Scheduling the **South American** Qualifiers to the **2018 FIFA** **by Integer Programming**

Guillermo Durán
University of Buenos Aires

Mario Guajardo
NHH Norwegian School of Economics

Denis Sauré
University of Chile

CLAIO 2016, Santiago, Chile

CONMEBOL's South American World Cup Qualifiers

- 10 teams compete for 4.5 slots in the finals
- Every team plays twice against every other team
- 2 years
- 18 rounds grouped into 9 double rounds

Round 1 Round 2 — **1st double round** — 8 – 13 Oct

Round 3 Round 4 — **2nd double round** — 12 – 17 Nov

…

Breaks within double rounds

	Round 1	Round 2	Round 3	Round 4	
ARGENTINA	Home	Away	Home	Away	…
BOLIVIA	Away	Home	Away	Away	

break

1st double round

2nd double round

Inconvenient for the teams/players

Breaks within double rounds

	Round 4	Round 5	Round 6	Round 7	
CHILE	Home	Away	Away	Home	...

break

3rd double round

Nov 2011 — June 2012 — Sept 2012

Almost a year without a home game

Inconvenient for the fans/local sponsors

Home vs Away start of a double round

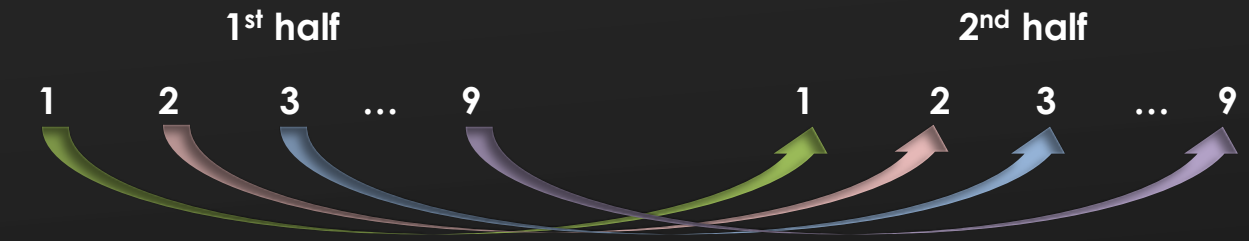
	Round 1	Round 2	Round 3	Round 4	
ARGENTINA	Home	Away	Home	Away	...
BOLIVIA	Away	Home	Away	Away	

1st double round

2nd double round

For logistical reasons, starting a double round with a home game is usually preferred

Traditional mirrored schedule (5 World Cups 1998-2014)



Round 1	Round 10
Argentina – Chile	Chile – Argentina
Uruguay – Bolivia	Bolivia – Uruguay
Ecuador – Venezuela	Venezuela – Ecuador
Peru – Paraguay	Paraguay – Peru
Colombia – Brazil	Brazil – Colombia

Old schedule 2002-2014 World Cups

Team	1	2	3	4	5	6	7	8	9
ARG	CHI	@VEN	BOL	@COL	ECU	@BRA	PAR	@PER	URU
BOL	@URU	COL	@ARG	@VEN	CHI	PAR	@ECU	@BRA	PER
BRA	@COL	ECU	@PER	URU	@PAR	ARG	@CHI	BOL	@VEN
CHI	@ARG	PER	@URU	PAR	@BOL	@VEN	BRA	COL	@ECU
COL	BRA	@BOL	VEN	ARG	@PER	@ECU	URU	@CHI	PAR
ECU	VEN	@BRA	@PAR	PER	@ARG	COL	BOL	@URU	CHI
PAR	@PER	URU	ECU	@CHI	BRA	@BOL	@ARG	VEN	@COL
PER	PAR	@CHI	BRA	@ECU	COL	@URU	VEN	ARG	@BOL
URU	BOL	@PAR	CHI	@BRA	VEN	PER	@COL	ECU	@ARG
VEN	@ECU	ARG	@COL	BOL	@URU	CHI	@PER	@PAR	BRA

...

1) Could we find a mirrored schedule without breaks within double rounds?

