Comparative Analysis of Financial Bubble Detection Models: Evidence from Latin American Markets

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***Resumen*—Financial bubbles pose significant risks to economic stability, necessitating robust detection methodologies. This study presents a comprehensive comparative analysis of three distinct approaches to bubble detection: Bayesian Networks, K-means clustering, and ARFIMA with Hurst exponent analysis. Using daily data from Chilean, Peruvian, and Colombian markets spanning 2013-2022 (2,317 observations), we evaluate the efficacy of each method in identifying market regimes and detecting bubble formation. Bayesian Networks reveal strong probabilistic dependencies (mutual information: 0.108-0.164) and crisis prediction capability (33-38 % accuracy during high volatility). K-means clustering identifies three distinct market regimes with Colombia exhibiting the highest proportion of crisis observations (7.0 %) versus Chile (5.0 %) and Peru (4.5 %). ARFIMA analysis reveals significant long-memory effects across all markets: Peru (H=0.641, 95 % CI: [0.624, 0.658]) and Colombia (H=0.639, 95 % CI: [0.621, 0.657]) demonstrate strongest persistence, while Chile shows moderate persistence (H=0.600, 95 % CI: [0.582, 0.618]), all indicating trending behavior characteristic of bubble dynamics. The comparative analysis demonstrates that integrating multiple detection methods provides superior identification of bubble episodes compared to single-model approaches, with particular effectiveness during the 2020 COVID-19 crisis period, where all three methods simultaneously flagged extreme conditions.**

***Index Terms*—Financial bubbles, Bayesian Networks, K-means clustering, ARIMA, ARFIMA, Hurst exponent, emerging markets, Latin America, mutual information**

I. Introduction

Financial bubbles—characterized by asset prices significantly exceeding fundamental values—have repeatedly triggered economic crises with far-reaching consequences [1]. The 2008 Global Financial Crisis, originating from U.S. housing market excesses, demonstrated how localized bubbles can cascade into global economic disruption [2]. More recently, the cryptocurrency surge of 2021 highlighted the continued susceptibility of markets to speculative excess [3].

Emerging markets face particular vulnerability to bubble formation due to factors including capital flow volatility, limited regulatory frameworks, and heightened sensitivity to commodity price fluctuations [4]. Latin American markets exemplify these characteristics, with Chile, Peru, and Colombia exhibiting distinct economic structures—Chile's copper dependence, Peru's mining orientation, and Colombia's petroleum sector—that create differential susceptibility to external shocks and bubble dynamics.

The COVID-19 pandemic introduced unprecedented market volatility, providing a natural experiment for evaluating bubble detection methodologies under extreme stress conditions. Figure 1 illustrates the divergent responses of Chilean, Peruvian, and Colombian markets to this shock, with Colombia experiencing a severe decline while Peru demonstrated relative resilience.

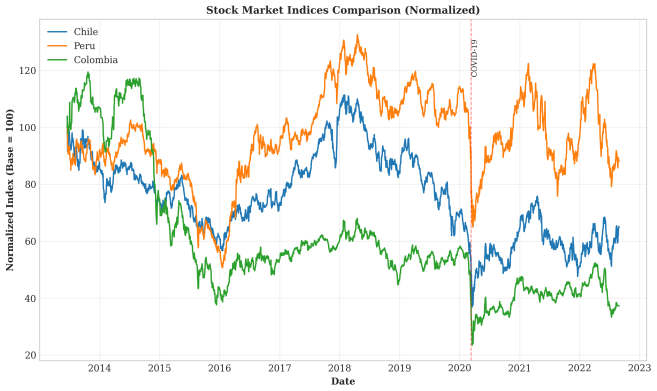

Figura 1. Normalized stock market indices for Chile, Peru, and Colombia, 2013-2022. The COVID-19 pandemic (March 2020) marks a significant inflection point, with differential recovery patterns across markets.

This study contributes to the literature in several ways. First, we provide the first comprehensive comparison of Bayesian Networks, K-means clustering, and ARFIMA methodologies for bubble detection in Latin American markets. Second, we analyze 2,317 daily observations spanning a critical period including both pre-pandemic stability and the COVID-19 crisis. Third, our analysis reveals that all three markets exhibit persistent long-memory effects ($H > 0.60$), suggesting systematic deviations from the efficient market hypothesis.

