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Estimation and prediction of the OD matrix in uncongested urban road network based on traffic flows using deep learning

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ABSTRACT

In this article, we propose a new method for OD (Origin–Destination) matrix prediction based on traffic data using deep learning. The input values of the developed model were determined based on data on the structure of the road network, origin and destination points of trips, as well as data on traffic intensity on road network sections recorded by video-sensing devices. The advantage of the method is that the complex process of data acquisition and processing is not required for the estimation and prediction of the matrix. Historical data and the iterative method of estimating a prior OD matrix were used only to generate training sequences for the neural network. The proposed method using deep learning neural networks with the long short-term memory (LSTM) or autoencoders layers (DLNa — deep learning networks with autoencoders) is characterized by relatively high accuracy and resistance to temporary missing data from several measurement points located in the urban road network. The case study was conducted for a network of a medium-sized city in Poland. The results show (average MAPE = 7.18% (LSTM), 6.80% (DLNa)) that the proposed method can have a practical implementation in real-time dynamic traffic assignment (DTA) systems for ITS applications. The proposed method of short-term forecasting the OD matrix does not require questionnaire research or detailed information on spatial development. Therefore, it is not as expensive and time-consuming as the methods based on these data.

1. Introduction

Analyses related to planning traffic flows, preparing traffic forecasts, and designing organizational changes require knowledge of the origin–destination (OD) matrix. It reflects the transport demand presented in the system of pairs of relations (OD pairs). With effective traffic management, when in critical cases (i.e., accidents, road works, breakdowns, or other unusual situations) it is necessary to know about the trip destinations of road users, dynamic OD matrices are often used, which are built and updated in successive periods. Only on this basis, optimal detour routes in urban road networks can be determined.

The OD matrix provides a picture of the spatial distribution of the traffic. Individual cells in the two-dimensional OD matrix represent the traffic flow expressed as the number of trips made between a pair of traffic analysis zones (TAZs). Centroids designated for each zone represent the place of accumulation of flows produced and attracted by the zone. In practical solutions, these centroids are usually transferred, using the so-called connectors, to the nearest nodes of the technical network (e.g., road or rail network). Therefore, the estimation of the OD matrix between centroids of the TAZs can be reduced to the estimation of the OD matrix between the nodes of the technical transport network.

OD matrices can be built for various modes of transport, for specific periods, for various travel purposes, and for groups of traffic participants. In this way, individual demand strata are formed, expressed in an OD pair structure. These matrices can be determined both for the existing state and for the future state. Individual elements of the OD matrix can be represented in absolute or relative units with the interpretation of the number of trips or the share of traffic flow moving between particular OD pairs, respectively.

The classic methods for estimating the OD matrix require large-scale surveys and information on the sociodemographic situation and spatial development of the area. Therefore, such methods are expensive and time-consuming, and the results obtained with their use may quickly become outdated due to the rapidly developing transport system in urban areas. Additionally, transport demand models built based on classical methods do not take into account temporary disturbances in trip distribution resulting from time of day, seasonality, organization of mass events, road works, or weather conditions. Therefore, more and more often, additional sources of information and modern techniques are used to build OD matrices.

Currently, in times of more and more common monitoring of urban areas, the use of data on the volume of traffic flow on sections of the transport network for the construction of the OD matrix is becoming

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an easy and cheap way to obtain additional data and therefore an increasingly attractive alternative to classical methods.

Both static and dynamic models can be used in planning a transport system. On the other hand, in traffic control and management, particular attention is paid to the necessity and possibility of frequent updating of the input data following changes in the volume of actual traffic flow in the urban road network. In such situations, the estimation of the OD matrix should be based on dynamic models and use up-to-date information on the intensity of traffic in sections of the transport network obtained from traffic detectors (Żochowska, 2012, 2014).

Traffic prediction plays a fundamental role in intelligent transport systems. Reliable forecasts of the structure and volume of traffic flows can help plan the route, guide vehicles, minimize congestion, and ensure smoothness of traffic flow. This is especially important when unexpected events occur on the road (e.g., accidents, vehicle breakdowns, damage of road infrastructure, etc.). Then it is necessary to know the structure of traffic flow in subsequent time intervals. This allows detours and diverting all or only some of the vehicles to alternative paths. However, this problem is difficult due to the complex and dynamic space–time relationships between traffic flows in different parts of the urban road network. In recent years, much research has been done in this area, especially in terms of the use of deep learning, which greatly improves the possibilities of traffic prediction.