- A mirrored schedule with no breaks in double rounds for CONMEBOL'S qualifiers is infeasible
- Minimum: 16
- Old schedule 2002-2014: 18
- Tradition: mirrored hard to change (same schedule used for 4 last World Cups)

2) Could we find a schedule *as mirrored as possible* without breaks within double rounds?

Integer programming

$$x_{i,j,k} = \begin{cases} 1 & \text{team } i \text{ plays at home against team } j \text{ in round } k, \\ 0 & \sim . \end{cases}$$

$$w_{i,k} = \begin{cases} 1 & \text{team } i \text{ has a double round break in round } k, \\ 0 & \sim . \end{cases}$$

$$y_{i,k} = \begin{cases} 1 & \text{team } i \text{ has a H-A sequence in round } k, \\ 0 & \sim . \end{cases}$$

Double round-robin constraints

- Double round-robin split in two single-round robin halves

$$\sum_{k \in K : k \leq n-1} (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall i \in I, j \in I : i \neq j$$

$$\sum_{k \in K : k > n-1} (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall i \in I, j \in I : i \neq j$$

- Home-away balance with opponent

$$\sum_{k \in K} x_{i,j,k} = 1 \quad \forall i \in I, j \in I : i \neq j$$

- Compactness

$$\sum_{i \in I : i \neq j} (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall j \in I, k \in K$$

Balance constraints

- Home-away balance within double rounds

$$n/2 - 1 \leq \sum_{k \in K_{odd}} y_{i,k} \leq n/2, \quad \forall i \in I,$$

- Logical relationships

$$\sum_{j \in I : i \neq j} (x_{i,j,k} + x_{j,i,k+1}) \leq 1 + y_{i,k}, \quad \forall i \in I, k \in K_{odd},$$

$$y_{i,k} \leq \sum_{j \in I : i \neq j} x_{i,j,k}, \quad \forall i \in I, k \in K_{odd},$$

$$y_{i,k} \leq \sum_{j \in I : i \neq j} x_{j,i,k+1}, \quad \forall i \in I, k \in K_{odd}.$$

Objective function

- Minimize double breaks

$$\min z = \sum_{i \in I} \sum_{k \in K_{odd}} w_{i,k}.$$

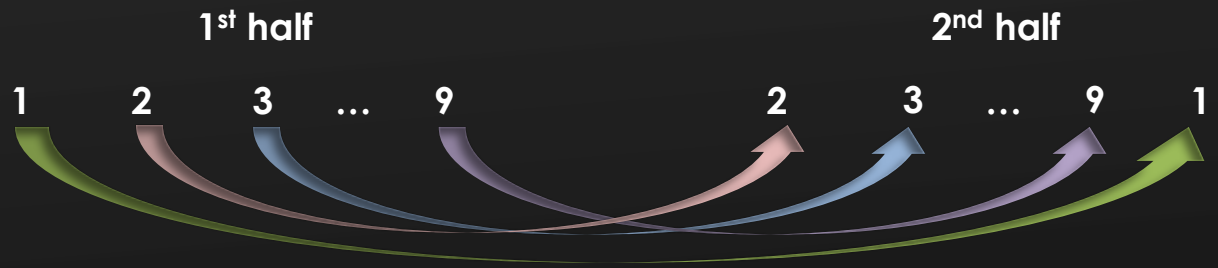
- Logical relationships

$$\sum_{j \in I : i \neq j} (x_{j,i,k} + x_{j,i,k+1}) \leq 1 + w_{i,k}, \quad \forall i \in I, k \in K_{odd},$$

$$w_{i,k} \leq \sum_{j \in I : i \neq j} x_{j,i,k}, \quad \forall i \in I, k \in K_{odd},$$

$$w_{i,k} \leq \sum_{j \in I : i \neq j} x_{j,i,k+1}, \quad \forall i \in I, k \in K_{odd}.$$