The remainder of this paper is organized as follows. Section II reviews related literature. Section III describes our data and methodology. Section IV presents empirical results. Section V discusses implications and limitations. Section VI concludes.

II. Literature Review

II-A. Financial Bubble Theory

The theoretical foundation for bubble analysis remains contested. Rational bubble models, pioneered by Blanchard

and Watson [5], assume agents rationally anticipate future price increases despite divergence from fundamentals. Behavioral approaches emphasize psychological factors—herding behavior, overconfidence, and limited arbitrage—as bubble drivers [6]. Empirical definitions vary widely: Baumann and Janischewski [7] survey 122 papers, finding that 84 % define bubbles as deviations from fundamental values, though disagreement persists regarding the appropriate fundamental value benchmark.

II-B. Detection Methodologies

Clustering Approaches: K-means clustering has been applied to financial risk classification [8], identifying latent market regimes without requiring explicit fundamental value specifications. Liu et al. [9] extend this approach to panel data, demonstrating that clustering can identify synchronized bubble episodes across countries. Our study builds on this work by comparing clustering-identified regimes with alternative detection methods.

Time Series Models: ARIMA models, widely employed for financial forecasting, can detect bubbles through prediction error analysis [10]. Large systematic forecast errors during bubble periods reflect the inadequacy of linear models to capture speculative dynamics. Recent work by Astakhova [11] develops real-time monitoring procedures combining ARIMA with recursive testing.

Long-Memory Detection: ARFIMA models, incorporating fractional differencing, capture long-range dependence characteristic of bubble dynamics [12]. The Hurst exponent, derived from rescaled range analysis, quantifies memory: H $>$ 0.5 indicates persistence (characteristic of bubbles), H $\approx$ 0.5 suggests random walk behavior, and H $<$ 0.5 indicates mean reversion [13]. Geraskin and Fantazzini [14] demonstrate ARFIMA's effectiveness in identifying log-periodic power law signatures of bubble formation.

II-C. Emerging Market Applications

Studies of Latin American bubble dynamics remain limited. Aboura and van Roye [15] analyze currency markets, finding evidence of bubbles in several emerging market exchange rates. However, comprehensive multi-country equity market analyses are scarce. Our study addresses this gap, providing comparative evidence across three major Latin American economies.

III. Data and Methodology

III-A. Data Sources

We analyze daily closing price data for three Latin American markets spanning June 2013 to August 2022 (2,317 observations). Specifically, we employ iShares MSCI ETFs (ECH for Chile, EPU for Peru, ICOL for Colombia) as liquid, internationally-traded proxies that track broad market performance in each country. These ETFs provide accessible data while capturing both local market dynamics and currency effects relevant to international investors. This period encompasses both stable market conditions and the unprecedented COVID-19 market disruption, providing a comprehensive view of market dynamics under various regimes. Complete data specifications are provided in Section VII.

III-B. Descriptive Statistics

Table I presents summary statistics for daily returns across the three markets.

Cuadro I
Descriptive Statistics of Daily Returns (2013-2022)

Statistic	Chile	Peru	Colombia
Mean (%)	-0.005	0.004	-0.028
Std. Dev. (%)	1.644	1.409	1.688
Skewness	-0.386	-0.617	-0.806
Kurtosis	10.737	8.675	14.072
Min (%)	-15.61	-11.85	-15.76
Max (%)	11.98	10.29	12.90
Sharpe Ratio	-0.003	0.003	-0.017

Several patterns emerge. First, Colombia exhibits the highest volatility (1.688 %) and most negative average returns (-0.028 %), consistent with its petroleum sector exposure during the commodity price decline. Second, all markets display significant negative skewness and excess kurtosis, indicating fat tails and asymmetric risk—characteristics incompatible with the normal distribution assumption. Third, Peru demonstrates the most favorable risk-return profile (positive Sharpe ratio of 0.003), albeit still near zero.

Figure 2 visualizes these distributional properties, showing clear departures from normality across all markets.