Deep learning methods turned out to be more accurate and resistant to missing data and errors than previously used approaches based on traditional machine learning methods, such as Kalman filters, Bayesian networks, or Support Vector Machine (SVM). The authors of the publication conducted an interesting and detailed review of deep learning in transport systems (Yuan Wang et al., 2019).

This article describes the method for estimating and predicting the OD matrix using a long-short-term memory (LSTM) recursive network. This network allows the characteristics of the time series to be captured over short and long periods and is often used to forecast traffic volumes. Traffic forecasts made with the use of the LSTM network can achieve better reliability (Junyi Li et al., 2021; Wang et al., 2021). The detailed contribution of the authors is described in the next chapter.

The remainder of this article is structured as follows. Section 2 provides a general overview of the literature on methods for estimating dynamic OD matrices. It mainly covers articles from 2012 to 2021. Section 3 describes the method for estimating the prior OD matrix. It also includes a description of the prediction model using the LSTM network. Section 4 presents a case study of the road network in the city of Gliwice (Poland). Section 5 analyses and discusses the results obtained. Finally, Section 6 contains the conclusions and plans for future work.

2. Methods of OD matrix estimation and traffic prediction — literature review

The methods of estimating the OD matrix with the use of traffic volumes differ mainly in the data acquisition process and the techniques used to build trip distribution. Traffic flow data used to build the OD matrix can be obtained in various ways. The most commonly used include surveying travellers, the use of vehicle registration plates (Antoniou et al., 2006; Zhou and Mahmassani, 2007; Jing Liu et al., 2020; Sánchez-Cambronero et al., 2021), analysis of data from mobile phones (Zin et al., 2018; Tolouei et al., 2017; Alexander et al., 2015; Liu and Fu, 2021), floating car data (FCD) (Nigro et al., 2018; Vogt et al., 2018), measuring traffic volumes and speeds (Krishnakumari et al., 2020), and recording of vehicle GPS position (Yang et al., 2017). Data obtained from video sensing detectors are also an important source of information on traffic flow in a road section, especially when it is necessary to estimate the OD matrix in real time.

The OD matrix can be obtained from a macroscopic demand model for a city or a larger area. However, if the area under study is too small and is located in only one or more TAZs, or if the macroscopic model is missing, estimating the OD matrix becomes a problem for practitioners. In such cases, the OD matrix can be estimated from information on traffic volumes in road sections. Table 1 presents a list of selected publications that deal with the issue of estimating the OD matrix based on traffic flows (Żochowska, 2012).

As can be seen in Table 1, many approaches are used to estimate an OD matrix from traffic flow count data. Usually, the choice of the method depends on assumptions about the quality of the final results, the structure of the existing transport network, and the treatment of congestion in the estimation process (Żochowska, 2012; Savrasovs and Pticina, 2017).

One of the first models based on the concept of traffic modelling assumed a proportional assignment that does not take into account congestion (Van Zuylen and Willumsen, 1980). The idea of extending the entropy model was presented only by Fisk (1988), who took into account traffic congestion by introducing boundary conditions in the form of equilibrium assignment that complies with the first Wardrop principle. The further development of these methods was directed mainly towards the use of both gravity techniques and the intervening opportunities models of the OD matrix estimation.

With the development of technology, these methods were modified. Sherali et al. (1994) worked on applying a linear programming approach to estimating the OD matrix based on traffic volumes. The optimization of the objective function introduced in his model was to minimize the sum of the travel costs and the deviations between the estimated and observed values of both the traffic volumes and the prior OD matrix. In subsequent publications, Sherali adapted his model to a situation where traffic volumes are not known in all sections (Sherali et al., 2003). A solution can only be obtained for fixed points that have been heuristically estimated by iteratively fitting a nonlinear model to a sequence of linear programming techniques. Sherali and Park also developed a model used to estimate the OD matrix with the dynamic approach (Sherali and Park, 2001). In their model, as a decision variable, they used traffic volumes in road sections instead of traffic flows for the OD pairs, and the problem of dynamic estimation of the OD matrix was formulated based on assumptions of least squares method. The path generation process in the transport network according to the Sherali and Park model is iterative.

