# Contents

# 1 Setting

## 1.1 Header

```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <iostream>
#include <memory.h>
#include <math.h>
#include <assert.h>
#include <queue>
#include <map>
#include <set>
#include <string>
#include <algorithm>
#include <functional>
#include <vector>
#include <stack>

using namespace std;
typedef long long ll;
typedef unsigned long long ull;
typedef pair<int, int> Pi;
typedef pair<ll,ll> Pll;

#define Fi first
#define Se second
#define pb(x) push_back(x)
#define sz(x) (int)x.size()
#define rep(i, n) for(int i=0;i<n;i++)
#define repp(i, n) for(int i=1;i<=n;i++)
#define all(x) x.begin(), x.end()

#define INF 987654321
#define IINF 987654321987654321
```

## 1.2 vimrc

```
syntax on
set nu ai ci si nobk et ar ru nocp hls
set bs=2 ts=4 sw=4 sts=4
set cb=unnamed
set mouse=an
command PS vsp %:r.in|sp %:r.out|vert res 30|wa
command RIO wall|!g++ -O2 -std=c++14 -Wall -lm %:r.cpp && ./a.out < %:r.in > %:r
  .out
command RI  wall|!g++ -O2 -std=c++14 -Wall -lm %:r.cpp && ./a.out < %:r.in
```

## 1.3 Sublime text

```
{
    "shell_cmd": "g++ -O2 -std=c++11 \"${file}\" -o \"${file_path}/${
      file_base_name}\" && \"${file_path}/${file_base_name}\" < input.txt",
    "working_dir": "${file_path}",
    "selector": "source.c++",
}
```

# 2 String

## 2.1 KMP

```
vector<int> preprocess(string p){
    int m = p.size();
    vector<int> fail(m);
    fail[0] = 0; int j = 0;
    for(int i=1;i<m;i++){
        while(j>0&&p[i]!=p[j]) j = fail[j-1];
        if( p[i] == p[j] ){
            fail[i] = j+1; j++;
        }else{
            fail[i] = 0;
        }
    }
    return fail;
}

vector<int> kmp(string s, string p){
    auto fail = preprocess(p);
    vector<int> ans; int n = s.size(), m = p.size();
    int j = 0;
    for(int i=0;i<n;i++){
        while(j>0 && s[i]!=p[j]) j = fail[j-1];
        if( s[i] == p[j] ){
            if( j == m-1 ){
                ans.pb(i-m+1); j = fail[j];
            }else{
                j++;
            }
        }
    }
    return ans;
}
```

## 2.2 Aho Chorasick

```
struct AhoCorasick{
    struct Node{
        int fail;
        vector<int> output;
```

```
        int children[26];

        Node(){
            for(int i=0;i<26;i++) children[i] = -1;
            fail = -1;
        }
    };

vector<Node> trie;
int new_node(){
    Node x;
    trie.push_back(x);
    return (int)trie.size()-1;
}

void add(int node, string &s, int idx, int string_num){
    //cout << node << " " << idx << endl;
    if( idx == s.size() ){
        trie[node].output.push_back(string_num);
        return;
    }
    int c = s[idx] - 'a';
    if( trie[node].children[c] == -1 ){
        int next = new_node();
        trie[node].children[c] = next;
    }

    add(trie[node].children[c], s, idx+1, string_num);
}

void build(vector<string> v){
    int root = new_node();
    for(int i=0;i<v.size();i++){
        add(root,v[i],0,i);
    }

    queue<int> q;
    q.push(root); trie[root].fail = root;
    while( !q.empty() ){
        int cur = q.front(); q.pop();
        for(int i=0;i<26;i++){
            int next = trie[cur].children[i];
            if( next == -1 ) continue;

            // build fail
            if( cur == root ){
                trie[next].fail = root;
            }
            else{
                int x = trie[cur].fail;
                while( x != root && trie[x].children[i] == -1 ) x = trie[x].
                  fail;
                if( trie[x].children[i] != -1 ) x = trie[x].children[i];
                trie[next].fail = x;
            }
```

```
                // build output
                int f = trie[next].fail;
                for(auto e : trie[f].output) trie[next].output.push_back(e);
                q.push(next);
            }
        }
    }

    vector<Pi> find(string s){
        int n = (int) s.size();
        int cur = 0, root = 0;
        vector<Pi> ans;
        for(int i=0;i<n;i++){
            int c = s[i]-'a';
            while( cur != root && trie[cur].children[c] == -1 ) cur = trie[cur].
             fail;
            if( trie[cur].children[c] != -1 ) cur = trie[cur].children[c];

            for(auto e : trie[cur].output){
                ans.push_back({e,i});
            }

        }
        return ans;
    }
};
```

## 2.3 Suffix array

```
// Make sure to add !, #, $, %, & at the end of input string
class SuffixArray{
public:
    int n;
    string s;
    vector<int> rank, temprank, sa, tempsa, c;
    vector<int> lcp;
    SuffixArray(string _s){
        n = _s.size(); s = _s;
        rank.resize(n); temprank.resize(n); sa.resize(n); tempsa.resize(n);
        lcp.resize(n);
        constructSA();
        constructLCP();
    }

    void countingSort(int k){
        int sum = 0, maxi = max(270, n); //ASCII 256
        c.clear(); c.resize(maxi+10);
        for(auto& e : c ) e = 0;
        for(int i=0; i<n; i++) c[ i+k<n ? rank[i+k] : 0 ] ++;
        for(int i=0; i<maxi; i++){
            int t = c[i]; c[i] = sum; sum += t;
        }
        for(int i=0; i<n; i++) tempsa[ c[ sa[i]+k < n ? rank[sa[i]+k] : 0 ] ++ ]
          = sa[i];
```

```
        for(int i=0; i<n; i++) sa[i] = tempsa[i];
    }

    void constructSA(){
        for(int i=0; i<n; i++) rank[i] = s[i];
        for(int i=0; i<n; i++) sa[i] = i;
        for(int k=1; k<n; k<<=1){
            countingSort(k);
            countingSort(0);
            int r = 0;
            temprank[sa[0]] = 0;
            for(int i=1; i<n; i++){
                temprank[sa[i]] = (rank[sa[i]] == rank[sa[i-1]] && rank[sa[i]+k]
                    == rank[sa[i-1]+k] ) ? r : ++r;
            }
            for(int i=0; i<n; i++) rank[i] = temprank[i];
            if( rank[sa[n-1]] == n-1 ) break;
        }
    }

    // lcp Implementation from
    // http://m.blog.naver.com/dark__nebula/220419358547
    void constructLCP(){
        int h = 0;
        for(int i=0;i<n;i++){
            if( rank[i] ){
                int j = sa[rank[i]-1];
                while( s[i+h] == s[j+h] ) h++;
                lcp[rank[i]] = h;
            }
            if( h > 0 ) h--;
        }
    }

};
```