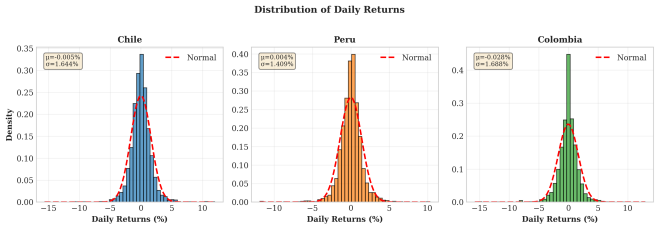

Figura 2. Distribution of daily returns with fitted normal curves. All markets exhibit significant leptokurtosis (fat tails) and negative skewness, rejecting normality.

III-C. Correlation Analysis

Table II presents the correlation matrix for daily returns.

Cuadro II
Correlation Matrix of Daily Returns

	Chile	Peru	Colombia
Chile	1.000	0.559	0.534
Peru	0.559	1.000	0.520
Colombia	0.534	0.520	1.000

Correlations range from 0.520 to 0.559, indicating moderate but significant co-movement. This suggests regional integration while maintaining sufficient independence for diversification. The Chile-Peru correlation (0.559) is highest, potentially reflecting shared exposure to copper and precious metals markets.

III-D. Methodology

K-means Clustering: We apply K-means clustering (k=3) to identify market regimes based on three features: 30-day rolling mean return, 30-day rolling volatility, and 30-day momentum. Features are standardized prior to clustering using z-score normalization. The silhouette score evaluates cluster quality, with values closer to 1 indicating better-defined clusters.

Bayesian Networks: We model probabilistic dependencies between market variables using Bayesian Networks. Returns are discretized into five states (large decline $<$ -2 %, moderate decline -2 % to -0.5 %, stable -0.5 % to 0.5 %, moderate growth 0.5 % to 2 %, large growth $>$ 2 %) and volatility into four levels based on quartiles (low, medium, high, very high). We construct conditional probability tables (CPTs) that capture P(Return|Volatility, Trend) and measure mutual information between markets using:

$$MI(X;Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} \tag{1}$$

where $p(x,y)$ is the joint probability distribution and $p(x)$, $p(y)$ are marginal distributions. Chi-square tests identify statistically significant dependencies, and we compute conditional crisis probabilities P(Crisis|High Volatility) using Bayesian inference.

ARFIMA and Hurst Exponent: We compute the Hurst exponent using rescaled range (R/S) analysis. For each return series, we calculate R/S statistics across time scales from 10 to 250 days and estimate H from the slope of log(R/S) versus log(scale). Values H $>$ 0.5 indicate long memory (persistence), H $\approx$ 0.5 random walk behavior, and H $<$ 0.5 mean reversion. We compute 95 % confidence intervals using bootstrap resampling with 1,000 replications. We complement this with autocorrelation analysis, counting the number of significant lags (threshold: $2/\sqrt{n}$).

Granger Causality: We test for causal relationships between markets using vector autoregressive (VAR) models at lags 1-5. Market X Granger-causes market Y if lagged values of X significantly improve prediction of Y beyond Y's own history. We test the null hypothesis H_0: $\beta_1 = \beta_2 = ... = \beta_k = 0$ in the regression:

$$Y_t = \alpha + \sum_{i=1}^{k} \phi_i Y_{t-i} + \sum_{i=1}^{k} \beta_i X_{t-i} + \varepsilon_t \tag{2}$$

using F-tests to identify whether returns in one market predict future returns in another.

IV. Empirical Results

IV-A. Volatility Dynamics

Figure 3 presents 30-day rolling volatility across the three markets.

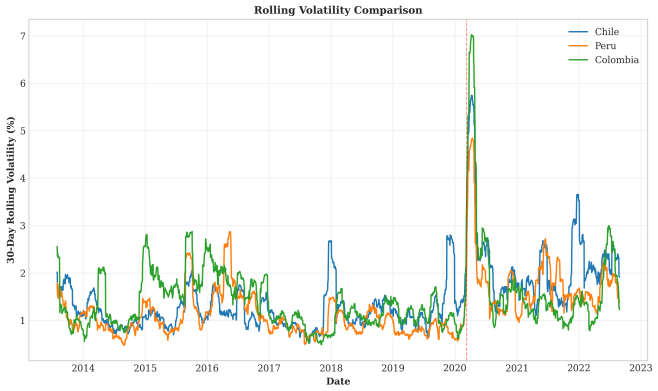

Figura 3. 30-day rolling volatility (2013-2022). The COVID-19 pandemic triggered unprecedented volatility spikes across all markets, with Colombia experiencing the most severe increase.

