HW1 Key

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Chapter 1

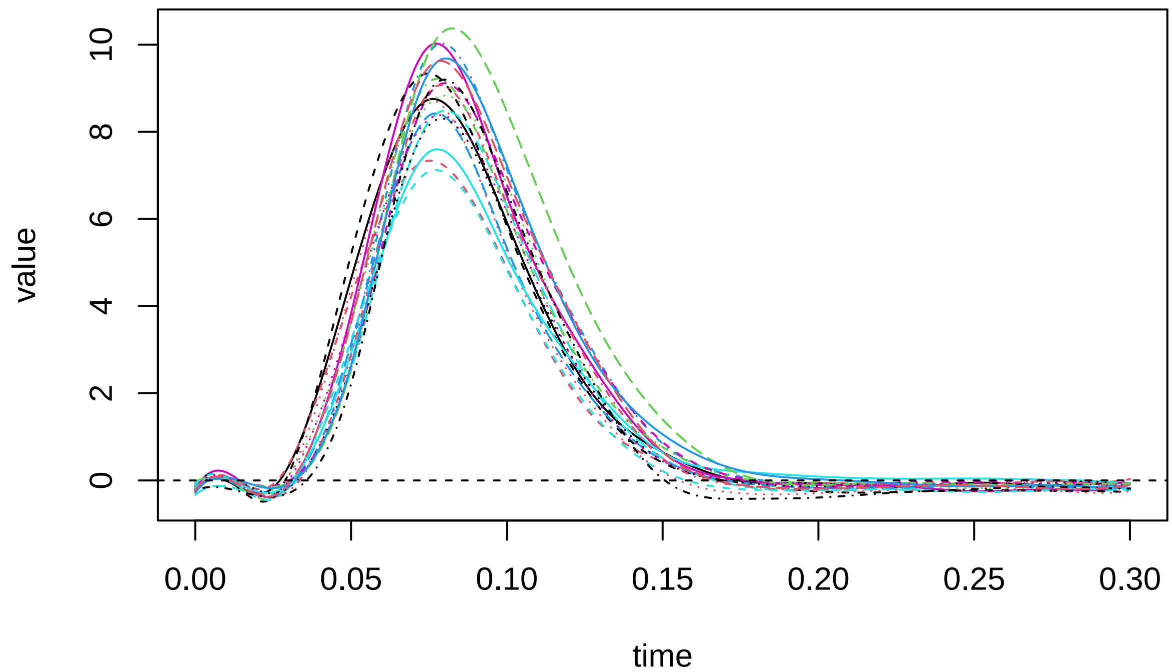
Load packages.

```r
library(fda)
library(plot3D)
library(YieldCurve)
library(fields)
library(expm)
```

1.6.1.

1.6.1.(a)

```r
my_basis <- create.bspline.basis(c(0,0.3),nbasis=15,norder=4)
pinch.F <- Data2fd(pinchtime,pinch,my_basis)
plot(pinch.F)
```




```
## [1] "done"
```

plot(pinch.F)

1.6.1.(b)

```
mu.F <- mean.fd(pinch.F)
sd.F <- sd.fd(pinch.F)

plot(pinch.F,col="grey")
```

```
## [1] "done"
```

```
plot(mu.F,lwd=4,add=TRUE)
```

```
## [1] "done"
```

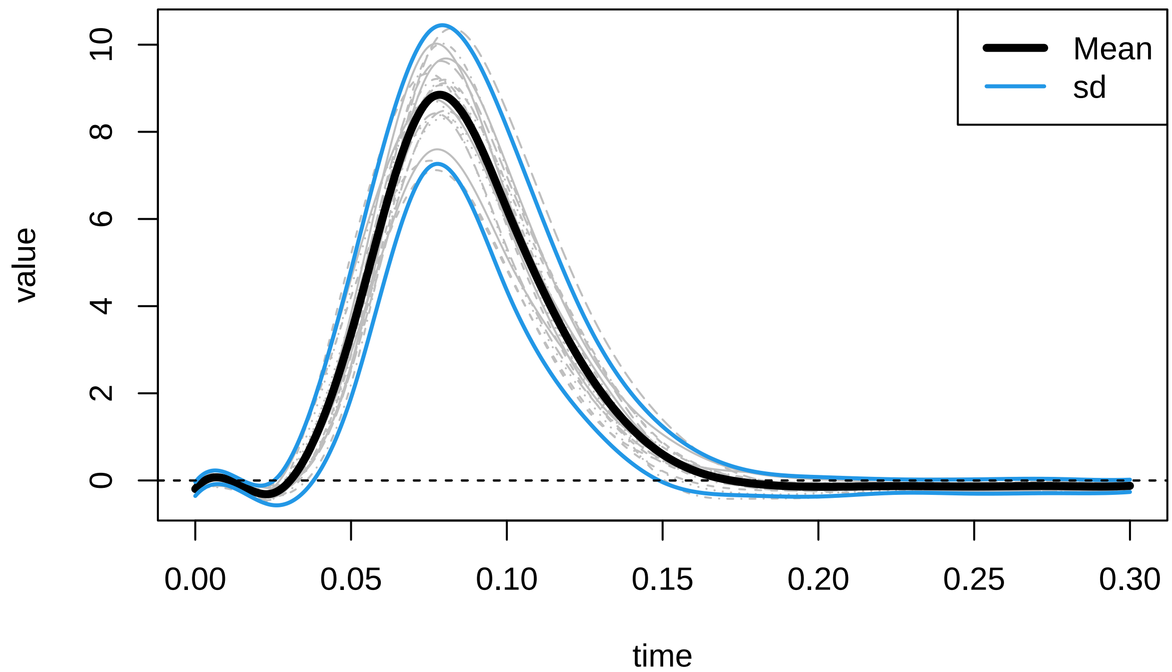
```
plot(mu.F+2*sd.F,add=TRUE,col=4,lwd=2)
```

```
## [1] "done"
```

```
plot(mu.F-2*sd.F,add=TRUE,col=4,lwd=2)
```