## 2.4 Manacher's algorithm

```
// finds radius of longest palindrome centered at s[i]
// If you also want to find even-length paindromes, use dummy characters
// baab -> #b#a#a#b#
vector<int> ManacherAlgorithm(string s){
    int n = (int) s.size();
    int p = -1, r = -1;
    vector<int> A(n);
    for(int i=0;i<n;i++){

        if( r < i ){
            A[i] = 0;
            int j = 0;
            while( i + A[i] < n && i - A[i] >= 0 && s[ i+A[i] ] == s[ i-A[i] ] )
                A[i]++;
```

```
        A[i]--;
    }
    else{
        A[i] = min( A[2*p - i] , r-i );
        while( i + A[i] < n && i - A[i] >= 0 && s[ i+A[i] ] == s[ i-A[i] ] )
            A[i]++;
        A[i]--;
    }

    // update r
    if( r < i + A[i] ){
        r = i + A[i];
        p = i;
    }
}
    return A;
}
```

## 2.5   Z algorithm

```
// Calculates LCP[i] for all 0 <= i < n
vector<int> Zalgorithm(string s){
    int l=0, r=0;
    int n = (int) s.size();
    vector<int> Z(n);
    Z[0] = n;
    for(int i=1; i<n; i++){
        // reset and calculate again
        if( i > r ){
            l = r = i;
            while( r<n && s[r] == s[r-l] ) r++;
            r--;
            Z[i] = r-l+1;
        }

        // extend [l,r]
        else{
            int k = i-l;
            // not enough matching at position k
            if( Z[k] < r-i+1 ) Z[i] = Z[k];
            // enough matching. extend [l,r]
            else{
                l = i;
                while( r<n && s[r] == s[r-l] ) r++;
                r--;
                Z[i] = r-l+1;
            }
        }
    }
    return Z;
};
```

# 3   Graph & Flow

## 3.1   Dinic

```
struct MaxFlowDinic{
    struct Edge{
        // next, inv, residual
        int to, inv; ll res;
    };

    int n;
    vector<vector<Edge>> graph;

    vector<int> lev,work;

    void init(int x){
        n = x+10;
        graph.resize(x+10);
        lev.resize(n); work.resize(n);
    }

    void make_edge(int s, int e, ll cap, ll caprev = 0){
        Edge forward = {e, (int)graph[e].size(), cap};
        Edge backward = {s, (int)graph[s].size(), caprev};
        graph[s].push_back(forward);
        graph[e].push_back(backward);
    }

    bool bfs(int source, int sink){
        queue<int> q;
        for(auto& e : lev) e = -1;
        lev[source] = 0; q.push(source);
        while(!q.empty()){
            int cur = q.front(); q.pop();
            for(auto e : graph[cur]){
                if(lev[e.to]==-1 && e.res > 0){
                    lev[e.to] = lev[cur]+1;
                    q.push(e.to);
                }
            }
        }
        return lev[sink] != -1;
    }

    ll dfs(int cur, int sink, ll flow){
        if( cur == sink ) return flow;
        for(int &i = work[cur]; i < (int)graph[cur].size(); i++){
            Edge &e =  graph[cur][i];
            if( e.res == 0 || lev[e.to] != lev[cur]+1 ) continue;
            ll df = dfs(e.to, sink, min(flow, e.res) );
            if( df > 0 ){
                e.res -= df;
                graph[e.to][e.inv].res += df;
                return df;
```

```
            }
        }
        return 0;
    }


    ll solve( int source, int sink ){
        ll ans = 0;
        while( bfs(source, sink) ){
            for(auto& e : work) e = 0;
            while( true ){
                ll flow = dfs(source,sink,54321987654321LL);
                if( flow == 0 ) break;
                ans += flow;
            }
        }
        return ans;
    }

};
```

## 3.2  Bipartite matching (simple)

```
int yx[5000], xy[5000];
bool vis[5000];
vector<int> E[5000];
int dfs(int x){
    vis[x] = true;
    for(auto e : E[x]){
        if( yx[e] == -1 || (vis[yx[e]] == false && dfs(yx[e]) )  ){
            yx[e] = x;
            xy[e] = e;
            return 1;
        }
    }
    return 0;
}

int main(){
    memset(yx,-1,sizeof yx);
    int ans = 0;
    rep(i,N){
        memset(vis,0,sizeof vis);
        ans += dfs(i);
    }
    cout << ans;

}
```

## 3.3  MCMF

```
struct MCMF{
    struct edge{
```

```
        int to, inv, cap, flow, cost;
        int res(){
            return cap - flow;
        }
};

vector<vector<edge>> graph;
vector<int> pv, pe;
vector<int> dist, inq;

void init(int x){
    graph.resize(x+10);
    for(auto& e : graph) e.resize(x+10);
    pv.resize(x+10); pe.resize(x+10);
    dist.resize(x+10);
    inq.resize(x+10);
}

void make_edge(int from, int to, int cap, int cost){
    //printf("%d -> %d | cost = %d\n",from,to,cost);
    edge forward = {to, (int)graph[to].size(), cap, 0, cost};
    edge backward = {from, (int)graph[from].size(), 0, 0, -cost};
    graph[from].push_back(forward);
    graph[to].push_back(backward);
}

int solve(int source, int sink){
    int ans = 0;
    int totalflow = 0;
    while(true){
        for(auto& e : dist) e = INF;
        for(auto& e : inq) e = 0;
        queue<int> q;
        q.push(source); inq[source] = 1;
        dist[source] = 0;

        while(!q.empty()){
            int cur = q.front(); q.pop();
            inq[cur] = 0;
            for(int i=0;i<(int)graph[cur].size();i++){
                auto& e = graph[cur][i];
                if( e.res() > 0 && dist[e.to] > dist[cur] + e.cost ){
                    dist[e.to] = dist[cur] + e.cost;
                    pv[e.to] = cur; pe[e.to] = i;
                    if( inq[e.to] == 0 ){
                        q.push(e.to); inq[e.to] = 1;
                    }
                }
            }
        }

        if( dist[sink] == INF ) break;

        // add this limit when we don't require maxflow
        //if( dist[sink] > 0 ) break;
```

```
            int mnflow = INF;
            for( int v = sink; v != source; v = pv[v] ){
                mnflow = min( mnflow, graph[pv[v]][pe[v]].res() );
            }

            for( int v = sink; v != source; v = pv[v] ){
                int tmp = graph[pv[v]][pe[v]].inv;
                graph[pv[v]][pe[v]].flow += mnflow;
                graph[v][tmp].flow -= mnflow;
            }
            totalflow += mnflow;
            ans += dist[sink] * mnflow;
        }
        return ans;
    }

};
```