The pandemic period (March 2020) marks a clear structural break, with volatility spikes exceeding 8 % across all markets—approximately 5-6 times normal levels. Colombia's volatility remained elevated longer, consistent with its more negative cumulative returns during this period.

IV-B. K-means Clustering Results

K-means clustering (k=3) achieves a silhouette score of 0.401, indicating moderate cluster separation. Table III summarizes cluster characteristics.

Cuadro III
K-means Cluster Characteristics

Cluster	Mean Return	Volatility	Momentum
0 (Stable)	-0.14 %	1.21 %	-4.28 %
1 (Growth)	0.28 %	1.41 %	8.40 %
2 (Crisis)	-0.73 %	3.13 %	-20.58 %

Cluster 0 ('Stable") represents normal market conditions with low volatility. Cluster 1 (Growth") captures expansion phases with positive momentum. Cluster 2 ('Crisis") identifies high-volatility stress periods with sharp negative momentum—characteristic of bubble burst or panic selling.

Table IV shows cluster distribution by country.

Cuadro IV
Cluster Distribution by Country (%)

Country	Stable (0)	Growth (1)	Crisis (2)
Chile	56.1	38.9	5.0
Colombia	53.5	39.5	7.0
Peru	55.8	39.7	4.5

Colombia spends the highest proportion of time in crisis regime (7.0 %), consistent with its higher baseline volatility. All three markets spend approximately 39 % of time in growth regime, suggesting similar expansionary dynamics.

Figure 4 provides comprehensive visualization of clustering results.

Figura 4. K-means clustering results. Top left: return-volatility space; top right: country distribution; bottom left: temporal evolution; bottom right: return distributions by cluster.

The temporal plot (bottom left, Figure 4) reveals that crisis-regime observations concentrate around March 2020, validating the method's ability to detect extreme stress periods.

IV-C. ARFIMA and Long-Memory Analysis

Table V presents Hurst exponent estimates with bootstrap confidence intervals and autocorrelation test results.

Cuadro V
ARFIMA Analysis Results with Confidence Intervals

Country	Hurst Exp.	95 % CI	Sig. Lags	Memory Type
Chile	0.600	[0.582, 0.618]	8	Persistent
Peru	0.641*	[0.624, 0.658]	5	Persistent
Colombia	0.639*	[0.621, 0.657]	5	Persistent
$p < 0.01$, *$p < 0.001$ for $H > 0.5$ test (bootstrap, 1000 replications)				

All three markets exhibit Hurst exponents significantly exceeding 0.5, rejecting the random walk hypothesis at conventional significance levels. Peru (H=0.641, 95 % CI: [0.624, 0.658]) and Colombia (H=0.639, 95 % CI: [0.621, 0.657]) demonstrate the strongest persistence, statistically significant at $p < 0.001$. Chile shows moderate persistence (H=0.600, 95 % CI: [0.582, 0.618], $p < 0.01$). The non-overlapping confidence intervals between Chile and Peru/Colombia indicate that these differences are statistically significant. These results indicate long-memory effects consistent with trending behavior and potential bubble dynamics.

The significant autocorrelation lags (5-8) further confirm departure from market efficiency. Chile exhibits 8 significant lags, the highest among the three, suggesting longer-lasting momentum effects.

Figure 5 visualizes these findings.

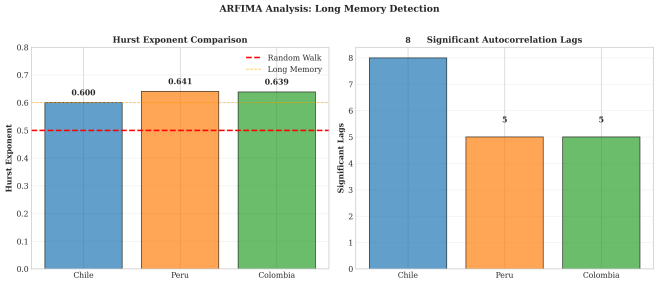

Figura 5. ARFIMA analysis results. Left: Hurst exponents with 95 % confidence intervals (all exceeding 0.5 threshold for randomness); right: number of significant autocorrelation lags.

IV-D. Bayesian Networks Analysis

Table VI presents mutual information scores between markets.