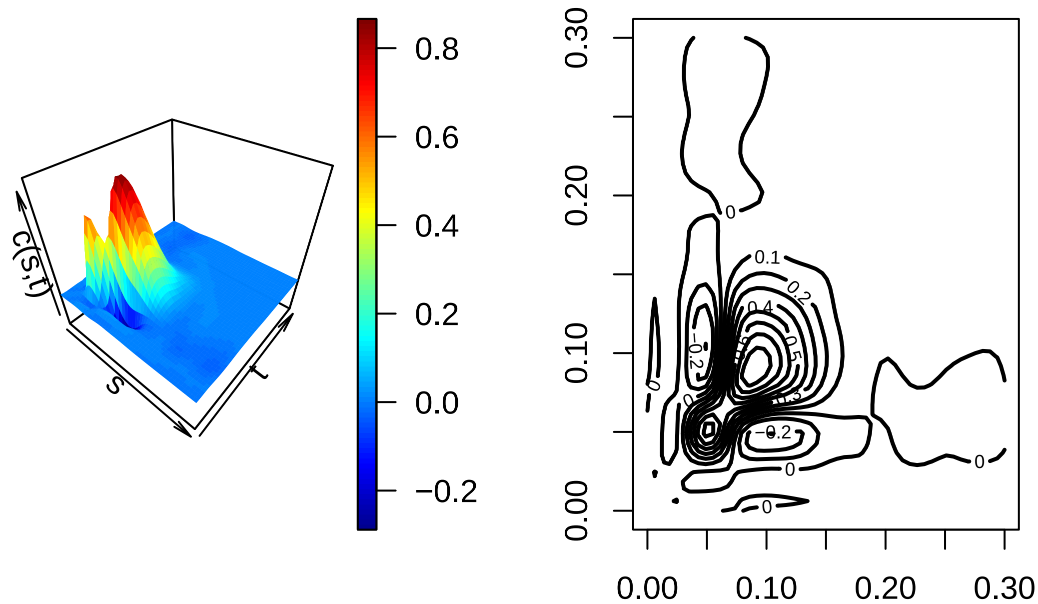
```
## [1] "done"
```

```
legend("topright",c("Mean","sd"),col=c(1,4),lwd=c(4,2))
```



1.6.1.(c)

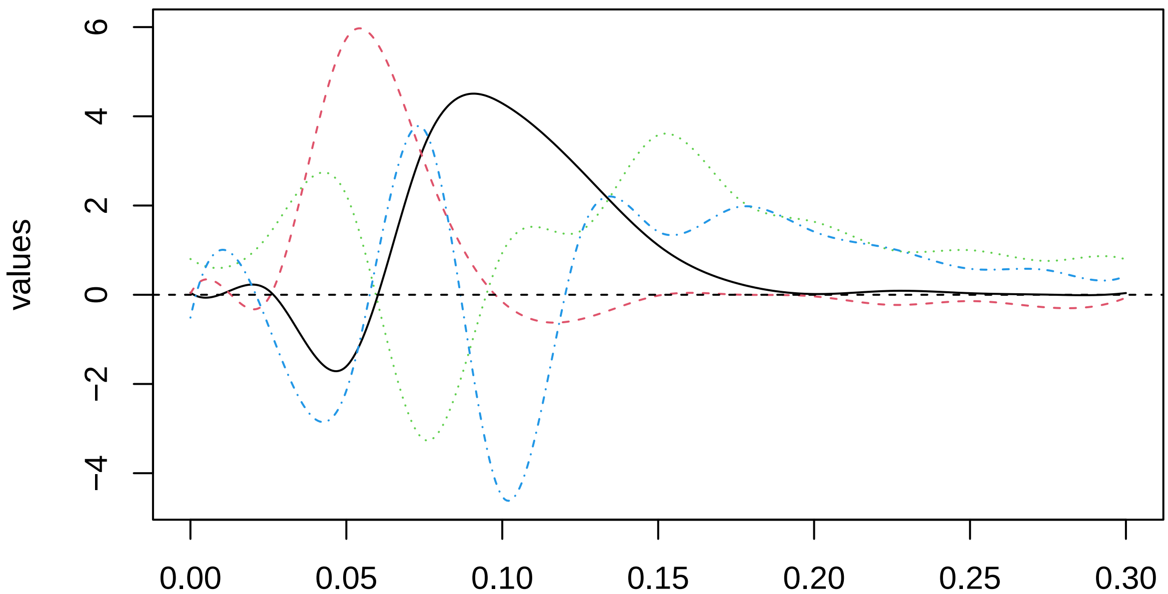
```
par(mfrow=c(1,2))
pinch_var <- var.fd(pinch.F)
pts <- seq(from=0,to=0.3,length=50)
pinch_mat <- eval.bifd(pts,pts,pinch_var)
persp3D(pts,pts,pinch_mat, xlab="s", ylab="t", zlab="c(s,t)")
contour(pts, pts, pinch_mat, lwd=2)
```




```
par(mfrow=c(1,1))
```

1.6.1.(d)

```
pinch_pca <- pca.fd(pinch.F, nharm=4)
plot(pinch_pca$harmonics)
```




```
## [1] "done"
```

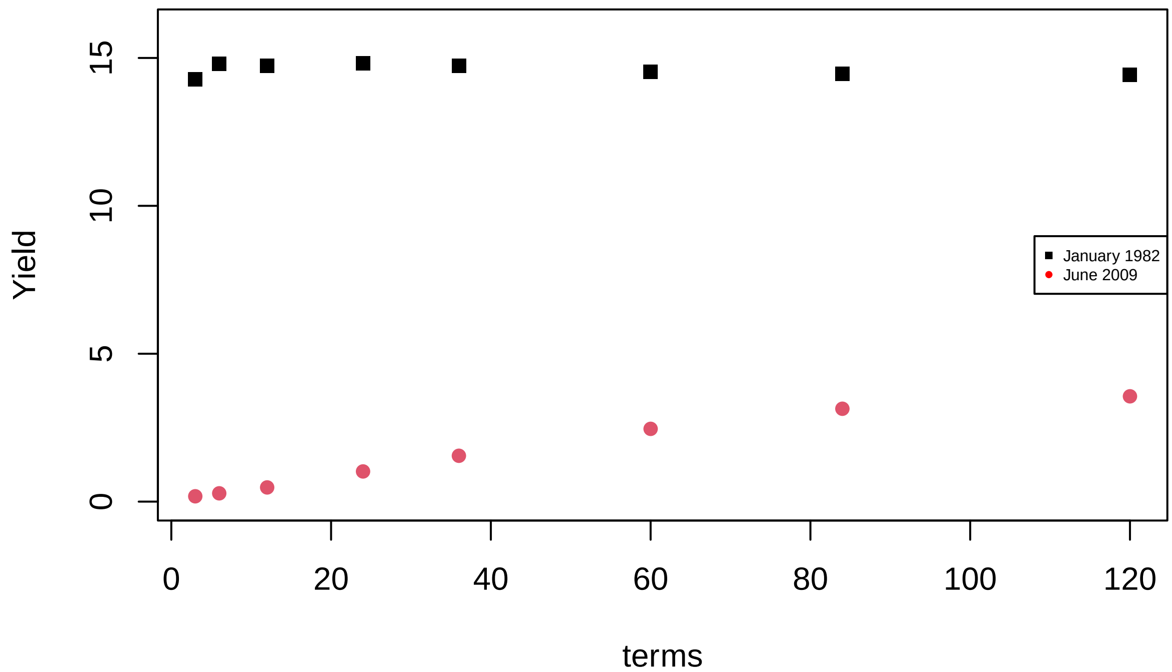
```
which(cumsum(pinch_pca$varprop)>0.9)[1]
```

```
## [1] 2
```