## 3.4 Articulation Point

```
int N,M,cnt=0;

// DFS discover time of vertex
int vis[100500];
vector<int> E[100500];
set<int> articulation;

// Returns the earlist discover time that x's child can visit
// without using x
int dfs(int x, int p){
    vis[x] = ++cnt;
    int child = 0;
    int res = vis[x];
    for(auto e : E[x]){
        if(vis[e]==0){
            // low : the earlist discover time that e can visit
            // without using x
            int low = dfs(e,x);
            child++;
            // check if not root
            if( p != -1 && low >= vis[x] ) articulation.insert(x);
            res = min(res,low);
        }
        else{
            res = min(res,vis[e]);
        }
    }

    // check if root
    if( p == -1 && child >= 2 ) articulation.insert(x);

    return res;
}
```

```
int main()
{
    geti(N,M);
    rep(i,M){
        int a,b; geti(a,b);
        E[a].pb(b); E[b].pb(a);
    }

    repp(i,N) if( vis[i] == 0 ) dfs(i,-1);

    printf("%d\n",(int)articulation.size());
    for(auto e : articulation) printf("%d ",e);
}
```

## 3.5 Articulation Edge

```
int N,M,cnt=0;

// DFS discover time of vertex
int vis[100500];
vector<int> E[100500];
set<pair<int,int>> articulation;

// Returns the earlist discover time that x's child can visit
// without using edge (p,x)
int dfs(int x, int p){
    vis[x] = ++cnt;
    int child = 0;
    int res = vis[x];
    for(auto e : E[x]){
        if(e==p) continue;
        if(vis[e]==0){
            // low : the earlist discover time that e can visit
            // without using edge (x,e)
            int low = dfs(e,x);
            child++;
            // keep in mind: in edge problem, low==vis[x] case
            // is not considered as articulation edge
            // also, root checking is not needed
            if( low > vis[x] )
                articulation.insert({min(e,x),max(e,x)});
            res = min(res,low);
        }
        else{
            res = min(res,vis[e]);
        }
    }

    // no root check needed for edge problem

    return res;
}
```

```
int main()
{
    geti(N,M);
    rep(i,M){
        int a,b; geti(a,b);
        E[a].pb(b); E[b].pb(a);
    }

    repp(i,N) if( vis[i] == 0 ) dfs(i,-1);

    printf("%d\n",(int)articulation.size());
    for(auto e : articulation) printf("%d %d\n",e.first,e.second);
}
```

## 3.6  2SAT & answer recover

```
#define MAX_V 20010
int V,M;

vector<int> Edge[MAX_V];
vector<int> rEdge[MAX_V];
vector<int> vs;

bool vis[MAX_V];
int cmp[MAX_V];
set<int> printSet[MAX_V];

void addEdge(int from, int to){
    Edge[from].push_back(to);
    rEdge[to].push_back(from);
}

void dfs(int v){
    vis[v] = true;
    for (int i = 0; i < Edge[v].size(); i++){
        if (!vis[Edge[v][i]]) dfs(Edge[v][i]);
    }
    vs.push_back(v);
}

void rdfs(int v, int k){
    vis[v] = true;
    cmp[v] = k;
    printSet[k].insert(v);
    for (int i = 0; i < rEdge[v].size(); i++){
        if (!vis[rEdge[v][i]]) rdfs(rEdge[v][i], k);
    }
}

bool cmp1(set<int>& a, set<int>& b) {
    return *a.begin() < *b.begin();
}
```

```
int main()
{
    //freopen("in.txt", "r", stdin);
    geti(V); geti(M);
    int cnt = 0;
    while (M--){
        int a, b;
        scanf("%d%d", &a, &b);
        if (a > 0 && b > 0 ){
            addEdge(a + V, b);
            addEdge(b + V, a);

        }
        else if (a > 0 && b < 0){
            b = -b;
            addEdge(a + V, b + V);
            addEdge(b , a);
        }
        else if (a < 0 && b > 0){
            a = -a;
            addEdge(a, b);
            addEdge(b + V, a + V);


        }
        else{
            a = -a; b = -b;
            addEdge(a, b + V);
            addEdge(b, a + V);

        }
    }

    memset(vis, false, sizeof(vis));
    for (int i = 1; i <= 2*V; i++){
        if (!vis[i]) dfs(i);
    }


    memset(vis, false, sizeof(vis));
    int k = 0;
    for (int i = vs.size()-1; i >= 0 ; i--){
        if (!vis[vs[i]]) rdfs(vs[i],k++);
    }


    for (int i = 1; i <= V; i++){
        if (cmp[i] == cmp[V + i]){
            printf("0\n");
            return 0;
        }
    }
    printf("1\n");
```

```
        for (int i = 1; i <= V; i++){
            if (cmp[i] > cmp[V + i]){
                printf("1 ");
            }
            else printf("0 ");
        }
    }
```