For the methods using statistical inference, it is assumed that the traffic volumes in the sections are treated as realizations of independent Poisson random variables. These models often use maximum likelihood techniques, generalized least squares, and Bayesian inference. The case of an uncongested transport network was presented by Hazelton (2000). He uses an approach based on the maximization of multivariate normal approximations to the likelihood. In turn, Cheng et al. (2014) presented a Bayesian network model for the estimation of the OD matrix, assuming a normal distribution for traffic flows. The proposed Bayesian network model uses historical traffic volumes. The results show that the proposed method has high accuracy and practical application.

In the article Zhou and Mahmassani's (2006), information from the automatic vehicle identification system (AVI count) was used to estimate the OD matrix. A non-linear ordinary least squares estimation model is presented that systematically combines the AVI counts, the link counts, and historical OD demand information into a multi-objective optimization framework.

Kalman filtering methods are still used to estimate the OD matrix. Ashok (1996) and Barcelo et al. (2010) provide ad hoc Kalman filtering procedures that use data from Bluetooth sensors in mobile devices in vehicles to estimate time-dependent OD matrices. Barcelo et al. (2010) also investigated the quality of the data obtained from Bluetooth devices and their usefulness for the prediction of travel time and the estimation of time-dependent OD matrices. The authors found that the use of mixed data sources to estimate the OD matrix can reduce the computational complexity and thus solve the problem to estimate the OD matrix in real time.

Table 1
Selected techniques of O–D matrix estimation based on traffic counts.

Research technique	Authors
Approaches based on traffic modelling including: • Approach based on gravity models • Gravity-opportunity-based models • Information minimization and entropy maximization approach • Linear programming techniques	• Van Zuylen and Willumsen (1980) • Willumsen (1984) • Fisk (1988) • Sherali et al. (1994) • Sherali and Park (2001) • Sherali et al. (2003)
Statistical inference approaches including: • Maximum-likelihood method • Generalized least squares method • Bayesian inference • Gaussian elimination method	• Cascetta et al. (1993) • Hazelton (2000) • Zhou and Mahmassani (2006) • Cheng et al. (2014) • Yang et al. (2017) • Hualan et al. (2020)
Approaches based on mathematical techniques and models including • Gradient-based solution techniques • Kalman filtering techniques • PCA — Principal Component Analysis • Bilevel programming approach • PFE — Path Flow Estimator	• Okutani (1987) • Spiess (1990) • Ashok and Ben-Akiva (1993) • Ashok (1996) • Bell and Shield (1995) • Ashok and Ben-Akiva (2000) • Maher et al. (2001) • Ashok and Ben-Akiva (2002) • Chen et al. (2004) • Chen et al. (2005) • Zhou and Mahmassani (2007) • Djukic et al. (2012) • Djukic et al. (2012) • Lu et al. (2015)
Genetic Algorithms based approaches	• Kim et al. (2001) • Yun and Park (2005)
Multi-Vehicle ODM Approach including using aggregate data: • GPS tracking data • mobile phone data • FCD — floating car data • Smartcard data • Bee colony-based optimization • Bluetooth traffic data	• Barcelo et al. (2010) • Caggiani et al. (2012) • Barcelo et al. (2013) • Alexander et al. (2015) • Ge and Fukuda (2016) • Tolouei et al. (2017) • Nigro et al. (2018) • Zin et al. (2018) • Vogt et al. (2018) • Brzeziński and Dybicz (2021) • Hussain et al. (2021) • Ceccato et al. (2022) • Ros-Roca et al. (2022)
Approaches based on vehicle identification data including: • License plate surveys • Video recordings • Path travel times	• Zhou and Mahmassani (2006) • Antoniou et al. (2006) • Cipriani et al. (2014) • Savrasovs and Pticina (2017) • Jing Liu et al. (2020) • Sánchez-Cambronero et al. (2021)
Approaches based on a neural network including **deep learning approaches**: • HA — Historical Average method • LSTM — Long Short-Term Memory network • TCN — Temporal Convolutional Network • ConvLSTM — Convolutional LSTM Network • ST-GCN — Spatial-Temporal Graph Convolutional Network • MLP — Multi-Layer Perceptron Network • MGC — Multi-Graph Convolutional Network • ED-MGC network — Encoder-Decoder Multi-Graph Convolutional Network • DySAT — Dynamic Self-Attention Network • DNEAT — Dynamic Node-Edge Attention Network • RMGC — Residual Multi-Graph Convolutional Network • ST-ED-RMGC — Spatio-Temporal Encoder–Decoder Residual Multi-Graph Convolutional Network • MF-ResNet — Multi-Fused Residual Network • CAS-CNN — Channel-wise Attentive Split-Convolutional Neural Network	• Krishnakumari et al. (2020) • Jinlei Zhang et al. (2021) • Dapeng Zhang et al. (2021) • Jintao Ke et al. (2021) • Liu and Fu (2021) • He et al. (2022)

To simplify the problem, some researchers have used principal component analysis (PCA) to estimate high-dimensional OD matrices. This approach was used by the Djukic et al. (2012, 2021) who dealt with the problem of dimensionality reduction and approximation of OD demand. In their work, they showed an improvement in the quality of the OD estimates using the so-called ‘‘colour’’ Kalman filter.