1.6.2

1.6.2.(a)

```
data(FedYieldCurve)
terms <- c(3,6,12,24,36,60,84,120)
yield <- t(FedYieldCurve)
plot(terms, yield[,2],pch=15, ylab="Yield", ylim=c(0,16))  # 1982 January
points(terms, yield[,331], pch=16, col=2)                     # 2009 June
legend("right",
       legend = c("January 1982", "June 2009"),
       pch = c(15, 16),
       col = c("black", "red"),
       cex = 0.5)
```



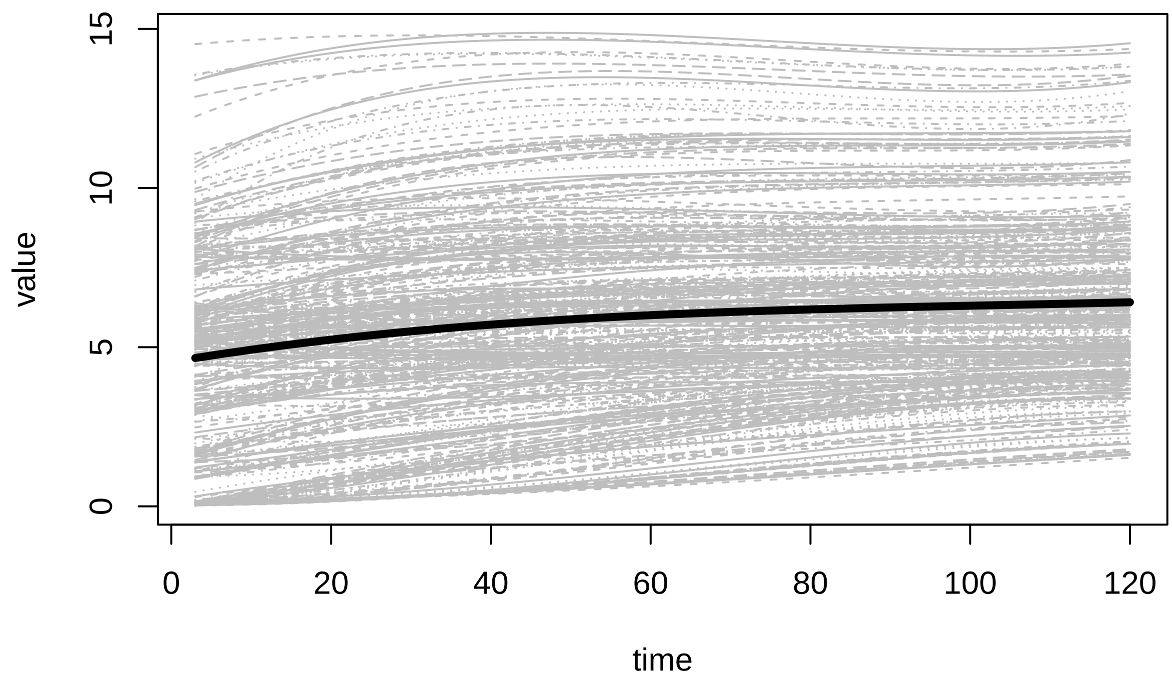
1.6.2.(b)

```
my_basis <- create.bspline.basis(c(3,120),nbasis=4)
yield.F <- Data2fd(terms,yield,my_basis)

mu.F <- mean.fd(yield.F)
plot(yield.F,col="grey")
```

```
## [1] "done"
```

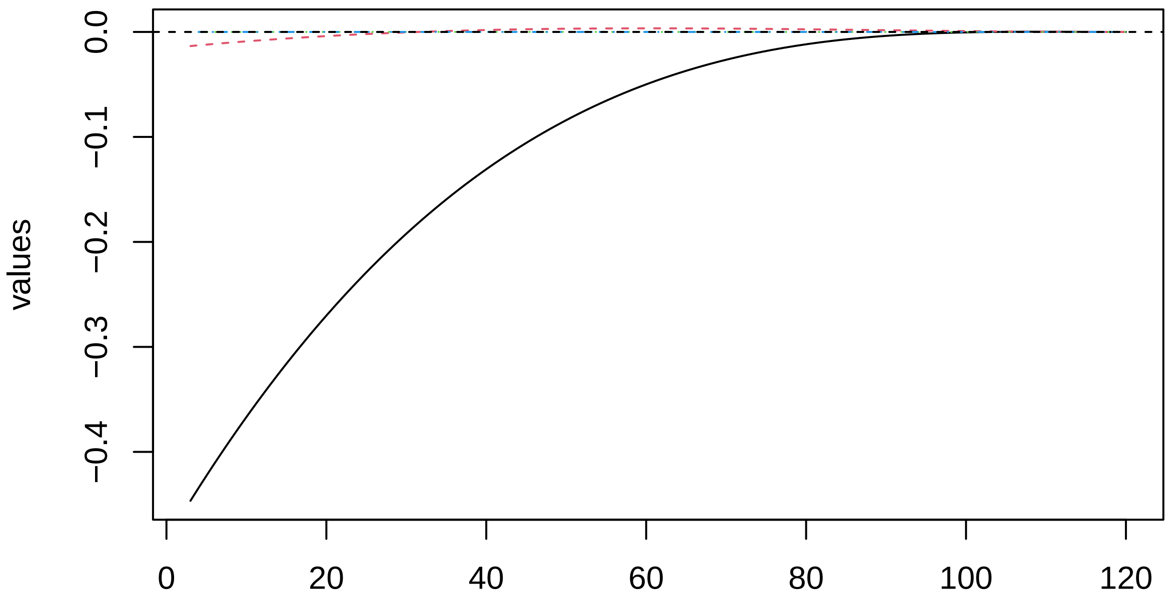
```
plot(mu.F,lwd=4,add=TRUE)
```




```
## [1] "done"
```

1.6.2.(c)

```r
yield_pca <- pca.fd(yield.F, nharm=4)
plot(yield_pca$harmonics)
```




```
## [1] "done"
```

```r
yield_pca$varprop
```

```
## [1] 9.999905e-01 9.469593e-06 2.176403e-11 2.882496e-17
```

1.6.6.

$$
\begin{aligned}
\bar{x}_N(t) &= \frac{1}{N}\sum_{n=1}^{N} x_n(t) \\
&= \frac{1}{N}\sum_{n=1}^{N}\sum_{m=1}^{M} c_{nm}B_m(t) = \sum_{m=1}^{M}\left(\sum_{n=1}^{N}\frac{1}{N}c_{nm}\right)B_m(t) = \sum_{m=1}^{M} a_m B_m(t)
\end{aligned}
$$