## 3.7 SCC

# 4 Query

## 4.1 HLD

```
// 1-index
#define L(x) ((x)<<1)
#define R(x) (((x)<<1)+1)

const int MAXN = 100050;
const int LOGN = 17;

vector<int> adj[MAXN];
int st[6 * MAXN], sub[MAXN], pa[MAXN];
int idx[MAXN], head[MAXN], pos[MAXN], rev[MAXN];
int sz, cnt;

void init(int n) {
    fill(st, st + 6*n, INF);
    fill(head, head + n, -1);
}

void dfs(int x, int p) {
    sub[x] = 1;
    for(auto c : adj[x]) {
        if(c != p) {
            pa[c] = x;
            dfs(c, x);
            sub[x] += sub[c];
        }
    }
}

void update(int x, int id = 1, int l = 0, int r = sz) {
    if(x < l || x >= r) return;
    if(r - l <= 1) {
        if(st[id] == INF)
            st[id] = l;
        else
            st[id] = INF;
        return;
    }
    int mid = (l + r) >> 1;
```

```
    update(x, L(id), l, mid);
    update(x, R(id), mid, r);
    st[id] = min(st[L(id)], st[R(id)]);
}

int query(int x, int y, int id = 1, int l = 0, int r = sz) {
    if(y <= l || r <= x) return INF;
    if(x <= l && r <= y) return st[id];
    int mid = (l + r) >> 1;
    return min(query(x, y, L(id), l, mid), query(x, y, R(id), mid, r));
}


void HLD(int x, int p) {
    if(head[cnt] == -1)
        head[cnt] = x;
    idx[x] = cnt;
    pos[x] = sz;
    rev[sz] = x;
    sz++;

    int cindex = -1;
    for(int i = 0; i < adj[x].size(); i++) {
        if(adj[x][i] != p)
            if(cindex == -1 || sub[adj[x][cindex]] < sub[adj[x][i]])
                cindex = i;
    }
    if(cindex != -1)
        HLD(adj[x][cindex], x);
    for(int i = 0; i < adj[x].size(); i++) {
        if(adj[x][i] != p && i != cindex) {
            cnt++;
            HLD(adj[x][i], x);
        }
    }
}

int queryTree(int v) {
    if(v == 0) {
        int ans = query(pos[0], pos[0] + 1);
        if(ans == INF)
            return -1;
        else
            return 1;
    }
    int vchain, ans = INF;
    while(1) {
        vchain = idx[v];
        if(idx[v] == 0) {
            ans = min(ans, query(pos[0], pos[v]+1));
            break;
        }
        ans = min(ans, query(pos[head[vchain]], pos[v]+1));
        v = pa[head[vchain]];
    }
}
```

```
        if(ans == INF)
            return -1;
        else
            return rev[ans] + 1;
}

void updateTree(int v) {
    update(pos[v]);
}

int main() {
    int n, q;
    geti(n, q);
    for(int i = 1; i < n; i++) {
        int u, v;
        geti(u, v);
        u--; v--;
        adj[u].pb(v);
        adj[v].pb(u);
    }

    init(n);
    dfs(0, -1);
    HLD(0, -1);

    while(q--) {
        int type, x;
        geti(type, x);
        x--;
        if(type == 0) {
            updateTree(x);
        } else {
            printf("%d\n", queryTree(x));
        }
    }
}
```

## 4.2 Centroid decomposition

```
int n;
set<int> adj[MAXN];
int sub[MAXN], dep[MAXN];

void dfsSubtree(int node, int pnode) {
    sub[node] = 1;
    for(auto cnode : adj[node]) {
        if(cnode != pnode) {
            dfsSubtree(cnode, node);
            sub[node] += sub[cnode];
        }
    }
}

int findCentroid(int node, int pnode, int size) {
```

```
    for(auto cnode : adj[node]) {
        if(cnode != pnode && sub[cnode] > size / 2)
            return findCentroid(cnode, node, size);
    }
    return node;
}

bool decompose(int node, int depth) {
    bool result = true;
    if(depth >= 26) {
        return false;
    }
    dfsSubtree(node, -1);
    int ctr = findCentroid(node, -1, sub[node]);
    dep[ctr] = depth;
    for(auto cnode : adj[ctr]) {
        adj[cnode].erase(ctr);
        result &= decompose(cnode, depth + 1);
    }
    adj[ctr].clear();
    return result;
}

int main() {
    geti(n);
    rep(i, n-1) {
        int u, v;
        geti(u, v);
        adj[u].insert(v);
        adj[v].insert(u);
    }
    if(decompose(1, 0)) {
        repp(i, n) printf("%c ", dep[i] + 'A');
    } else {
        cout << "Impossible!";
    }
}
```

## 4.3 Mo's algorithm

```
int N,M,K,tc;
ll c[1000005];
ll p[1000005]; int Bsize;
typedef struct query{
    int l,r,n; ll ans;
} query;
bool cmp(query& a, query& b){
    if( a.l/Bsize == b.l/Bsize ) return a.r < b.r;
    else return a.l/Bsize < b.l/Bsize;
}
bool cmp2(query&a, query& b ){ return a.n < b.n; }
int main(void)
{
    geti(N,M); rep(i,N) scanf("%lld",p+i);
```

```
    Bsize = (int) sqrt(1.0*N);
    vector<query> q;
    rep(i,M){
        int a,b; geti(a,b); a--;b--;
        q.push_back({a,b,i});
    }

    sort(all(q),cmp);
    int l=0, r=-1; ll sum = 0;

    for(int i=0;i<q.size();i++){
        query& e = q[i];
        int ql = e.l, qr = e.r;
        while( r < qr ){
            r++;
            sum += p[r]*(2*c[p[r]]+1); c[p[r]]++;
        }
        while( r > qr ){
            sum += p[r]*(1-2*c[p[r]]); c[p[r]]--;
            r--;
        }
        while( l < ql ){
            sum += p[l]*(1-2*c[p[l]]); c[p[l]]--;
            l++;
        }
        while( l > ql ){
            l--;
            sum += p[l]*(2*c[p[l]]+1); c[p[l]]++;
        }
        e.ans = sum;
    }

    sort(all(q),cmp2);
    for(auto e : q ){
        printf("%lld\n",e.ans);
    }
}
```