Table 2
Characteristics of selected deep learning-based methods used in traffic prediction.

Method	Description
Convolutional Neural Network (CNN)	• Used to extract the spatial correlation in road networks from two-dimensional temporal—spatial traffic data, • Requires transforming the structure of the traffic network at different times into images and dividing the images into standard grids each representing a region, • Limited to modelling Euclidean data,
Graph Convolutional Network (GCN)	• Extends the convolution operation to graphically structured data that is more suitable for representing traffic, • Used for modelling non-Euclidean data with a spatial structure that may be consistent with the structure of the road network,
Recurrent Neural Network (RNN) including LSTM, GRU	• Commonly used to model time dependencies, • Captures the non-linear relation of traffic flows, • Relies on the order of data in data processing, • When modelling long sequences, their ability to remember may deteriorate,
KNN-LSTM	• Selects mainly related objects by computing the spatial correlations of the traffic flows using the KNN method, • Predicts OD flows by analysing the temporal changes in the traffic flow using the LSTM,
Grid-embedding based Multi-task Learning (GEML)	• Models the spatial pattern of passenger traffic and the neighbouring relationships of different CNN regions by embedding of the grid, • Models the attributes of time and captures the goals of the OD matrix prediction problem using the multitasking educational part,
Residual CNN (RCNN)	• Avoids gradient disappearance and captures spatial–temporal and external travel requirements,
Spatial Interaction GCN model (SI-GCN)	• Combines GCN and a mapping function to predict the flow information
Fusion Line GCN (FL-GCN)	• Transforms the selected road network into a line graph, • Recognizes the spatiotemporal correlations and predicts OD flows,
Multi-perspective GCN (MPGCN)	• Captures the complex temporal–spatial correlations of OD flows, • Learns temporal characteristics of flows on OD pairs using LSTM model, • Extracts the OD flows spatial associations using GCN model,
Channel-wise Attentive Split CNN (CAS-CNN)	• Introduces a gated mechanism to solve data sparsity and dimensionality problems.

One of the possible solutions that takes into account the interaction of traffic flows during the assignment procedure is bi-level programming approach, which consists in determining the proportion of the use of paths when estimating the OD matrix (Maher et al., 2001). In this method, one of the statistical techniques (e.g., the generalized least squares method) can be used to solve the higher-level problem regarding the selection of the optimal OD matrix. In turn, in the problem of the lower level, in which the assignment of the traffic flow to the paths is determined endogenously, a procedure based on the assumptions of the equilibrium traffic assignment using the results from previous estimates can be applied.

Bell and Shield (1995) used a non-linear path flow estimator (PFE) model to estimate the OD matrix, based on the assumption of stochastic user equilibrium (SUE). This gives clearly defined volumes of traffic flow in the road sections and does not require knowledge of the volumes in all sections.

Genetic algorithms are also used to estimate the O–D matrix based on traffic flows. The authors of Kim et al. (2001) compared the results obtained with the use of this technique with the results obtained with the use of bi-level programming for the static model and noticed a high level of convergence. There were also attempts to apply this technique in dynamic models and for large networks (Yun and Park, 2005).

More and more publications are in the literature in which the neural network was used to estimate the OD matrix. Krishnakumari et al. (2020) proposed a data-driven method to estimate the OD matrix in cases where the supply pattern that covers the velocities and volume of traffic flows is available. They used a simple multilayer perceptron neural network to predict production and attraction in their experiments. They showed that with these input data, no iterative dynamic procedure that leads to equilibrium assignment is needed.