Thus $a_m = \frac{1}{N}\sum_{n=1}^{N} c_{nm}$

$$
\begin{aligned}
\hat{c}(t,s) &= \frac{1}{N-1}\sum_{n=1}^{N}(x_n(t)-\bar{x}_n(t))(x_n(s)-\bar{x}_n(s)) \\
&= \frac{1}{N-1}\sum_{n=1}^{N}\left(\sum_{m=1}^{M}(c_{nm}-a_m)B_m(t)\right)\left(\sum_{k=1}^{M}(c_{nk}-a_k)B_k(t)\right) \\
&= \sum_{m=1}^{M}\sum_{k=1}^{M}B_m(t)B_k(s)\left(\frac{1}{N-1}\sum_{n=1}^{N}(c_{nm}-a_m)(c_{nk}-a_k)\right) \\
&= \sum_{m=1}^{M}\sum_{k=1}^{M}b_{mk}B_m(t)B_k(s),\ \text{where } b_{mk}=\frac{1}{N-1}\sum_{n=1}^{N}(c_{nm}-a_m)(c_{nk}-a_k)
\end{aligned}
$$

Chapter 2

2.5.1.

$$
\begin{aligned}
x(t) &= c_o + \sum_{j=1}^{J}[a_j sin(wjt) + b_j cos(wjt)], \\
x^{(1)}(t) &= \sum_{j=1}^{J}[wja_j cos(wt) - wjb_j sin(wt)] \\
&= \sum_{j=1}^{J}[(-wjb_j)sin(wt) + (wja_j)cos(wt)] \\
x^{(3)}(t) &= \sum_{j=1}^{J}[(w^3j^3b_j)sin(wt) + (-w^3j^3a_j)cos(wt)] \\
L(x)(t) &= w^2x^{(1)}(t) + x^{(3)}(t) \\
&= w^3\sum_{j=2}^{J}(j(j^2-1)b_j sin(wjt) + j(1-j^2)a_j cos(wjt))
\end{aligned}
$$

$$
\begin{aligned}
\int_0^T (L(x)(t))^2 &= w^6\sum_{j=2}^{J}j^2(1-j^2)^2(a_j^2+b_j^2)\frac{T}{2} \\
&= \pi w^5\sum_{j=2}^{J}j^2(j^2-1)^2(a_j^2+b_j^2)
\end{aligned}
$$

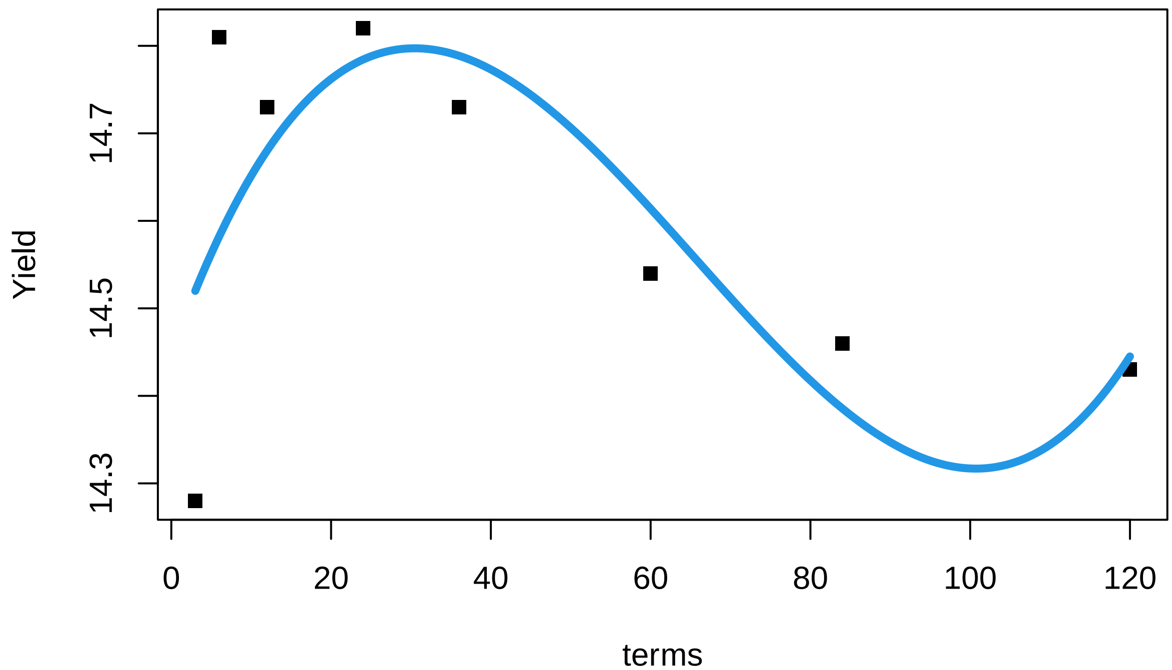
2.5.2

2.5.2.(a)

```
terms <- c(3,6,12,24,36,60,84,120)
yield.1982J <- t(FedYieldCurve[2,])

my_basis <- create.bspline.basis(c(3,120),nbasis=4)
yield.1982J.S <- smooth.basis(terms,yield.1982J,my_basis)

plot(terms, yield.1982J,pch=15, ylab="Yield")  # 1982 January
plot(yield.1982J.S,lwd=4,add=TRUE,col=4)
```




```
## [1] "done"
```