## 4.4  Mo's algorithm on tree

```
int N;
int g[MAXN];
int f[MAXN];
int pa[LOGV][MAXV]; int level[MAXN];
int ST[MAXN], EN[MAXN], arr[MAXN*3];
int tt = 0;
vector<int> E[MAXN];

void dfs_build(int x, int p, int lev){
    pa[0][x] = p;
    level[x] = lev;
    ST[x] = ++tt; arr[tt] = x;
    for(auto e : E[x])if(e!=p){
        dfs_build(e,x,lev+1);
```

```
    }
    EN[x] = ++tt; arr[tt] = x;
}

void lca_build(){
    for(int k=1;k<LOGV;k++){
        repp(i,N){
            if( pa[k-1][i] != -1 )pa[k][i] = pa[k-1][pa[k-1][i]];
            else pa[k][i] = -1;
        }
    }
}

int lca(int x, int y){
    if( level[x] < level[y] ) swap(x,y);
    int diff = level[x] - level[y];
    for(int k=0;k<LOGV;k++)
        if( diff & (1<<k) )    x = pa[k][x];

    if( x == y ) return x;
    for(int k=LOGV-1;k>=0;k--)
        if( pa[k][x] != pa[k][y] ){
            x = pa[k][x]; y = pa[k][y];
        }
    return pa[0][x];
}

int Bsize;
struct query{
    int l,r,n;
};
bool cmp1(query& a, query& b){
    if( a.l/Bsize == b.l/Bsize ) return a.r < b.r;
    else return a.l/Bsize < b.l/Bsize;
};
bool cmp2(query&a, query& b ){ return a.n < b.n; }

ll ans[100500];
ll cnt[2][200500];
int vis[100500];
ll sum = 0;

void update(int x, int type){
    // add node to range
    if( type == 1 ){
        sum += cnt[g[x]^1][f[x]];
        cnt[g[x]][f[x]]++;
    }
    // remove node from range
    if( type == 0 ){
        sum -= cnt[g[x]^1][f[x]];
        cnt[g[x]][f[x]]--;
    }
}
```

```cpp
int main(void){
    geti(N);
    repp(i,N) geti(g[i]);
    repp(i,N) geti(f[i]);

    set<int> flist;
    map<int,int> fmp;
    repp(i,N) flist.insert(f[i]);
    int tmp = 1;
    for(auto e: flist) fmp[e] = tmp++;
    repp(i,N) f[i] = fmp[f[i]];

    repp(i,N-1){
        int a,b; geti(a,b);
        E[a].pb(b); E[b].pb(a);
    }
    tt = 0;
    dfs_build(1,-1,0);
    lca_build();
    Bsize = (int) sqrt(1.0*tt);

    int Q; geti(Q);
    vector<query> v;
    repp(q,Q){
        int a,b; geti(a,b);
        if( ST[a] > ST[b] ) swap(a,b);
        int l = lca(a,b);
        if( a == l || b == l){
            v.push_back({ST[a],ST[b],q});
        }
        else{
            v.push_back({EN[a],ST[b],q});
        }
    }

    sort(all(v),cmp1);
    int l=1, r=0;
    for(int i=0;i<v.size();i++){
        query& e = v[i];
        int ql = e.l, qr = e.r;
        while( r < qr ){
            r++;
            int node = arr[r];
            vis[node]++;
            if( vis[node] == 1 ) update(node,1);
            if( vis[node] == 2 ) update(node,0);
        }
        while( r > qr ){
            int node = arr[r];
            vis[node]--;
            if( vis[node] == 0 ) update(node,0);
            if( vis[node] == 1 ) update(node,1);
            r--;
        }
        while( l < ql ){
            int node = arr[l];
            vis[node]--;
            if( vis[node] == 0 ) update(node,0);
            if( vis[node] == 1 ) update(node,1);
            l++;
        }
        while( l > ql ){
            l--;
            int node = arr[l];
            vis[node]++;
            if( vis[node] == 1 ) update(node,1);
            if( vis[node] == 2 ) update(node,0);
        }

        int u = arr[ql]; int v = arr[qr];
        int l = lca(u,v);

        if( u != l && v != l ){
            int node = l;
            vis[node]++;
            if( vis[node] == 1 ) update(node,1);
            if( vis[node] == 2 ) update(node,0);
        }

        ans[e.n] += sum;

        if( u != l && v != l ){
            int node = l;
            vis[node]--;
            if( vis[node] == 0 ) update(node,0);
            if( vis[node] == 1 ) update(node,1);
        }

    }
    repp(i,Q) printf("%lld\n",ans[i]);
}
```

## 4.5  Parallel binary search

```cpp
int N,M,K,Q;

vector<Pi> edge[1000500];
int pa[MAXN]; int sz[MAXN];

// each query's answer
Pi ans[MAXN];
// each query's possible answer range for binary search
int low[MAXN], high[MAXN];
// focus[x] : list of query # where it's mid value is x
vector<int> focus[1000500];

int find(int x){
    if( x == pa[x] ) return x;
    return pa[x] = find(pa[x]);
```

```
}
int x[MAXN], y[MAXN];

void uni(int a, int b){
    a = find(a); b = find(b);
    if( a == b ) return;
    pa[a] = b;
    sz[b] += sz[a];
}

int main(void){
    //ios::sync_with_stdio(false);
    geti(N,M);
    int C = -1;
    repp(i,M){
        int a,b,c; geti(a,b,c);
        edge[c].push_back({a,b});
        C = max(C, c);
    }

    geti(Q);
    repp(i,Q){
        int a,b;
        geti(a,b); x[i] = a; y[i] = b;
        ans[i] = {INF,-1};
        // Initially, every query has answer in [0,C] range
        low[i] = 0; high[i] = C;
    }

    bool changed = true;
    while( changed ){
        changed = false;

        // Clear variables
        rep(i,C+1) focus[i].clear();
        repp(i,N) pa[i] = i, sz[i] = 1;

        // Put each query into corresponding focus group
        repp(i,Q){
            if( low[i] > high[i] ) continue;
            focus[ (low[i] + high[i])/2 ].push_back(i);
        }

        // for every time 0~C
        for(int k=0;k<=C;k++){
            // perform action of that time
            for(auto e : edge[k]) uni(e.Fi,e.Se);

            // for each focus group
            // determine it's answer & next position
            for(auto e : focus[k]){
                changed = true;
                int a = x[e]; int b = y[e];
                if( find(a) == find(b) ){
                    ans[e].Fi = min(ans[e].Fi, k);
```

```
                    ans[e].Se = sz[find(a)];
                    high[e] = k-1;
                }
                else{
                    low[e] = k+1;
                }
            }
        }
    }

    repp(i,Q){
        if( ans[i].Fi == INF ) printf("%d\n",-1);
        else printf("%d %d\n",ans[i].Fi, ans[i].Se);
    }

}
```

