Assignment NO.4 FTS

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1.
$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + c + \varepsilon_t \tag{1}$$

(a) Eigenequation of (1) is

$$\lambda^2 - \alpha_1 \lambda - \alpha_2 = 0$$

Stationarity requires that the root of eigenequation fall within the unit circl, i.e.

$$\left| \frac{\alpha_1 \pm \sqrt{\alpha_1^2 + 4\alpha_2}}{2} \right| < 1$$

(b) Use the lag operator to rewrite (1) and we get

$$(1 - \alpha_1 L - \alpha_2 L) y_t = c + \varepsilon_t \tag{2}$$

Calculate expectation on both sides of (2) and we get the conclusion proved.

$$E(y_t) = (1 - \alpha_1 L - \alpha_2 L)^{-1} c$$

(c) Subtract μ from both sides of (1)

$$y_t - \mu = \alpha_1 (y_{t-1} - \mu) + \alpha_2 (y_{t-2} - \mu) + c + \varepsilon_t \tag{3}$$

Multiply both sides of (3) by $y_{t-j} - \mu$, and calculate the expectation

$$\gamma_j = \alpha_1 \gamma_{j-1} + \alpha_2 \gamma_{j-2} \tag{4}$$

Divide both sides of (4) by γ_0

$$\rho_j = \alpha_1 \rho_{j-1} + \alpha_2 \rho_{j-2}$$

(d)

$$\rho_1 = \alpha_1 \rho_0 + \alpha_2 \rho_{-1} = \alpha_1 \rho_0 + \alpha_2 \rho_1 = \frac{\alpha_1}{1 - \alpha_2} \tag{5}$$

$$\rho_2 = \alpha_1 \rho_1 + \alpha_2 \rho_0 = \frac{\alpha_1^2 - \alpha_2^2 + \alpha_2}{1 - \alpha_2} \tag{6}$$

(e) When $\alpha_1 = 0.6, \alpha_2 = 0.3$, use (5) and (6) we $\rho_1 = 0.857, \rho_2 = 0.814$