2.5.2.(b)

```r
length(terms) # as many data points
```

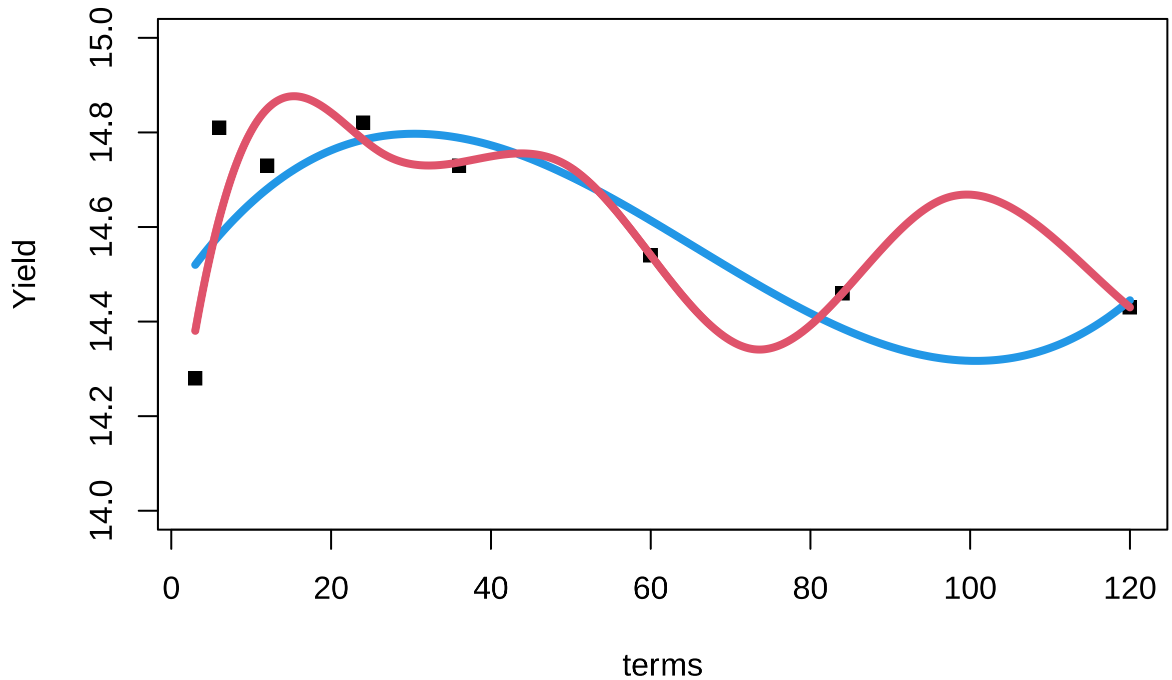
```
## [1] 8
```

```r
my_basis <- create.bspline.basis(c(3,120),nbasis=8)
my_par <- fdPar(my_basis,Lfdobj=2,lambda=1)
yield.1982J.S.P <- smooth.basis(terms,yield.1982J,my_par)

plot(terms, yield.1982J,pch=15, ylab="Yield",ylim=c(14,15))  # 1982 January
plot(yield.1982J.S,lwd=4,add=TRUE,col=4)
```

```
## [1] "done"
```

```r
plot(yield.1982J.S.P,lwd=4,add=TRUE,col=2)
```




```
## [1] "done"
```

2.5.2.(c)

```
my_par1 <- fdPar(my_basis,Lfdobj=2,lambda=1)
yield.1982J.S.P1 <- smooth.basis(terms,yield.1982J,my_par1)
my_par2 <- fdPar(my_basis,Lfdobj=2,lambda=10)
yield.1982J.S.P2 <- smooth.basis(terms,yield.1982J,my_par2)
my_par3 <- fdPar(my_basis,Lfdobj=2,lambda=100)
yield.1982J.S.P3 <- smooth.basis(terms,yield.1982J,my_par3)
my_par4 <- fdPar(my_basis,Lfdobj=2,lambda=1000)
yield.1982J.S.P4 <- smooth.basis(terms,yield.1982J,my_par4)

yield.1982J.S.P1$gcv
```

```
## 1982-01-31
##  0.4374292
```

```
yield.1982J.S.P2$gcv
```

```
## 1982-01-31
##  0.2316358
```

```
yield.1982J.S.P3$gcv
```

```
## 1982-01-31
##  0.1033746
```

```r
yield.1982J.S.P4$gcv  # minimal gcv at lambda=1000
```

```
## 1982-01-31
## 0.06979254
```

```r
plot(terms, yield.1982J,pch=15, ylab="Yield",ylim=c(14,15))  # 1982 January
plot(yield.1982J.S.P1,lwd=4,add=TRUE,col=2)
```

```
## [1] "done"
```

```r
plot(yield.1982J.S.P2,lwd=4,add=TRUE,col=3)
```

```
## [1] "done"
```

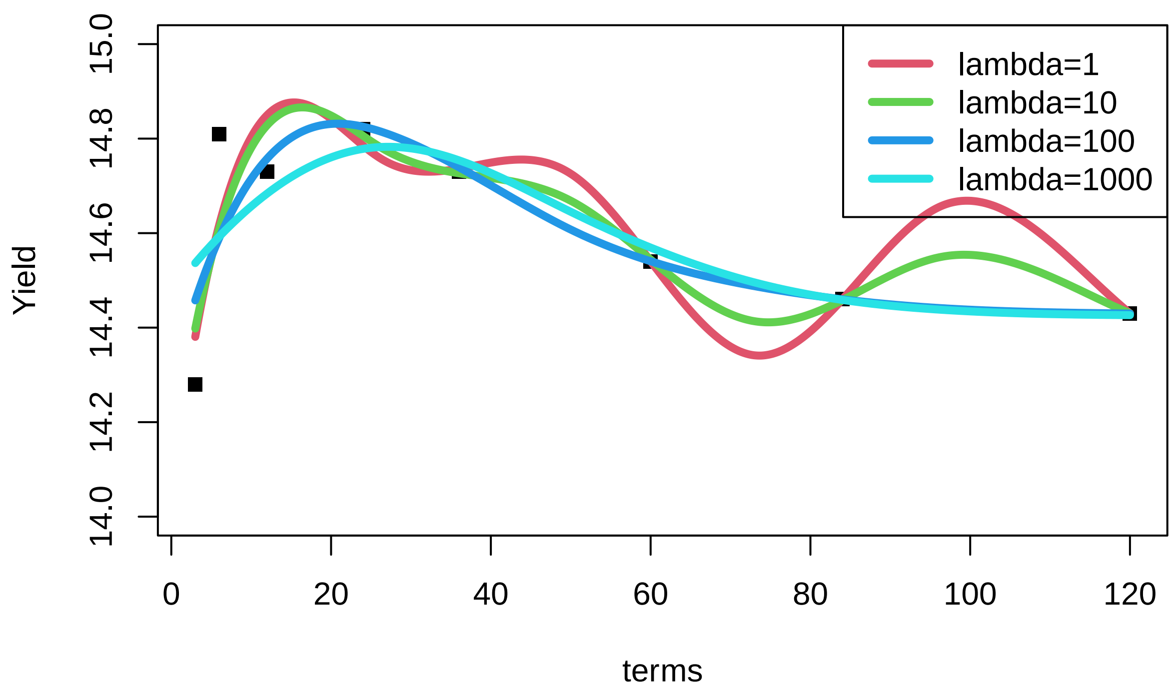
```r
plot(yield.1982J.S.P3,lwd=4,add=TRUE,col=4)
```

```
## [1] "done"
```

```r
plot(yield.1982J.S.P4,lwd=4,add=TRUE,col=5)
```

```
## [1] "done"
```

```r
legend("topright",c("lambda=1","lambda=10","lambda=100","lambda=1000"),col=2:5,lwd=rep(4,4))
```



Note on Simulating Matern Process

Matern process is a Gaussian process. Thus, given $t = t_1, \ldots, t_m$, $(X(t_1), \ldots, X(t_m))$ follows a multivaraite normal distribution. If you are generating mean zero Matern process given time points, $t_1, \ldots, t_m$, - Compute $C = (c_{ij})$ where $c_{ij} = c(t_i, t_j) = cov(X(t_i), X(t_j))$ - Generate $Z \sim N_m(0, I_m)$ - $X = C^{1/2}Z$ Then