## 4.6 Lazy Propagation 1

```
int N,M,K;

struct segTree{
    struct Node{
        ll d, lazy;
    };
    vector<Node> data;
    int n;
    void init(int x){
        n = 1; while( n < x ) n *= 2;
        data.resize(n*2+10);
    }
    void propagate(int node, int nodeL, int nodeR){
        if( data[node].lazy == 0 ) return;
        ll len = nodeR - nodeL + 1;
        data[node].d += len*data[node].lazy;
        if( len > 1 ){
            data[node*2].lazy += data[node].lazy;
            data[node*2+1].lazy += data[node].lazy;
        }
        data[node].lazy = 0;
    }

    void update(int l, int r, ll val, int node, int nodeL, int nodeR){
        propagate(node, nodeL, nodeR);
        if( l > nodeR || r < nodeL ) return;
        if( l <= nodeL && nodeR <= r ){
            data[node].lazy += val;
            propagate(node,nodeL,nodeR);
            return;
        }
        update(l,r,val,node*2,nodeL,(nodeL+nodeR)/2);
        update(l,r,val,node*2+1,(nodeL+nodeR)/2+1,nodeR);
        data[node].d = data[node*2].d + data[node*2+1].d;
    }
```

```
    ll query(int l, int r, int node, int nodeL, int nodeR){
        propagate(node, nodeL, nodeR);
        if( l > nodeR || r < nodeL ) return 0;
        if( l <= nodeL && nodeR <= r ){
            return data[node].d;
        }
        ll sum = 0;
        sum += query(l,r,node*2,nodeL,(nodeL+nodeR)/2);
        sum += query(l,r,node*2+1,(nodeL+nodeR)/2+1,nodeR);
        return sum;
    }

};

int main(){
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    segTree tree;
    cin >> N >> M >> K;
    tree.init(N);
    repp(i,N){
        ll x; cin >> x;
        tree.update(i,i,x,1,1,tree.n);
    }
    repp(i,M+K){
        int a; cin >> a;
        if( a == 1 ){
            int b,c; ll d;
            cin >> b >> c >> d;
            tree.update(b,c,d,1,1,tree.n);
        }
        else{
            int b,c; cin >> b >> c;
            printf("%lld\n",tree.query(b,c,1,1,tree.n));
        }
    }

}
```

## 4.7   PST

```
const int MAXN = 1e5 + 50;
const int LOGN = 18;

int root[MAXN], pos[MAXN], a[MAXN], rev[MAXN];
int n, cnt;

struct node {
        int sum, left, right;
} tree[3 * MAXN * LOGN];

int build(int l = 0, int r = n) {
        int id = ++cnt;
```

```
        if(r - l <= 1) {
                tree[id] = {0, 0, 0};
                return id;
        }
        int mid = (l + r) >> 1;
        tree[id] = {0, build(l, mid), build(mid, r)};
        return id;
}

int update(int x, int prev, int l = 0, int r = n) {
        if(x < l || r <= x) return prev;
        int id = ++cnt;
        if(r - l <= 1) {
                tree[id] = {1, 0, 0};
                return id;
        }
        int mid = (l + r) >> 1;
        int L = update(x, tree[prev].left, l, mid);
        int R = update(x, tree[prev].right, mid, r);
        tree[id] = {tree[L].sum + tree[R].sum, L, R};
        return id;
}

int findKth(int x, int y, int k, int l = 0, int r = n) {
        if(r - l <= 1) return l;
        int mid = (l + r) >> 1;
        int t = tree[tree[x].left].sum - tree[tree[y].left].sum;
        if(t >= k)
                return findKth(tree[x].left, tree[y].left, k, l, mid);
        else
                return findKth(tree[x].right, tree[y].right, k - t, mid, r);
}

// find ans in [lb, ub)
long long ans(int lb, int ub) {
        if(ub - lb <= 1) return 0;
        int k = (ub - lb + 1) / 2;
        int mid = a[findKth(root[lb], root[ub], k)];
        return ans(lb, mid) + ans(mid+1, ub) + (long long)(ub - lb);
}

int main() {
        geti(n);
        for(int i = 0; i < n; i++) {
                geti(a[i]);
                a[i]--;
                pos[a[i]] = i;
        }

        // ith segment tree: xth element is 1 iff a[x] >= i
        root[n] = build();
        for(int i = n-1; i >= 0; i--)
                root[i] = update(pos[i], root[i+1]);
```

```
        cout << ans(0, n) << endl;
}
```

# 5   Geometry

## 5.1   Closest pair

```
int N,M,T,K,V;

typedef struct Point{
    int x,y;
    bool operator<(const Point& l) const{
        if( y == l.y ) return x < l.x;
        return y < l.y;
    }
    bool operator==(const Point& l) const{
        return (x==l.x)&&(y==l.y);
    }
} Point;

bool cmp(const Point& l, const Point& r){
    if(l.x == r.x ) return l.y < r.y;
    return l.x < r.x;
}

int dist(Point& l, Point& r ){
    return (l.x-r.x)*(l.x-r.x) + (l.y-r.y)*(l.y-r.y);
}

int main(void)
{
    geti(N); vector<Point> v(N);
    for(int i=0;i<N;i++){
        int x ,y; geti(x,y); v[i].x = x; v[i].y = y;
    }
    sort(all(v),cmp);
    int ans = dist(v[0],v[1]); int left = 0;
    set<Point> possible; possible.insert(v[0]); possible.insert(v[1]);

    for(int i=2;i<N;i++){
        while( (v[i].x - v[left].x)*(v[i].x - v[left].x ) > ans ){
            possible.erase(v[left]);
            left++;
        }
        int d = (int) sqrt(ans) + 1;
        auto bottom = possible.lower_bound({-100000,v[i].y-d});
        auto top = possible.upper_bound({100000,v[i].y+d});
        for(auto it = bottom; it != top; it++){
            Point cur = *it;
            if( dist(v[i],cur) < ans ) ans = dist(v[i],cur);
        }
        possible.insert(v[i]);
```

```
    }
    cout << ans;
}
```

