Assignment NO.2 FTS

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1. A

   (a) $Pf.$

$$
\begin{aligned}
y_t &= c_0 + \alpha y_{t-1} \\
&= c_0 + \alpha(c_0 + \alpha y_{t-2}) \\
&= \ldots\ldots \\
&= c_0 \sum_{i=0}^{t-1} \alpha^i + \alpha^t
\end{aligned}
$$

   (b) $Pf.$ i.e. to prove that

$$
\sum_{i=0}^{t-1} \alpha^i = \frac{1-\alpha^t}{1-\alpha} \tag{1}
$$

   It's obvious that equation (1) holds if and only if $|\alpha| \neq 1$.

   (c) A random walk with intercept terms.

2. $y_t = c_0 +_{t-1} + b_1\epsilon_t + b_2\epsilon_{t-1}$

   (a)

$$
\begin{aligned}
y_t &= (1-\alpha L)^{-1}c_0 + (1-\alpha L)^{-1}(b_1\varepsilon_t + b_2\varepsilon_{t-1} \\
&= \ldots\ldots \\
&= (1-\alpha)^{-1}c_0 + \sum_{i=0}^{t-1} \alpha^i (b_2 + \alpha b_1)\varepsilon_{t-1-i}
\end{aligned}
$$

   (b)

$$
y_{t+j} = (1-\alpha)^{-1}c_0 + b_1\varepsilon_{t+j} + \ldots + \alpha^{j-1}(\alpha b_1 + b_2)\varepsilon_t + \ldots
$$

   Thus, the dynamic multiplier is $\frac{\partial y_{t+j}}{\partial \varepsilon_t} = \alpha^{j-1}(\alpha b_1 + b_2)$

   (c)

$$
CIRF = \frac{\partial y_{t+j}}{\partial \varepsilon_t} + \frac{\partial y_{t+j}}{\partial \varepsilon_{t+1}} + \ldots + \frac{\partial y_{t+j}}{\partial \varepsilon_{t+j}} = \sum_{i=0}^{j-1} \alpha^i (b_2 + \alpha b_1)
$$

3. (a) The characteristic equation is $\lambda^2 - 1.2\lambda + 0.2 = 0$, and the solutions are $\lambda_1 = 1$ and $\lambda_2 = 0.2$; meanwhile the solutions of inverse characteristic equation are $\lambda_1' = 1$ and $\lambda_2' = 5$. It is not a stationary series.

   (b) The characteristic equation is $\lambda^2 - 1.2\lambda + 0.4 = 0$, and the solutions are $\lambda_1 = 0.6 + 0.2i$ and $\lambda_2 = 0.6 - 0.2i$; meanwhile the solutions of inverse characteristic equation are $\lambda_1' = 1.5 + 0.5i$ and $\lambda_2' = 1.5 - 0.5i$. It is a stationary series.