$(X(t_1), \ldots, X(t_m)) = (X_1, \ldots, X_m)$ is one realization of mean-zero Matern process observed on $t_1, \ldots, t_m$.

```
# Materh cov. function
cov.ft <- function(d, nu, ssq=1){ # when nu > 0
  td <- sqrt(2*nu)*d
  ssq * 1/(gamma(nu)*2^(nu-1)) * (td)^nu * besselK(td,nu)
}
```

```
# moore penrose power of a matrix.
mppower <- function(matrix,power,ignore=1e-14){ # symmetric matrix
  eig <- eigen(matrix, symm=TRUE)
  eval <- eig$values
  evec <- eig$vectors
  m <- length(eval[eval>ignore])
  tmp <- evec[,1:m]%*%diag(eval[1:m]^power)%*% t(evec[,1:m])
  return(tmp)
}
```

```
# generate matern process
matern.ftn <- function(J,N, nu, ssq=1){
  # Input: J : # of time points, N : number of curves
  # Output: J X N matrix. out[,1] is one curve
  cov.mat <- matrix(0, J, J)
  tt <- 1:J/J
  cov.vec <- cov.ft(d=tt[2:J], nu=nu, ssq=ssq)
  for(j in 1:J-1){
    cov.mat[j,((j+1):J)] <- cov.vec[1:(J-j)]
  }
  cov.mat <- cov.mat + t(cov.mat) + diag(1, J)
  cov.mat.sq <- mppower(cov.mat,1/2,10^(-14))
  return((cov.mat.sq %*% matrix(rnorm(N*J), J, N)))
}
```

2.5.5

2.5.5.(a)

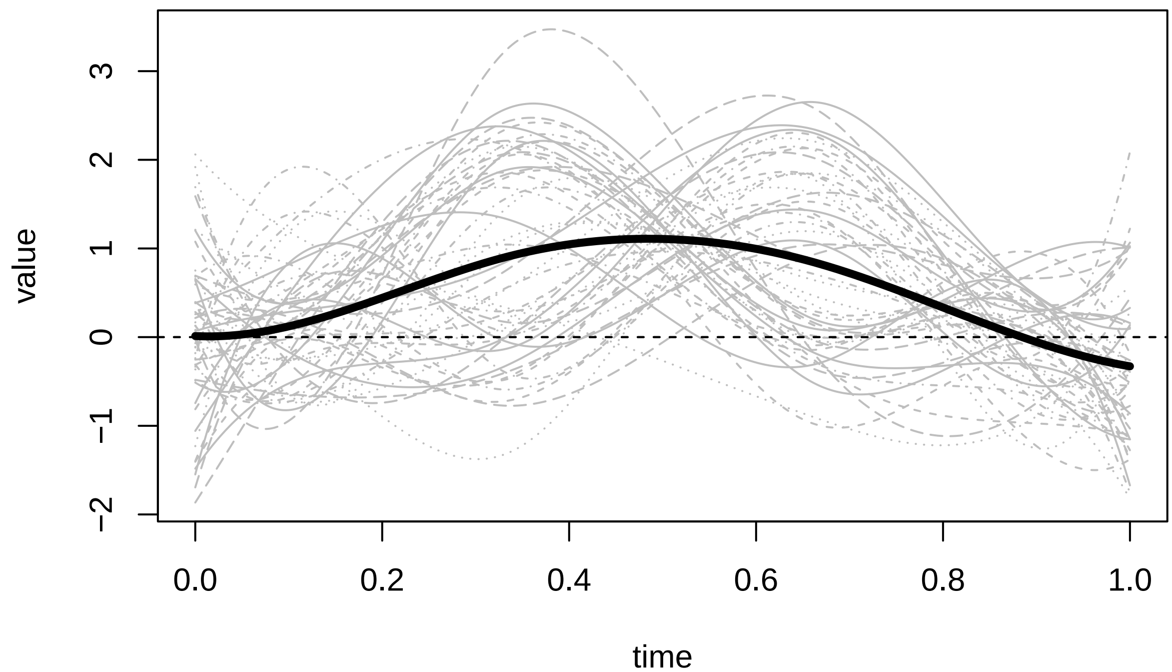
```
set.seed(0)
Bump <- function(x, c, r = 0.25, a = 5) {
  result <- ifelse(abs(x - c) < r,
                   a * exp(-1/(1-((x - c) / r)^2)), 0)
  return(result)
}
N <- 50
J <- 20
x <- seq(0, 1, len = J)
m0_matern = matern.ftn(J, N, 1,.1)
matern5 = matrix(0, J,N)
matern5[,1:25] = m0_matern[,1:25] + Bump(x, c=3/8)
matern5[,26:50] = m0_matern[,26:50] + Bump(x, c=5/8)


my_basis <- create.bspline.basis(c(0,1),nbasis=6)
Data.F <- Data2fd(x,matern5,my_basis)
mu.F <- mean.fd(Data.F)
```

```r
plot(Data.F,col="grey")
```

```
## [1] "done"
```

```r
plot(mu.F,lwd=4,add=TRUE)
```




```
## [1] "done"
```