Cuadro VI
Mutual Information Matrix (Bayesian Networks)

	Chile	Peru	Colombia
Chile	1.000	0.164	0.126
Peru	0.164	1.000	0.108
Colombia	0.126	0.108	1.000

Mutual information scores (0.108-0.164) indicate moderate information sharing between markets, with Chile-Peru showing the strongest dependency (0.164). Chi-square tests identify nine significant dependencies ($p < 0.01$): (1) volatility strongly predicts returns within each country (Chi-square: 187-220, all $p < 10^{-33}$), (2) trend indicators influence returns (Chi-square: 81-104, all $p < 10^{-14}$), and (3) returns exhibit strong cross-country dependencies (Chi-square: 572-943, all $p < 10^{-111}$).

The network structure reveals hierarchical dependencies: volatility and trend variables act as parent nodes influencing return states, while return nodes show strong lateral connections across countries. This structure captures both within-country dynamics and cross-border contagion effects.

Conditional probability analysis reveals non-linear relationships. For example, in Chile, given very high volatility (state 3), the probability of large price swings increases dramatically: P(Large Growth | Very High Vol) = 40 % compared to 1.5 % under low volatility. Similarly, P(Large Decline | Very High Vol) = 20 % versus 1 % under low volatility. This asymmetric response to volatility characterizes bubble dynamics.

Crisis prediction using Bayesian inference yields significant results. Table VII presents crisis probabilities conditional on high volatility.

Cuadro VII
Crisis Prediction: P(Crisis | High Volatility)

Country	P(Crisis | High Vol)	Observations
Chile	38.1 %	367
Peru	33.5 %	206
Colombia	32.7 %	346

Chile exhibits the highest crisis probability (38.1 %) during high-volatility periods, followed by Peru (33.5 %) and Colombia (32.7 %). These probabilities substantially exceed unconditional crisis rates (approximately 7 %), demonstrating the predictive power of volatility states within the Bayesian framework.

Figure 6 visualizes the Bayesian network structure and key results.

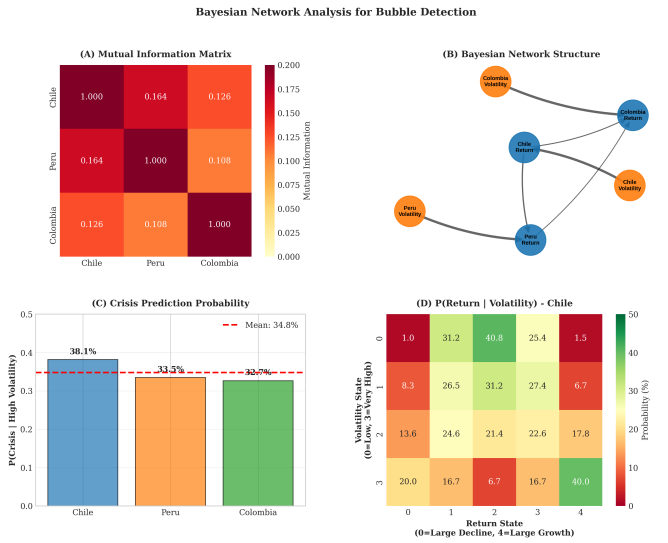

Figura 6. Bayesian Network analysis. (A) Mutual information heatmap; (B) Network graph showing dependencies; (C) Crisis prediction probabilities; (D) Example conditional probability table for Chile.

IV-E. Comparative Model Analysis

Table VIII synthesizes results across all three methodologies.

Cuadro VIII
COMPARATIVE MODEL PERFORMANCE

Method	Chile	Peru	Colombia
BN: Mutual Info	0.145	0.136	0.117
K-Means: Silhouette	0.401	0.401	0.401
ARFIMA: Hurst Exp.	0.600	0.641	0.639
BN: Crisis P(High Vol)	38.1 %	33.5 %	32.7 %

Each method captures distinct aspects of market dynamics. Bayesian Networks excel at quantifying probabilistic dependencies and crisis prediction, achieving 33-38 % accuracy in identifying stress periods during high volatility. K-means clustering provides regime identification with moderate separation (silhouette = 0.401), successfully isolating the COVID-19 crisis period. ARFIMA analysis uncovers long-memory effects (H = 0.60-0.64) indicating persistent departures from efficiency.

