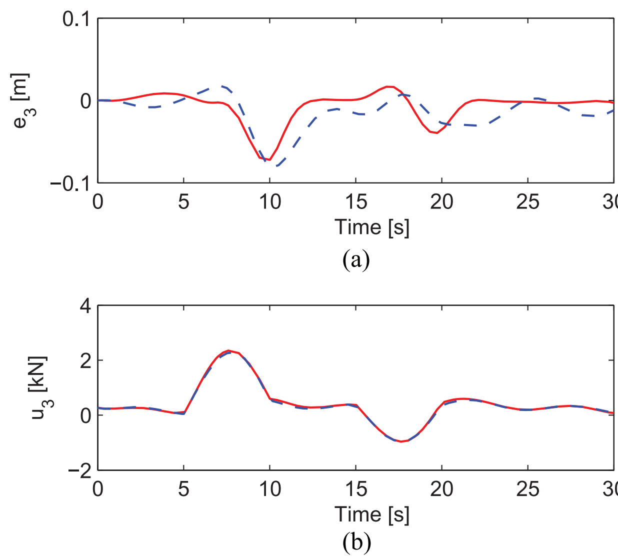

Fig. 8. Spacing error and control force histories of the third follower under algebraic (solid line) and logarithmic (dashed line) controllers, respectively. (a) Space error history. (b) Control force history.

to simulate precisely as they are uncertain themselves. We randomly chose the above uncertainties to show the algorithm is adequate in dealing with external disturbances that are not counted in the model. Suppose Assumption 1 is fulfilled with $\rho_{E_1} = \rho_{E_2} = \rho_{E_3} = -0.1$ and Assumption 2 is met with

$$\Pi_i = 0.1\dot{e}_i^2 + 0.2e_i^2 + 0.5, \;\; i = L, 1, 2, 3. \tag{62}$$

Suppose the leading vehicle is under the following algorithm:

$$\bar{u}_L := \bar{c}_i \dot{x}_i |\dot{x}_i| + \bar{F}_i \tag{63}$$

$$u_L = \begin{cases} \bar{u}_L + 2500 \sin 0.1\pi(t-15) & \text{if } 15 < t \leq 25 \\ \bar{u}_L - 1500 \sin 0.1\pi(t-35) & \text{if } 35 < t \leq 45 \\ \bar{u}_L & \text{otherwise.} \end{cases} \tag{64}$$

The density functions of the algebraic (D_a) and logarithmic (D_l) transformations are as follows:

$$\begin{aligned} D_a := \frac{\partial z_i}{\partial e_i} &= a_i{}^{-1}\Big[\Delta_1^2 - (e_i + \Delta_2)^2\Big]^{-\frac{1}{2}} \\ &\quad + a_i{}^{-1}\Big[\Delta_1^2 - (e_i + \Delta_2)^2\Big]^{-\frac{3}{2}}(e_i + \Delta_2)^2 \end{aligned} \tag{65}$$

$$D_l := \frac{\partial z_i}{\partial e_i} = \lambda_i^{-1}\Lambda_1\Big[\Lambda_1(e_i + \Lambda_2)^{-1} - \Lambda_3\Big]^{-1}(e_i + \Lambda_2)^{-2}. \tag{66}$$

For numerical demonstration, choosing $a_i = 0.2$, $b_i = 1.8$, $\underline{\Delta}_i = 10$, $\bar{\Delta}_i = 5$, we get $\Delta_1 = 7.5$, $\Delta_2 = 2.5$, $\Delta_3 = 0.3536$, $\Lambda_1 = 30$, $\Lambda_2 = 10$, $\Lambda_3 = 2$. The density comparison is shown in Fig. 5. It shows that for $-8.27 < e_i < 3.28$, $D_a < D_l$;

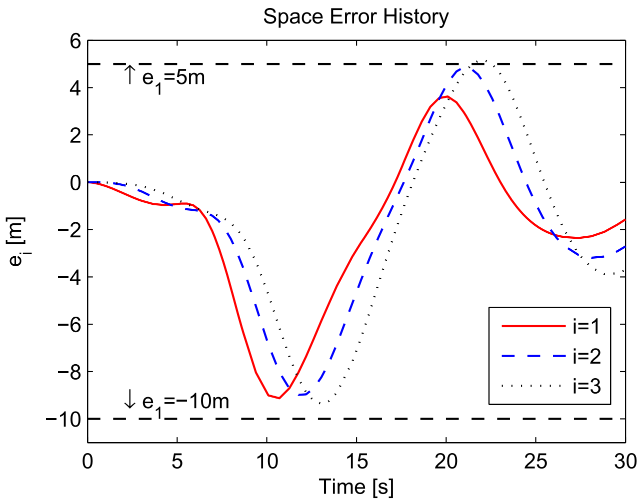

Fig. 9. Spacing error history of the three followers in the platoon under PD controller.