Recently, a great deal of research has been done on methods for predicting traffic volume or estimating the OD matrix using deep learning. These methods greatly increase the possibilities of traffic prediction. The article by Yin et al. (2021) provides a comprehensive review of deep learning approaches in traffic forecasting from multiple perspectives. Existing methods of traffic forecasting are summarized. Deep learning methods give good results and are currently the most widely used for traffic forecasts. The authors found that the accuracy of the forecast depends mainly on the data set used and the forecast horizon. Table 2 presents the characteristics of selected deep learning-based methods based on Shuai et al. (2022).

Good results on the forecast of short-term traffic flow were obtained by the authors of publications (Junyi Li et al., 2021; Pamuła, 2019) by applying the machine learning method or transferring models to machine learning methods. Research results have shown that machine learning-based models can increase accuracy in the case of missing data. Deep learning networks have been successfully used by the authors (Wang et al., 2021; Pamula, 2018) to predict traffic congestion and conditions in the city and on highways. They used convolutional networks and the LSTM network with good results.

In the article by Jinlei Zhang et al. (2021) a channel-wise attentive split-convolutional neural network (CAS-CNN) was proposed for the short-term forecast of the OD flows. The model was tested on two real large-scale data sets from the Beijing metro. The authors found that the method was superior to other comparative methods. In turn, Jinlei Zhang et al. (2021) proposed an innovative neural network architecture called the Dynamic Node-Edge Attention Network (DNEAT) to model separate features in forecasting OD demand. The experiments were carried out on two real travel demand data sets (from Chengdu, China, and New York) to compare the proposed model with another benchmark. The results show that, compared to the graph-based models, the convolutional and LSTM network model performs better in terms of the results of RMSE (Root Mean Square Error) and MAPE (Mean Absolute Percentage Error) and is more resistant to data of high sparsity.

The authors of another article Jintao Ke et al. (2021) present a model in which several different deep learning networks were used to estimate the OD matrix, such as a space–time encoder–decoder network, a convolutional network, and a long-short-term memory network (LSTM). After testing on real for-hire vehicle data sets in Manhattan, New York, the authors concluded that the proposed deep learning framework is far superior to state-of-the-art solutions. However, the calculated MAPE error was 0.38. The developed model requires a lot of computing power, and its accuracy is not too high. The convolutional network for the prediction of the OD matrix was also used by the authors (Liu and Fu, 2021). Data from mobile phones were entered into the model.

In summarizing the literature review, it can be stated that the use of deep learning to predict data, especially those that make up time series, is still a valid and appropriate method. In the presented publications, the authors determined OD matrices using various methods, also using deep learning. Hybrid models were often used, which contained many parameters and required very high computing power. However, we have not encountered the prediction of the OD matrix based on traffic volume and the direct use of the LSTM network.

The authors' contribution is presented below:

- we propose a new model of direct OD matrix prediction based on traffic counts using deep learning neural networks with the long short-term memory (LSTM), or autoencoders layers (DLNa),
- an OD matrix can be predicted directly from the traffic flow data without prior OD matrix,
- promising results were obtained for various time horizons (15, 30 and 45 min).

3. Methodology

3.1. Assumptions and general description of the method

The proposed method to estimate and predict the OD matrix in the urban road network uses data obtained from traffic monitoring devices. Video-sensing devices that record information on traffic volume are located in key locations in the area under study. For the prediction of the OD matrix, a 15-minute analysis period was adopted. Furthermore, it was assumed that there was no traffic congestion. In such a case, it is not necessary to consider a path change for an OD pair in dependence on the traffic flow, which makes it possible to adopt fixed paths at all periods.

In the proposed method, the OD pair includes two vertices at which the single trip begins and ends. They correspond to the vertices of the transport network located on the border of the analysis area in places where the detectors that record the intensity of traffic in the entry and exit directions in this area have been installed. Therefore, the structure of the OD matrix corresponds to the connections between these points.

A general scheme of the method is presented in Fig. 1. The method consists of four steps. In the first stage, a road network model should be built to allow further analysis. For this purpose, information is necessary on the network structure and the location of detectors that record the data used to estimate the OD matrix. Elements of graph theory were used to build the road network model.

The urban road network and data obtained from traffic monitoring devices are the basis for the development of the prior OD matrix in the second stage. This matrix is constructed according to the iterative procedure. It was assumed that after assigning the estimated OD matrix to the section of the network, the deviations between the real and estimated traffic flows in the individual sections should be as small as possible. Due to the assumed lack of congestion, the paths for individual OD pairs are fixed and independent of the period.

The results obtained from the iterative method together with the detector data were used to train the neural network in stage 3. After reaching the acceptable error level, the neural network was used to predict the OD matrix in the last stage of the method.