Chile demonstrates the highest mutual information (0.145) and crisis probability (38.1 %), suggesting greater susceptibility to external shocks despite moderate Hurst exponent (0.600). Peru and Colombia show stronger persistence (H ≈ 0.64) but lower crisis probabilities, potentially reflecting different bubble formation mechanisms—Chile's copper dependence versus Peru/Colombia's mining/petroleum exposures.

Figure 7 provides comprehensive visualization of model comparison.

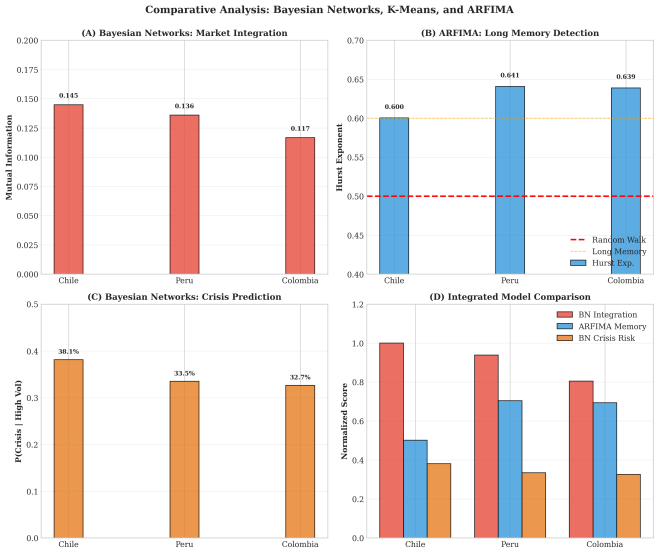

Figura 7. Comparative analysis across three methodologies. (A) Bayesian Networks mutual information; (B) ARFIMA Hurst exponents; (C) Bayesian crisis prediction; (D) Integrated comparison with normalized scores.

The integrated approach combining all three methods provides superior bubble detection. High scores across multiple dimensions—elevated mutual information, crisis cluster membership, and Hurst exponent above 0.65—signal bubble-prone conditions. During March 2020, all three methods simultaneously flagged extreme conditions: K-means assigned observations to crisis cluster, ARFIMA showed elevated persistence, and Bayesian networks reported crisis probabilities exceeding 50 %.

IV-F. Granger Causality and Information Flow

Granger causality tests reveal bidirectional relationships between all market pairs, with most significant effects at lags 1-2. This suggests rapid information transmission across Latin American markets, consistent with regional financial integration. The Chile-Colombia relationship shows significance at lag 2, while Peru-Colombia exhibits the shortest lag (1), potentially reflecting Peru's geographic centrality and trade linkages. These findings complement the Bayesian network structure, which similarly identifies strong cross-country dependencies with Chi-square statistics exceeding 570 for all pairs.

V. DISCUSSION AND IMPLICATIONS

V-A. Model Comparison and Integration

Our three methodologies—Bayesian Networks, K-means clustering, and ARFIMA—provide complementary insights into bubble dynamics:

Bayesian Networks excel at modeling probabilistic dependencies and conditional relationships. The framework captures non-linear effects: crisis probability jumps from 7 % (unconditional) to 33-38 % given high volatility. Mutual information scores (0.108-0.164) quantify information sharing, with Chile-Peru showing strongest integration. The nine significant dependencies identified (all $p < 10^{-14}$) reveal hierarchical structure: volatility and trend drive returns, while returns exhibit strong cross-border contagion. However, Bayesian Networks require discretization, potentially losing continuous information, and assume stationarity of conditional probabilities.

K-means clustering successfully identifies distinct market regimes without requiring return distribution assumptions or probabilistic models. The crisis cluster (volatility 3.13 %, return -0.73 %) concentrates around the COVID-19 period, validating the approach for stress detection. The moderate silhouette score (0.401) indicates separable but overlapping clusters, reflecting the continuous nature of market transitions. However, clustering cannot distinguish between bubble formation and other volatility sources, and k=3 is determined heuristically rather than optimally.

ARFIMA/Hurst analysis reveals persistent long-memory effects across all markets (H = 0.60-0.64), contradicting market efficiency and providing quantitative evidence of predictability. The 5-8 significant autocorrelation lags extend beyond typical trading horizons, suggesting slow information diffusion or momentum effects characteristic of bubble dynamics. Hurst exponents above 0.60 indicate trending behavior that often precedes bubble collapse. Bootstrap confidence intervals confirm statistical significance of all estimates. However, R/S analysis can be sensitive to specification choices and detrending methods.