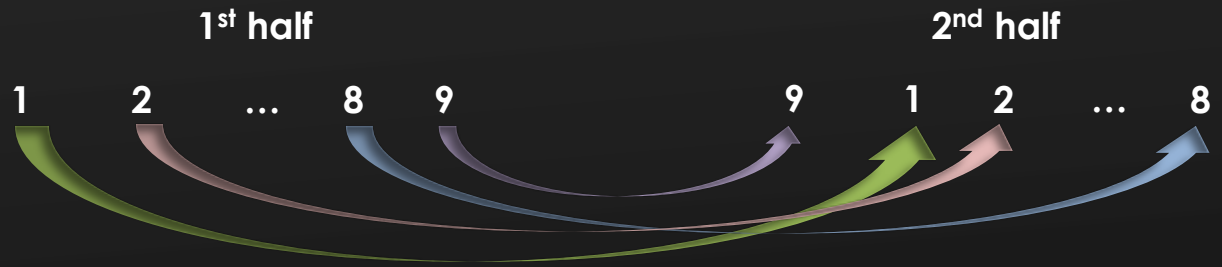
Symmetry: French scheme



$$x_{i,j,1} = x_{j,i,2n-2},$$

$$x_{i,j,k} = x_{j,i,k+n-2}, \quad \forall\, i, j \in I : i \neq j,\ 2 \leq k \leq n-1.$$

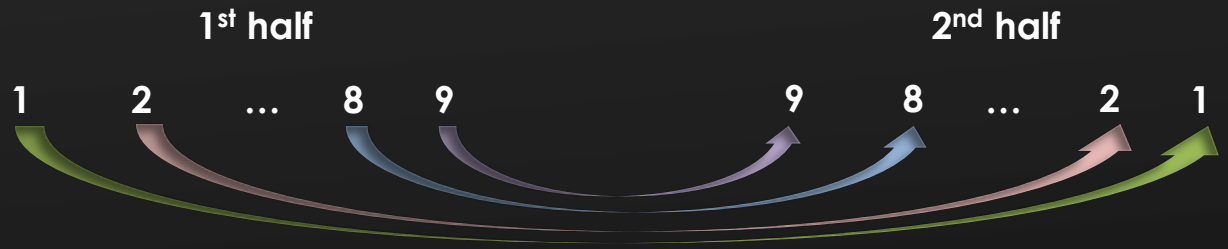
Symmetry: English scheme



$$x_{i,j,n-1} = x_{j,i,n},$$

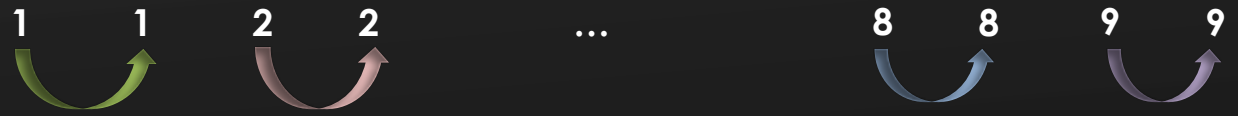
$$x_{i,j,k} = x_{j,i,k+n}, \quad \forall\, i, j \in I : i \neq j,\ 2 \leq k \leq n-2.$$

Symmetry: Inverted scheme



$$x_{i,j,k} = x_{i,j,2n-1-k} \quad \forall i \in I, j \in I, k \in K : i \neq j, 1 \leq k \leq n-1$$

Symmetry: back-to-back scheme



$$x_{i,j,k} = x_{j,i,k+1}, \quad \forall\, i, j \in I : i \neq j,\ k \in K_{odd}.$$

Symmetry: Min-Max separation scheme

- Every team plays against every other team at most once in c consecutive rounds

$$\sum_{\bar{k}\in K:k\leq\bar{k}\leq k+c} (x_{i,j,\bar{k}} + x_{j,i,\bar{k}}) \leq 1, \quad \forall\, i,j \in I : i \neq j,\ k \leq |K| - c,$$

- Every team plays against every other team at least once in d consecutive rounds

$$\sum_{\bar{k}\in K:k-d\leq\bar{k}\leq k+d:\bar{k}\neq k} x_{i,j,\bar{k}} \geq x_{j,i,k}, \quad \forall\, i,j \in I : i \neq j,\ d < k \leq |K| - d.$$