3.2. Road network model

The road network has been mapped in the form of a directed graph (N, L), where N corresponds to a set of nodes and L — to a set of links of this graph. When building the structure of the road network model, the places where the main traffic flows divide, and merge were taken into account. The network model also includes the locations of video-sensing devices that record the intensity of traffic.

Therefore, the set of all nodes N has been divided into three subsets:

NB — a set of border nodes, located on the border of the analysis area; for these elements, data from detectors containing information on the recorded volumes of traffic entering and exiting the analysis area are available,

ND — a set of nodes located inside the area (i.e., not border nodes) for which information from video-sensing detectors is available,

NU — set of nodes inside the area for which no traffic information is available.

These subsets are disjunctive and complementary. Thus, the set of all nodes used to build the road network model can be described as follows:

$$N = \{\dots, i, \dots, j, \dots : \quad i, j \in NB \cup ND \cup NU, \quad i \neq j, \\ NB \cap ND \cap NU = \emptyset\} \tag{1}$$

Each section of the road network has been mapped in the form of a pair of nodes between which there is a direct connection to the road infrastructure, i.e.: (i, j), with $i \in N,\ j \in N,\ i \neq j$. Therefore, the set of links can be described as follows:

$$L = \{\dots, (i, j), \dots : \quad i \in N, \quad j \in N, \quad i \neq j\} \tag{2}$$

Using the possibility of obtaining data on traffic volumes registered in specific locations (i.e., in nodes of the road network included in the set NB or ND) using video-sensing detectors, the set of links L can be decomposed into two disjunctive and complementary subsets:

LD — a set of sections (graph links) for which information about the traffic is available; it is a set of links for which the volume of traffic flows was recorded for at least one of the nodes constituting the start or the end of the link, i.e.:

$$LD = \{(i, j) : \quad i \in (NB \cup ND) \vee j \in (NB \cup ND), \quad i \neq j\} \tag{3}$$

LU — a set of sections (graph links) for which information about the traffic is not available; it is a set of links for which the volume of traffic flows was not recorded for any of the nodes constituting the start or the end of the link, i.e.:

$$LU = \{(i, j) : \quad i \in NU \wedge j \in NU, \quad i \neq j\} \tag{4}$$

The nodes included in the NB set were used to build the structure of the OD matrix. By connecting the nodes between which there are connections, the OD pairs described as (o, d) were obtained, where $o \in NB,\ d \in NB,\ o \neq d$.

3.3. Description of the iterative method for estimating the prior OD matrix based on traffic flows on road network sections

Using the previously entered notation, the OD matrix can be written in a general way as follows:

$$\left[x^{od}(t)\right] = \begin{bmatrix} x^{11}(t) & \cdots & x^{1n_d}(t) \\ \vdots & \ddots & \vdots \\ x^{n_o 1}(t) & \cdots & x^{n_o n_d}(t) \end{bmatrix} \tag{5}$$