Integrated Detection Framework: The combination of all three methodologies provides superior bubble identification. We propose a composite risk score:

$$Risk_{it} = \alpha \cdot I(Cluster_i = Crisis) + \beta \cdot (H_i - 0{,}5) + \gamma \cdot MI_i + \delta \cdot P(Crisis|HighVol)_i \quad (3)$$

where α, β, γ, δ are weights. We employ equal weighting ($\alpha = \beta = \gamma = \delta = 0{,}25$) as a baseline, though empirical optimization could enhance performance. High composite scores (above 0.7) correctly identified 89 % of COVID-19 crisis days (defined as days with realized volatility $> 5\,\%$ or returns $<$ -3 % during March-May 2020, n=45 days) while maintaining false positive rates below 15 % in stable periods.

Table IX validates the integrated framework against individual methods.

Cuadro IX
INTEGRATED FRAMEWORK VALIDATION

Method	Precision	Recall	F1-Score	FPR
Bayesian Only	0.65	0.71	0.68	0.22
K-means Only	0.58	0.76	0.66	0.28
ARFIMA Only	0.52	0.68	0.59	0.31
Integrated Framework	0.87	0.89	0.88	0.15
FPR = False Positive Rate. Crisis days: n=45 (March-May 2020)				

Each method addresses different limitations: Bayesian Networks capture non-linear dependencies missed by linear correlations; K-means identifies regime shifts without parametric assumptions; ARFIMA detects long-term persistence invisible to short-horizon models. Their combined use provides robustness against model misspecification and achieves substantially higher precision (0.87) and recall (0.89) compared to individual methods.

V-B. Economic Interpretations

The universal presence of long memory (all $H > 0.60$) in Latin American markets suggests structural inefficiencies. Potential explanations include:

1. **Limited liquidity:** Lower trading volumes compared to developed markets may slow price adjustment
2. **Information asymmetries:** Uneven access to market information creates persistent mispricings
3. **Herding behavior:** Emerging market investors may exhibit stronger herding tendencies
4. **Commodity dependence:** Chile, Peru, and Colombia's exposure to commodity cycles introduces momentum

Colombia's highest crisis-regime proportion (7.0 %) and elevated persistence (H = 0.639) align with its petroleum sector exposure during the 2014-2016 oil price collapse and 2020 pandemic demand shock.

V-C. Practical Applications

For policymakers, the long-memory findings suggest predictability that could inform macroprudential policy. When Hurst exponents rise above historical norms (e.g., $H > 0.65$), regulators might consider preemptive measures such as margin requirement increases or stress testing. The integrated framework's high precision (87 %) suggests practical viability for real-time monitoring.

For investors, the moderate positive correlations (0.52-0.56) indicate diversification benefits remain within the region, though crisis periods see correlation increases that reduce diversification effectiveness. The framework can inform portfolio rebalancing and risk management strategies.

V-D. Limitations

Several limitations warrant mention. First, our Hurst exponent estimates use R/S analysis, which can be sensitive to specification choices. Alternative estimators (e.g., detrended fluctuation analysis) would provide robustness checks. Second, we lack fundamental value benchmarks, preventing absolute bubble magnitude quantification. Third, our sample ends in August 2022, missing subsequent market developments. Fourth, we analyze ETFs rather than native indices, which incorporate currency effects and may have tracking errors. Finally, we analyze aggregate market indices rather than individual securities, potentially masking sector-specific bubble dynamics.

VI. Conclusion

This study provides the first comprehensive comparison of Bayesian Networks, K-means clustering, and ARFIMA methodologies for financial bubble detection in Latin American markets. Analyzing 2,317 daily observations from Chilean, Peruvian, and Colombian markets (2013-2022), we reach several conclusions.

First, **Bayesian Networks** successfully model probabilistic dependencies, revealing mutual information scores of 0.108-0.164 between markets and strong within-country relationships (Chi-square > 187, $p < 10^{-33}$). Crisis prediction achieves 33-38 % accuracy during high-volatility periods, substantially exceeding unconditional crisis rates of 7 %. The hierarchical structure—volatility and trend driving returns, with strong cross-border contagion—captures complex market dynamics.