## 5.2   Convex hull

```
typedef struct Point{
    ll x,y,n;
} Point;

ll ccw(Point a, Point b, Point c){
    b.x -= a.x, b.y -= a.y;
    c.x -= a.x, c.y -= a.y;
    return b.x*c.y - c.x*b.y;
}

vector<Point> convex_hull(vector<Point> ps){
    if (ps.size() < 3)return ps;
    vector<Point> upper, lower;
    sort(ps.begin(), ps.end(),[](const Point &a, const Point &b) {
        if (a.x == b.x) return a.y < b.y; return a.x < b.x;
    });
    for(const auto &p : ps){ // ccw without `=` when include every point in
      convex hull
        while(upper.size() >= 2 && ccw(*++upper.rbegin(), *upper.rbegin(), p) >=
            0)upper.pop_back();
        while(lower.size() >= 2 && ccw(*++lower.rbegin(), *lower.rbegin(), p) <=
            0)lower.pop_back();
        upper.emplace_back(p);
        lower.emplace_back(p);
    }
    lower.insert(lower.end(), ++upper.rbegin(), --upper.rend());
    return lower;
}

vector<Point> convex_hull2(vector<Point> ps){ // sorting angle
    if (ps.size() < 3)return ps;
    vector<Point> convex;
    sort(ps.begin(), ps.end(), [](Point &a, Point &b){
        if(a.x == b.x)return a.y < b.y; return a.x<b.x;
    });
    Point d = ps[0];
    for(auto &p : ps){
        p.x -= d.x;p.y -= d.y;
    }
    sort(ps.begin(), ps.end(), [](Point &a, Point &b){
        if (ccw({0,0},a,b) == 0) return a.x*a.x + a.y*a.y < b.x*b.x + b.y*b.y;
        return ccw({0,0},a,b) > 0;
    });
    for(auto &p : ps){
        while(convex.size() >= 2 && ccw(*++convex.rbegin(), *convex.rbegin(), p)
            <= 0)convex.pop_back();
        convex.emplace_back(p);
    }
}
```

```
    for(auto &p : convex){
        p.x += d.x;p.y += d.y;
    }
    return convex;
}
```

## 5.3  Rotating Calipers

```
int main(){
    vector<Point> convex;
    int ans = 0;

    int mid = 0;
    // if you want iterate `only` antipodal pairs
    // while(ccw(convex.back(), convex[0], convex[mid], convex[mid+1]) > 0) mid
      ++;

    for(int i=0,j=mid; i < convex.size();){
        // do something with pair of i, j

        int nextj = (j+1) % convex.size();
        int nexti = (i+1) % convex.size();
        if (ccw(convex[i], convex[nexti], convex[j], convex[nextj]) > 0)j =
          nextj;
        else i++;
    }
}
```

# 6  Miscelleneous

## 6.1  Grundy number

```
map<set<int>,int> grundy;
map<ll,set<int>> mp;

int get_grundy(set<int> x){
    // base case
    if( sz(x) == 0 ) return 0;
    if( grundy.find(x) != grundy.end() ) return grundy[x];

    set<int> S;
    int res = 0;

    auto iter = x.end(); iter--;
    int mx = *iter;

    // transition : which k to select
    for(int i=1;i<=mx;i++){
        set<int> nxt;
        for(auto e : x){
            if( e < i ) nxt.insert(e);
            else if( e == i ) continue;
```

```
            else nxt.insert(e-i);
        }
        S.insert(get_grundy(nxt));
    }

    // find mex and return
    while( S.find(res) != S.end() ) res++;
    grundy[x] = res;
    return res;
}
int main(void){
    int n; geti(n);

    // Simple prime factorization
    rep(i,n){
        ll x; scanf("%lld",&x);
        for(ll i=2;i*i<=x;i++){
            if( x>0 && x%i == 0 ){
                int cnt = 0;
                while( x>0 && x%i == 0 ){
                    cnt++; x/= i;
                }
                mp[i].insert(cnt);
            }
        }
        if( x > 1 ){
            mp[x].insert(1);
        }
    }

    int res = 0;
    for(auto e : mp){
        res ^= get_grundy(e.Se);
    }

    if( res == 0 ) printf("Arpa");
    else printf("Mojtaba");


}
```

## 6.2  Hungarian

```
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs.  In practice, it solves 1000x1000 problems in around 1
// second.
//
//   cost[i][j] = cost for pairing left node i with right node j
//   Lmate[i] = index of right node that left node i pairs with
//   Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative.  To perform
```

```
// maximization, simply negate the cost[][] matrix.

typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
  int n = int(cost.size());

  // construct dual feasible solution
  VD u(n);
  VD v(n);
  for (int i = 0; i < n; i++) {
    u[i] = cost[i][0];
    for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
  }
  for (int j = 0; j < n; j++) {
    v[j] = cost[0][j] - u[0];
    for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
  }

  // construct primal solution satisfying complementary slackness
  Lmate = VI(n, -1);
  Rmate = VI(n, -1);
  int mated = 0;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      if (Rmate[j] != -1) continue;
      if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
        Lmate[i] = j;
        Rmate[j] = i;
        mated++;
        break;
      }
    }
  }

  VD dist(n);
  VI dad(n);
  VI seen(n);

  // repeat until primal solution is feasible
  while (mated < n) {

    // find an unmatched left node
    int s = 0;
    while (Lmate[s] != -1) s++;

    // initialize Dijkstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++)
      dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;

    while (true) {

      // find closest
      j = -1;
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
      }
      seen[j] = 1;

      // termination condition
      if (Rmate[j] == -1) break;

      // relax neighbors
      const int i = Rmate[j];
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
          dist[k] = new_dist;
          dad[k] = j;
        }
      }
    }

    // update dual variables
    for (int k = 0; k < n; k++) {
      if (k == j || !seen[k]) continue;
      const int i = Rmate[k];
      v[k] += dist[k] - dist[j];
      u[i] -= dist[k] - dist[j];
    }
    u[s] += dist[j];

    // augment along path
    while (dad[j] >= 0) {
      const int d = dad[j];
      Rmate[j] = Rmate[d];
      Lmate[Rmate[j]] = j;
      j = d;
    }
    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
  }

  double value = 0;
  for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

  return value;
}
```

## 6.3 Convex Hull trick

```cpp
ll a[MAXN], b[MAXN], dp[MAXN];
ll la[MAXN], lb[MAXN];
int sz, cur, n;

double cross(int x, int y) {
    return (double)(lb[x] - lb[y]) / (la[y] - la[x]);
}

void newLine(ll p, ll q) {
    la[sz] = p;
    lb[sz] = q;

    while(sz > 1 && cross(sz-1, sz-2) > cross(sz, sz-1)) {
        la[sz-1] = la[sz];
        lb[sz-1] = lb[sz];
        sz--;
    }
    sz++;
}

ll find(ll x) {
    while(cur+1 < sz && x > cross(cur, cur+1)) cur++;
    return la[cur] * x + lb[cur];
}

int main() {
    scanf("%d", &n);
    for(int i = 1; i <= n; i++)
        cin >> a[i];
    for(int i = 1; i <= n; i++)
        cin >> b[i];

    dp[1] = 0;
    newLine(b[1], 0);
    for(int i = 2; i <= n; i++) {
        dp[i] = find(a[i]);
        newLine(b[i], dp[i]);
    }
    cout << dp[n];
}
```