Implementation

- Java/CPLEX 12.6, 3.4 GHz Intel Core i5
- IP has 1800 variables, 2136 constraints
- Feasible solutions in a few seconds/minutes
- Several proposals discussed with the Chile's ANFP officials
- They chose one of the template schedules that we generated according to the French scheme

CONMEBOL meeting

http://conmebol.com/

Mirrored

Same schedule

Decision: new schedule by a random draw

Our schedule proposal

unanimously

agreed by the 10 CONMEBOL's countries

Qualifiers for the 2018 World Cup Russia

1998 **2002** **2014** **Jan-2015** **May-2015** **Oct-2015/Oct-2017**

TEAM NAME

TEAM NUMBER

2018 FIFA World Cup Russia™
Preliminary Competition Format & Draw Procedures
SOUTH AMERICAN ZONE

FIFA WORLD CUP RUSSIA 2018

Shaun Botterill/Getty Images Europe (zimbio.com/)

Ronaldo
Best World Cup
Player Award 1998

SCHEDULE

Matchday 1		Matchday 2	
Team 1	Team 9	Team 3	Team 4
Team 2	Team 5	Team 5	Team 7
Team 4	Team 6	Team 6	Team 8
Team 7	Team 3	Team 9	Team 2
Team 8	Team 10	Team 10	Team 1

...

Matchday 17		Matchday 18	
Team 1	Team 3	Team 9	Team 1
Team 2	Team 6	Team 5	Team 2
Team 4	Team 9	Team 6	Team 4
Team 7	Team 10	Team 3	Team 7
Team 8	Team 5	Team 10	Team 8

1st double-round

9th double-round

OPERATIONS RESEARCH

Shaun Botterill/Getty Images Europe (zimbio.com/)

Forlán
Best World Cup
Player Award 2010

New schedule 2018 World Cup

Number	Team	1	2	3	4	5	6	7	8	9
4	ARG	ECU	@ PAR	BRA	@ COL	@ CHI	BOL	URU	@ VEN	@ PER
8	BOL	URU	@ ECU	VEN	@ PAR	COL	@ ARG	PER	@ CHI	@ BRA
5	BRA	@ CHI	VEN	@ ARG	PER	URU	@ PAR	@ ECU	COL	BOL
2	CHI	BRA	@ PER	COL	@ URU	ARG	@ VEN	@ PAR	BOL	@ ECU
1	COL	PER	@ URU	@ CHI	ARG	@ BOL	ECU	VEN	@ BRA	@ PAR
6	ECU	@ ARG	BOL	URU	@ VEN	PAR	@ COL	BRA	@ PER	CHI
3	PAR	@ VEN	ARG	@ PER	BOL	@ ECU	BRA	CHI	@ URU	COL
9	PER	@ COL	CHI	PAR	@ BRA	VEN	@ URU	@ BOL	ECU	ARG
10	URU	@ BOL	COL	@ ECU	CHI	@ BRA	PER	@ ARG	PAR	VEN
7	VEN	PAR	@ BRA	@ BOL	ECU	@ PER	CHI	@ COL	ARG	@ URU

...

Comparison

	Schedule 2002-2014					Schedule 2018				
Team	B_h	B_a	B	H-A	A-H	B_h	B_a	B	H-A	A-H
ARG	0	0	0	9	0	0	0	0	5	4
BOL	2	2	4	2	3	0	0	0	5	4
BRA	0	0	0	0	9	0	0	0	4	5
CHI	1	1	2	1	6	0	0	0	5	4
COL	1	1	2	6	1	0	0	0	5	4
ECU	1	1	2	4	3	0	0	0	4	5
PAR	1	1	2	3	4	0	0	0	4	5
PER	1	1	2	6	1	0	0	0	4	5
URU	1	1	2	4	3	0	0	0	4	5
VEN	1	1	2	1	6	0	0	0	5	4
Total	9	9	18	36	36	0	0	0	45	45

Concluding remarks

- Less challenging than most national leagues schedules
- More impact than any other project we have worked
- Incredibly fun to do
- Focus on sport fairness, specially in ***double rounds*** breaks; connection with the *generalized break* concept
- Double round breaks from 18 to 0
- Balance in H-A occurrences
- Next target… the World (Cup)!!!

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