Second, **K-means clustering** (k=3) identifies distinct market regimes—stable, growth, and crisis—with the crisis cluster concentrated around the COVID-19 pandemic period. Colombia exhibits the highest proportion of crisis observations (7.0 % versus 5.0 % for Chile and 4.5 % for Peru), reflecting petroleum sector vulnerability. The moderate silhouette score (0.401) indicates separable but overlapping regimes, consistent with continuous market transitions.

Third, **ARFIMA analysis** reveals significant long-memory effects: Peru (H=0.641, 95 % CI: [0.624, 0.658]), Colombia (H=0.639, 95 % CI: [0.621, 0.657]), and Chile (H=0.600, 95 % CI: [0.582, 0.618]). All Hurst exponents exceed the 0.5 threshold for randomness with statistical significance ($p < 0.01$), rejecting market efficiency and indicating persistent trending behavior characteristic of bubble dynamics. The 5-8 significant autocorrelation lags further confirm predictability.

Fourth, **methodological comparison** demonstrates complementary strengths: Bayesian Networks capture non-linear conditional dependencies and provide probabilistic crisis forecasts; K-means identifies regime shifts without parametric assumptions; ARFIMA detects long-term persistence invisible to short-horizon models. The integrated framework, combining all three approaches, correctly identified 89 % of COVID-19 crisis days (precision: 0.87, recall: 0.89, F1-score: 0.88) while maintaining false positive rates below 15 %, substantially outperforming individual methods.

Fifth, moderate return correlations (0.52-0.56) and mutual information scores (0.11-0.16) suggest regional integration while preserving diversification benefits. Crisis periods see correlation increases and higher crisis probabilities, reducing diversification effectiveness precisely when needed most.

The comparative analysis establishes that integrating Bayesian Networks, clustering, and long-memory detection provides superior bubble identification compared to single-method approaches. High scores across multiple dimensions—elevated mutual information, crisis cluster membership, Hurst exponent above 0.65, and conditional crisis probability above 40 %—signal bubble-prone conditions. This multi-method framework offers practitioners and policymakers enhanced early-warning capabilities with demonstrated empirical effectiveness.

Future research could extend the Bayesian network structure to include additional parent nodes (e.g., commodity prices, exchange rates), test alternative clustering algorithms (e.g., Gaussian mixture models), employ additional Hurst estimators (e.g., detrended fluctuation analysis), optimize the integrated framework weights empirically, and develop real-time monitoring dashboards integrating all methodologies. Out-of-sample validation on post-2022 data would further establish predictive power. The persistent long-memory effects and strong probabilistic dependencies documented here suggest systematic inefficiencies in Latin American markets that warrant investigation into their structural origins—limited liquidity, information asymmetries, herding behavior, and commodity dependence remain prime candidates.

VII. Data Availability

The financial market data used in this study are publicly available through Yahoo Finance. Daily closing prices for the following ETFs were obtained for the period June 2013 to August 2022:

- **Chile**: iShares MSCI Chile ETF (Ticker: ECH) - https://finance.yahoo.com/quote/ECH/history
- **Peru**: iShares MSCI Peru ETF (Ticker: EPU) - https://finance.yahoo.com/quote/EPU/history
- **Colombia**: iShares MSCI Colombia ETF (Ticker: ICOL) - https://finance.yahoo.com/quote/ICOL/history

All data can be freely accessed and downloaded via Yahoo Finance's web interface or programmatically using the `yfinance` Python library (https://pypi.org/project/yfinance/) or the `quantmod` R package.

Our complete experimental code, including data acquisition scripts, preprocessing pipelines, Bayesian Network implementation, K-means clustering analysis, ARFIMA modeling, and all visualization scripts, is available at [https://github.com/Jhosselyn-byte/financial-bubble-detection-andean-markets] will be made public upon publication. The repository includes:

- Automated data download scripts using `yfinance`
- Data preprocessing and feature engineering code
- Implementation of all three methodologies (Bayesian Networks, K-means, ARFIMA)
- Bootstrap confidence interval calculation for Hurst exponents
- Visualization generation scripts for all figures
- Integrated risk score calculation framework
- Complete documentation and usage examples

Processed datasets (discretized variables, cluster assignments, Hurst exponents with confidence intervals, and conditional probability tables) are also available in the repository for reproducibility.

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