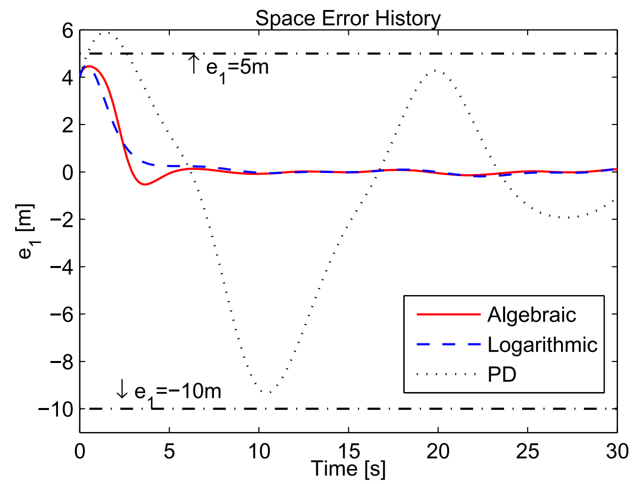

Fig. 11. Spacing error trajectories of the first vehicle in the platoon coordinated by the proposed control algorithm.

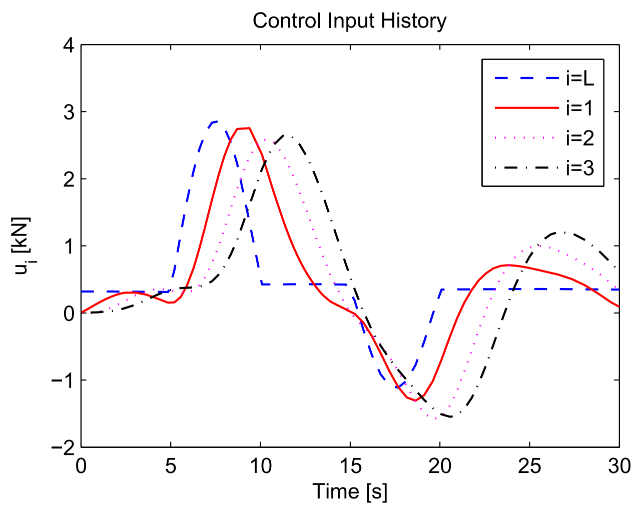

Fig. 10. Control force trajectories of the four vehicles in the platoon under a PD controller.

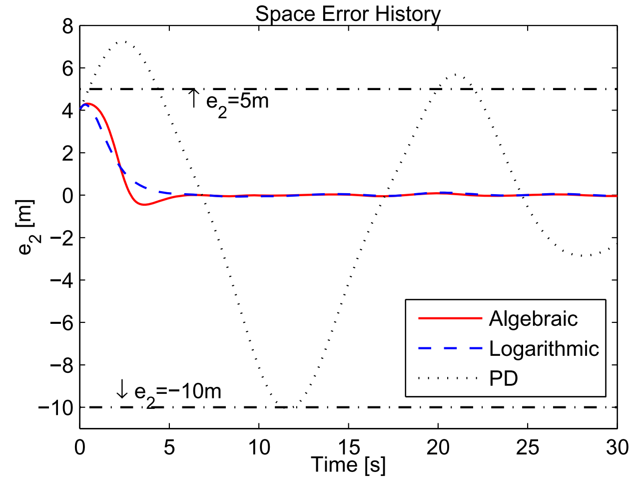

Fig. 12. Spacing error trajectories of the second vehicle in the platoon coordinated by the proposed control algorithm.

while for $-10 < e_i < -8.27$ and $3.28 < e_i < 5$, $D_a > D_l$. The transformation (algebraic or logarithmic) is a mapping from an interval in the error state space $e_i \in (\underline{\Delta}_i, \bar{\Delta}_i)$ to a 1-D space (i.e., the transformed state space $z_i \in \mathbf{R}$). Here, the density can be explained as the ratio of the corresponding lengths in z_i with each length of e_i in the interval $(\underline{\Delta}_i, \bar{\Delta}_i)$. The desired spacing are chosen as: $\Delta_1^d = \Delta_2^d = \Delta_3^d = 5$m.