b

```r
reg <- try(register.fd(Data.F),silent=TRUE)
```

```
##
##
## -------  Curve  1   --------
##
## Iter.    Criterion   Grad Length
## 0        0.3307      0.2951
## 1        0.0985      0.0029
## 2        0.0985      0.0029
##
## -------  Curve  2   --------
##
## Iter.    Criterion   Grad Length
## 0        0.383      0.2289
## 1        0.0972      0.0733
## 2        0.0805      0.0133
```

```
## 3        0.0805      0.0133
##
## -------  Curve  3   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2924      0.0602
## 1        0.2847      0.0015
## 2        0.2847      0.0016
##
## -------  Curve  4   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2573      0.1182
## 1        0.2074      0.0083
## 2        0.2074      0.0083
##
## -------  Curve  5   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2811      0.2412
## 1        0.1116      0.0147
## 2        0.1116      0.0147
##
## -------  Curve  6   --------
##
## Iter.    Criterion   Grad Length
## 0        0.4881      0.2599
## 1        0.338      0.0077
## 2        0.338      0.0077
##
## -------  Curve  7   --------
##
## Iter.    Criterion   Grad Length
## 0        0.457      0.3126
## 1        0.2953      0.0286
## 2        0.2953      0.0286
##
## -------  Curve  8   --------
##
## Iter.    Criterion   Grad Length
## 0        0.141      0.0412
## 1        0.1356      0.0016
## 2        0.1356      0.0016
##
## -------  Curve  9   --------
##
## Iter.    Criterion   Grad Length
## 0        0.5091      0.3562
## 1        0.026      0.0084
## 2        0.026      0.0084
##
## -------  Curve  10   --------
##
## Iter.    Criterion   Grad Length
```

```
## 0        0.0453      0.1022
## 1        0.0108      0.0024
## 2        0.0108      0.0022
##
## -------  Curve  11   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2888      0.2426
## 1        0.0593      4e-04
## 2        0.0593      4e-04
##
## -------  Curve  12   --------
##
## Iter.    Criterion   Grad Length
## 0        0.1455      0.1327
## 1        0.0709      0.0065
## 2        0.0709      0.0065
##
## -------  Curve  13   --------
##
## Iter.    Criterion   Grad Length
## 0        0.0406      0.0036
## 1        0.0405      3e-04
##
## -------  Curve  14   --------
##
## Iter.    Criterion   Grad Length
## 0        0.3094      0.2223
## 1        0.0431      0.0723
## 2        0.01      0.0013
## 3        0.01      0.0013
##
## -------  Curve  15   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2743      0.2651
## 1        0.0549      0.0014
## 2        0.0549      0.0014
##
## -------  Curve  16   --------
##
## Iter.    Criterion   Grad Length
## 0        0.5035      0.2205
## 1        0.1381      0.1326
## 2        0.0232      0.0131
## 3        0.0198      0.0016
## 4        0.019      0.0056
## 5        0.019      0.0056
##
## -------  Curve  17   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2064      0.2179
## 1        0.0765      0.0033
```

```
## 2        0.0765      0.0033
##
## -------  Curve  18   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2558      0.2401
## 1        0.0498      0.0292
## 2        0.0464      0.0026
## 3        0.0463      2e-04
##
## -------  Curve  19   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2068      0.1484
## 1        0.045      0.0356
## 2        0.0361      0.0028
## 3        0.0361      0.0028
##
## -------  Curve  20   --------
##
## Iter.    Criterion   Grad Length
## 0        0.4221      0.3057
## 1        0.2781      0.0171
## 2        0.2781      0.0171
##
## -------  Curve  21   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2993      0.2117
## 1        0.1758      0.0023
## 2        0.1758      0.0023
##
## -------  Curve  22   --------
##
## Iter.    Criterion   Grad Length
## 0        0.6771      0.3499
## 1        0.3602      0.0034
## 2        0.3602      0.0034
##
## -------  Curve  23   --------
##
## Iter.    Criterion   Grad Length
## 0        0.8894      0.27
## 1        0.1778      0.0346
## 2        0.1778      0.0346
##
## -------  Curve  24   --------
##
## Iter.    Criterion   Grad Length
## 0        0.1911      0.1579
## 1        0.0996      0.0141
## 2        0.0996      0.0141
##
## -------  Curve  25   --------
```

```
##
## Iter.    Criterion   Grad Length
## 0        0.4614      0.2961
## 1        0.2744      0.0184
## 2        0.2609      0.0098
## 3        0.2609      0.0098
##
## -------  Curve  26   --------
##
## Iter.    Criterion   Grad Length
## 0        0.1409      0.1906
## 1        0.0158      0.0081
## 2        0.0157      4e-04
## 3        0.0157      4e-04
##
## -------  Curve  27   --------
##
## Iter.    Criterion   Grad Length
## 0        0.4383      0.1064
## 1        0.4144      0.0097
## 2        0.4144      0.0097
##
## -------  Curve  28   --------
##
## Iter.    Criterion   Grad Length
## 0        0.3483      0.3097
## 1        0.1427      0.0032
## 2        0.1427      0.0032
##
## -------  Curve  29   --------
##
## Iter.    Criterion   Grad Length
## 0        0.6062      0.1935
## 1        0.4569      0.0153
## 2        0.4568      0.0063
##
## -------  Curve  30   --------
##
## Iter.    Criterion   Grad Length
## 0        0.6913      0.335
## 1        0.3148      0.0132
## 2        0.3148      0.0132
##
## -------  Curve  31   --------
##
## Iter.    Criterion   Grad Length
## 0        0.3472      0.0738
## 1        0.2633      0.0282
## 2        0.2633      0.0282
##
## -------  Curve  32   --------
##
## Iter.    Criterion   Grad Length
## 0        0.5347      0.3478
```

```
## 1        0.1868      0.0065
## 2        0.1868      0.0065
##
## -------  Curve  33   --------
##
## Iter.    Criterion   Grad Length
## 0        0.3116      0.2715
## 1        0.0601      0.0013
## 2        0.0601      0.0013
##
## -------  Curve  34   --------
##
## Iter.    Criterion   Grad Length
## 0        0.2083      0.1041
## 1        0.1844      0.0099
## 2        0.1844      0.0099
##
## -------  Curve  35   --------
##
## Iter.    Criterion   Grad Length
## 0        0.4346      0.3397
## 1        0.2009      0.0172
## 2        0.1886      0.0114
## 3        0.1886      0.0114
##
## -------  Curve  36   --------
##
## Iter.    Criterion   Grad Length
## 0        0.4959      0.3152
## 1        0.2717      0.0245
## 2        0.232      0.0263
## 3        0.232      0.0263
##
## -------  Curve  37   --------
##
## Iter.    Criterion   Grad Length
## 0        0.614      0.3405
## 1        0.2754      0.0034
## 2        0.2754      0.0034
##
## -------  Curve  38   --------
##
## Iter.    Criterion   Grad Length
## 0        0.214      0.2617
## 1        0.0254      0.0075
## 2        0.0254      0.0075
##
## -------  Curve  39   --------
##
## Iter.    Criterion   Grad Length
## 0        0.6503      0.1303
## 1        0.4635      0.0347
## 2        0.4409      0.01
## 3        0.434      0.0096
```

```
## 4        0.434      0.0096
##
## -------  Curve  40   --------
##
## Iter.    Criterion   Grad Length
## 0        0.1403      0.1879
## 1        0.0614      0.0044
## 2        0.0614      0.0044
##
## -------  Curve  41   --------
##
## Iter.    Criterion   Grad Length
## 0        0.3823      0.3167
## 1        0.1782      0.0014
## 2        0.1776      3e-04
## 3        0.1776      3e-04
##
## -------  Curve  42   --------
##
## Iter.    Criterion   Grad Length
## 0        0.5101      0.3215
## 1        0.1966      0.002
## 2        0.1966      0.002
##
## -------  Curve  43   --------
##
## Iter.    Criterion   Grad Length
## 0        0.3137      0.0671
## 1        0.2338      0.0078
## 2        0.2338      0.0078
##
## -------  Curve  44   --------
##
## Iter.    Criterion   Grad Length
## 0        0.3326      0.149
## 1        0.2224      0.009
## 2        0.2224      0.009
##
## -------  Curve  45   --------
##
## Iter.    Criterion   Grad Length
## 0        0.712      0.1073
## 1        0.3516      0.0099
## 2        0.3355      4e-04
## 3        0.3211      0.0092
## 4        0.3211      0.0092
##
## -------  Curve  46   --------
##
## Iter.    Criterion   Grad Length
## 0        0.4017      0.3097
## 1        0.0512      0.002
## 2        0.0512      0.002
##
```

```
## -------  Curve  47   --------
##
## Iter.    Criterion   Grad Length
## 0        0.1896      0.2264
## 1        0.0444      0.0078
## 2        0.0444      0.0078
##
## -------  Curve  48   --------
##
## Iter.    Criterion   Grad Length
## 0        0.9118      0.1931
## 1        0.7193      0.084
## 2        0.7192      0.0745
## 3        0.7192      0.0745
##
## -------  Curve  49   --------
##
## Iter.    Criterion   Grad Length
## 0        0.1555      0.0587
## 1        0.1368      0.0042
## 2        0.1368      0.0042
##
## -------  Curve  50   --------
##
## Iter.    Criterion   Grad Length
## 0        0.3075      0.2004
## 1        0.0592      0.0709
## 2        0.0146      0.0104
## 3        0.0146      0.0104
```

```
ls(reg)
```

```
## [1] "regfd"  "shift"  "warpfd" "Wfd"    "y0fd"   "yfd"
```

```
mu.reg <- mean.fd(reg$regfd)

par(mfrow=c(1,2))
plot(Data.F,col="grey",main="unaligned", ylim=c(-3,3))
```

```
## [1] "done"
```