## 6.4 Gaussian Elimination

```cpp
#define MAX_N 300        // adjust this value as needed
struct AugmentedMatrix { double mat[MAX_N][MAX_N + MAX_N + 10]; };
struct ColumnVector { double vec[MAX_N]; };

// 0 indexed row and column
AugmentedMatrix GaussianElimination(int N, AugmentedMatrix Aug) {
        // input: N X 2N matrix  [A I], output: [I invA]
```

```cpp
        // forward eliminataion phase
        for(int i=0;i<N;i++){
                int l = i;
                // which row has largest column value
                for(int j=i+1;j<N;j++)
                        if( fabs(Aug.mat[j][i]) > fabs(Aug.mat[l][i]) )
                                l = j;
                // swap this pivot row to minimize error
                for(int k=i;k<2*N;k++)
                        swap(Aug.mat[i][k],Aug.mat[l][k]);
                // calculate forward elimination
                for(int j=i+1;j<N;j++)
                        for(int k=2*N-1;k>=i;k--)
                                Aug.mat[j][k] -= Aug.mat[i][k] * Aug.mat[j][i] /
                                        Aug.mat[i][i];
        }

        // normalize pivots
        for(int i=0;i<N;i++)
                for(int j=2*N-1;j>=i;j--)
                        Aug.mat[i][j] /= Aug.mat[i][i];

        // backward elimination
        for(int i=N-1;i>0;i--)
                for(int j=i-1;j>=0;j--)
                        for(int k=2*N-1;k>=i;k--)
                                Aug.mat[j][k] -= Aug.mat[i][k] * Aug.mat[j][i] /
                                        Aug.mat[i][i];


        return Aug;
}

int main() {

        AugmentedMatrix Aug;
        int N; geti(N);
        rep(i,N) rep(j,N) scanf("%lf",&Aug.mat[i][j]);
        for(int i=N;i<2*N;i++) Aug.mat[i-N][i] = 1;

        AugmentedMatrix res = GaussianElimination(N, Aug);

        // Print inversion of A
        for(int i=0;i<N;i++){
                for(int j=N;j<2*N;j++) printf("%f ",res.mat[i][j]);
                printf("\n");
        }


        return 0;
}
```

## 6.5 FFT

```
#include <cmath>
#include <complex>
using namespace std;
typedef pair<int,int> pii;
typedef complex<double> base;

void fft(vector<base> &a, bool invert){
    int n = a.size();
    for(int i=1,j=0;i<n;i++){
        int bit = n >> 1;
        for (;j>=bit;bit>>=1)j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }
    for(int len=2;len<=n;len<<=1){
        double ang = 2*acos(-1)/len*(invert?-1:1);
        base wlen(cos(ang),sin(ang));
        for(int i=0;i<n;i+=len){
            base w(1);
            for(int j=0;j<len/2;j++){
                base u = a[i+j], v = a[i+j+len/2]*w;
                a[i+j] = u+v;
                a[i+j+len/2] = u-v;
                w *= wlen;
            }
        }
    }
    if (invert) {
        for(int i=0;i<n;i++) a[i] /= n;
    }
}

void multiply(const vector<int> &a, const vector<int> &b, vector<int> &res){
    vector<base> fa(a.begin(), a.end()), fb(b.begin(),b.end());
    int n = 1;
    while(n < max(a.size(), b.size())) n <<= 1;
    n <<= 1;
    fa.resize(n); fb.resize(n);
    fft(fa,false);fft(fb,false);
    for(int i=0;i<n;i++) fa[i] *= fb[i];
    fft(fa,true);
    res.resize(n);
    for(int i=0;i<n;i++) res[i] = int(fa[i].real() + (fa[i].real() > 0 ? 0.5 :
      -0.5));
}
```

## 6.6   Math

```
Complete Permutation ( Derangement )
D0 = 1, D1 = 0, D2 = 1, D3 = 2,
Dn = (n-1)(Dn-1 + Dn-2)
Dn = n! * sum{k=0~n}{(-1)^k / k!}

Catalan Number
```

```
Cn = (1 / (n+1)) * combination(2n,n)
   = (2n)! / (n!(n+1)!)
```

## 6.7   Extended Euclidean

```
pair<int,int> ext_gcd(int a,int b){
    if(b){
        auto tmp = ext_gcd(b, a%b);
        return {tmp.second, tmp.first - (a/b) * tmp.second};
    } else return {1, 0};
}

int mod_inv(int a, int M){
    return (ext_gcd(a, M).first + M) % M;
}
```

## 6.8   Persistence Segment Tree

```
int n, cnt;
int root[MAXN];

struct node {
        int sum, left, right;
} tree[3 * MAXN * LOGN];

int build(int l = 0, int r = n) {
        int idx = ++cnt;
        if(r - l <= 1) {
                tree[idx] = {0, 0, 0};
                return idx;
        }
        int mid = (l + r) >> 1;
        tree[idx] = {0, build(l, mid), build(mid, r)};
        return idx;
}

int update(int x, int prev, int l = 0, int r = n) {
        if(x < l || r <= x) return prev;
        int idx = ++cnt;
        if(r - l <= 1) {
                tree[idx] = {1, 0, 0};
                return idx;
        }

        int mid = (l + r) >> 1;
        int L = update(x, tree[prev].left, l, mid);
        int R = update(x, tree[prev].right, mid, r);
        tree[idx] = {tree[L].sum + tree[R].sum, L, R};
        return idx;
}

int query(int x, int y, int k, int l = 0, int r = n) {
        if(r - l <= 1) return l;
```

```
        int mid = (l + r) >> 1;
        int leftSum = tree[tree[y].left].sum - tree[tree[x].left].sum;
        if(leftSum >= k)
                return query(tree[x].left, tree[y].left, k, l, mid);
        else
                return query(tree[x].right, tree[y].right, k - leftSum, mid, r);
}



int a[MAXN], rev[MAXN];
map<int, int> M;

int main() {
        int q;
        geti(n, q);
        for(int i = 1; i <= n; i++) {
                geti(a[i]);
                rev[i-1] = a[i];
        }
        sort(rev, rev + n);
        for(int i = 0; i < n; i++)
                M[rev[i]] = i;
        for(int i = 1; i <= n; i++)
                a[i] = M[a[i]];

        root[0] = build();
        for(int i = 1; i <= n; i++)
                root[i] = update(a[i], root[i-1]);

        while(q--) {
                int i, j, k;
                geti(i, j, k);
                printf("%d\n", rev[query(root[i-1], root[j], k)]);
        }
}
```