For comparison, we also define a conventional PD control algorithm

$$u_i = -k_{i1}e_i - k_{i2}\dot{e}_i, \;\; i = 1, 2, 3 \tag{67}$$

where k_{i1} and k_{i2} are the proportional and derivative control gains. We choose the PD control algorithm as it is a very common used control algorithm in vehicle platooning, as well as in control of the ITS's [39].

The following parameters are specified: 1) The generated control algorithm: $\epsilon_1 = 800$, $\epsilon_2 = 600$, $\epsilon_3 = 400$ and 2) PD control algorithm: $k_{i1} = 220$, $k_{i2} = 500$, $i = 1, 2, 3$.

First, we consider the platoon starting from an initial condition without spacing errors with initial positions $x = [100\ 90\ 80\ 70]^T$ and initial velocities $\dot{x} = [20\ 20\ 20\ 20]^T$. The final results are shown in Figs. 6–10. Figs. 6–8 compare the spacing error and control force histories of the first, second, and third followers under the control algorithm with algebraic and logarithmic transformations, respectively. It shows that the spacing error under the algebraic controller is better than that of the logarithmic controller. This can be explained by the density of the logarithmic controller is larger than that of the algebraic controller. It also can be concluded that the spacing errors of the three followers stay in a region smaller than 0.3. Moreover, notice that $|e_1|_\infty < 0.3$, $|e_2|_\infty < 0.2$, $|e_3|_\infty < 0.1$, which means the error is attenuated when it propagates from followers 1 to 3. Figs. 9 and 10 show the spacing error and control force histories of the three followers under the PD controller. It shows that the spacing error is chattering and collision occurs ($e_3 > 5$) at around $t = 22.5$ for the third follower. The control force bounds of the robust controllers and PD controllers are almost the same.

Next, we consider the platoon started from a critical initial conditions with initial positions $x = [100\ 94\ 88\ 82]^T$ and

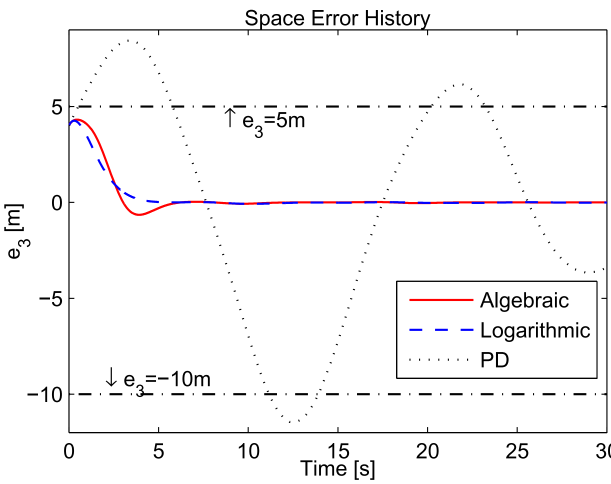

Fig. 13. Spacing error trajectories of the third vehicle in the platoon coordinated by the proposed control algorithm.

initial velocities $\dot{x} = [10\ 13\ 15\ 17]^T$. The final results are shown as follows. Figs. 11 and 13 compare the spacing error histories of the first, second, and third followers under the control algorithms with algebraic and logarithmic transformations and PD controller, respectively. It can be shown that the spacing error responses of the followers under the generated control algorithm stay in the region $-10 < e_i < 5$ and converge to a small region around $|e_i| < 0.2$ when $t > 5$; while the spacing errors response of the followers under PD control exceed the required region $-10 < e_i < 5$ (notice that $e_i > 5$ at around $t = 1$ for $i = 1, 2, 3$ and hence the vehicle collision occurs). However, it shows that under this conditions the spacing error with the logarithmic controller is better than that with the algebraic controller. This can be explained by the density of the algebraic controller is larger than that of the logarithmic controller when $e_i > 3.28$ (shown in Fig. 5).

VI. Conclusion

This paper addresses the problem of developing robust control algorithm for vehicle platoons under nonlinear and (potentially) fast time-varying uncertainties. A formal framework to derive the decentralized control algorithm is proposed. The resultant algorithm restraints the spacing error of each vehicle in the platoon with strict boundaries. Each vehicle in the platoon only needs the knowledge of the preceding vehicle. We prove that the proposed algorithm guarantees the uniformly boundedness and uniformly ultimate boundedness under any practical arbitrary initial condition in the transformed error dynamics. The effectiveness of the proposed control algorithm is validated with realistic scenarios.

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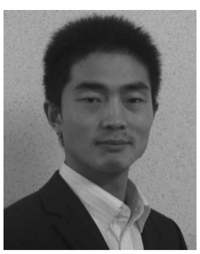

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