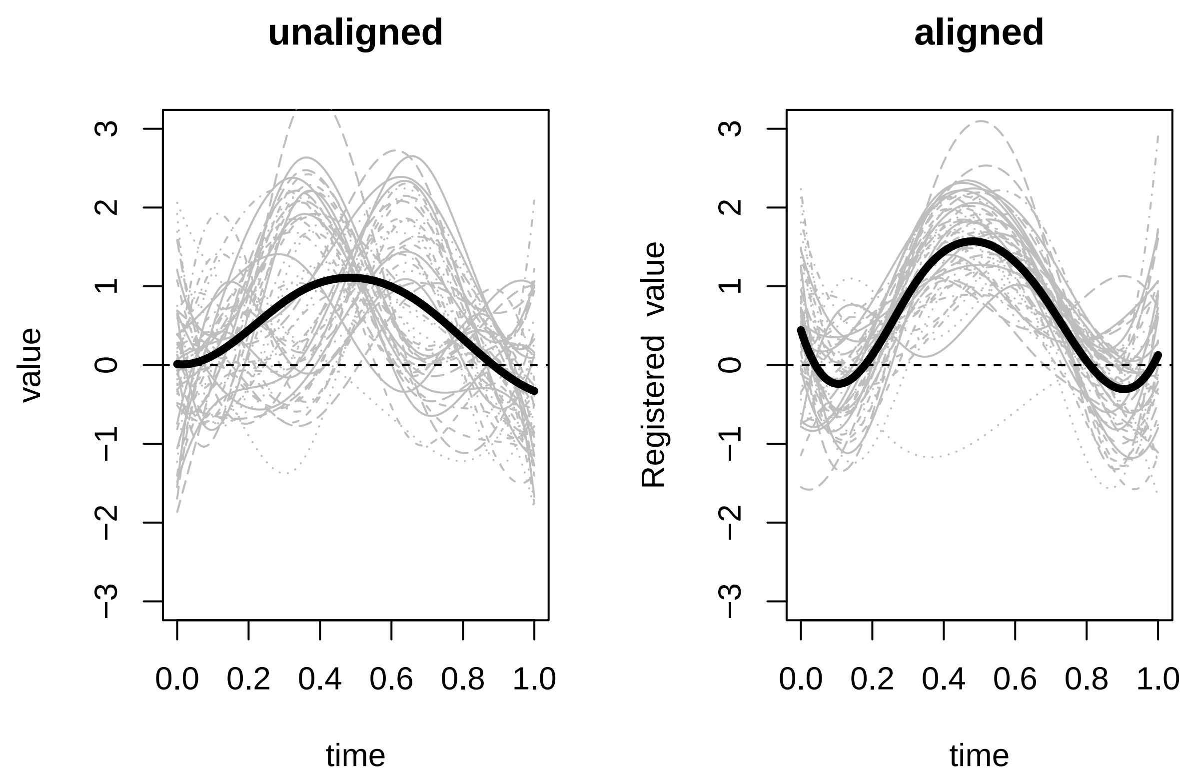
```
plot(mu.F,lwd=4,add=TRUE)
```

```
## [1] "done"
```

```
plot(reg$regfd,col="grey",main="aligned", ylim=c(-3,3))
```

```
## [1] "done"
```

```
plot(mu.reg,lwd=4,add=TRUE)
```




```
## [1] "done"
```

```r
# after aligned, just one peak left/aligned curve shows monotonic increase/decrease pattern
# similar to mean function
```

c

```r
# Carry out an FPCA with one PC on the unaligned and aligned curves separately.

Data.F_pc <- pca.fd(Data.F,nharm=1)
reg_pc <- pca.fd(reg$regfd,nharm=1)

# For each, do lm(score~dummy)
dummy <- c(rep(0,25),rep(1,25))
res1 <- lm(dummy ~ Data.F_pc$scores)
res2 <- lm(dummy ~ reg_pc$scores)

# Calculate a p-value to determine if the estimated slope parameters you get are signicant.
summary(res1)
```

```
##
## Call:
## lm(formula = dummy ~ Data.F_pc$scores)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.44003 -0.13336  0.02362  0.11539  0.41252
```

```
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        0.50000    0.02665   18.76   <2e-16 ***
## Data.F_pc$scores -0.80276    0.04604  -17.43   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1885 on 48 degrees of freedom
## Multiple R-squared:  0.8636, Adjusted R-squared:  0.8608
## F-statistic:   304 on 1 and 48 DF,  p-value: < 2.2e-16
```

```r
summary(res2)
```

```
##
## Call:
## lm(formula = dummy ~ reg_pc$scores)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.79988 -0.48719  0.01262  0.48317  0.59779
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)     0.5000     0.0714   7.003 7.28e-09 ***
## reg_pc$scores  -0.2031     0.1989  -1.021    0.312
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5049 on 48 degrees of freedom
## Multiple R-squared:  0.02125,    Adjusted R-squared:  0.0008594
## F-statistic: 1.042 on 1 and 48 DF,  p-value: 0.3124
```

```r
# Compare with the aligned and unaligned curves. What did aligning do to the p-value?
```

d

Open