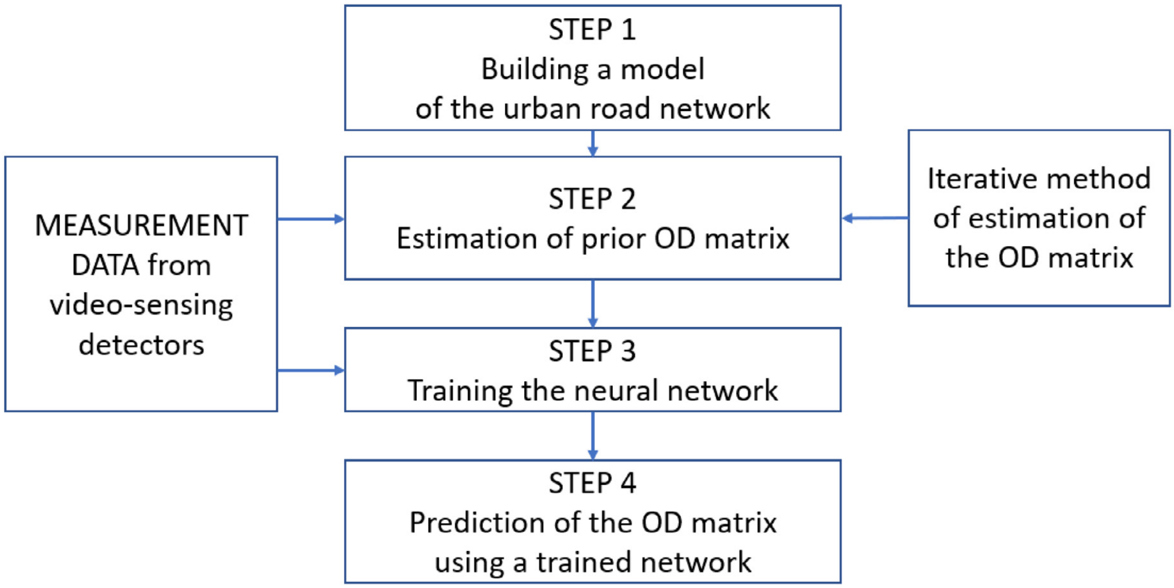


Fig. 1. Methodological framework.

where n_o is the number of nodes representing the origin of the trip and n_d is the number of nodes representing the destination of the trip.

In the proposed approach, it was assumed that the traffic detectors recording the traffic volumes at border nodes (i.e., included the NB set) are installed on two-way roads. Another assumption that the road network between the nodes $o \in NB$, $d \in NB$ is coherent—which means that at least one path can be provided between each pair of nodes—leads to the conclusion that the OD matrix is a square matrix, i.e., $n_o = n_d$. Furthermore, the presented research covered only OD flows with different origins and destinations, i.e., $o \neq d$. Then the OD matrix can be presented in the form:

$$\left[x^{od}(t)\right] = \begin{bmatrix} - & x^{12}(t) & \cdots & \cdots & x^{1n}(t) \\ x^{21}(t) & - & \cdots & \cdots & \vdots \\ \vdots & \cdots & - & \cdots & \vdots \\ \vdots & \cdots & \cdots & - & x^{n-1n}(t) \\ x^{n1}(t) & \cdots & \cdots & x^{nn-1}(t) & - \end{bmatrix} \tag{6}$$

where n is the number of elements of the set NB, that is, the order of the square OD matrix ($n = n_o = n_d$).

The problem of estimating the OD matrix based on traffic flows in sections of the road network consists of searching for a matrix that meets certain boundary conditions and an appropriately formulated optimization criterion. Therefore, for interval t, the optimization task can be defined as follows:

$$\widehat{OD}(t)^* = \arg\min F\left(\widehat{OD}(t), \hat{Q}(t)\right) \tag{7}$$

where $\widehat{OD}(t)^* \equiv \left[\hat{x}^{od}(t)\right]^*$ is the optimal estimated OD matrix containing demand flows (o, d), and $F\left(\widehat{OD}(t), \hat{Q}(t)\right)$ denotes a two-criteria objective function, in which one criterion $\left(f_1\right)$ is a measure of the deviations of the estimated OD matrix from the prior OD matrix, and the second criterion $\left(f_2\right)$ is a measure of the deviation of the real traffic volumes in the road sections from the estimated values determined based on the assignment of the estimated OD matrix to the transport network, which can be written as:

$$F\left(\widehat{OD}(t), \hat{Q}(t)\right) = \gamma_1 f_1\left(OD(t), \widehat{OD}(t)\right) + \gamma_2 f_2\left(Q(t), \hat{Q}(t)\right) \tag{8}$$

where $\widehat{OD}(t) \equiv \left[\hat{x}^{od}(t)\right]$ is estimated OD matrix and $OD(t) \equiv \left[x^{od}(t)\right]$ is prior OD matrix. In turn, matrices containing intensity values on the sections $(i, j) \in LD$, are denoted respectively $Q(t) \equiv \left[q_{ij}(t)\right]$ as the matrix recorded by video-sensing devices and $\hat{Q}(t) \equiv \left[\hat{q}_{ij}(t)\right]$ as the matrix estimated based on the assignment of the estimated OD matrix, i.e. $\widehat{OD}(t)$ to the road network.

Parameters γ_1 and γ_2 denote weights representing the degree of accuracy of the prior values. If the prior $OD(t)$ matrix (which is the reference matrix in the case) is reliable then the value of γ_1 should be large concerning γ_2. This will make the optimal estimated matrix $\widehat{OD}(t)^*$ more consistent with the prior matrix $OD(t)$. Then larger deviations between the $OD(t)$ and $\hat{Q}(t)$ matrices can be accepted. On the other hand, if the observed values of the intensity $Q(t)$ are more reliable than the information contained in the prior matrix $OD(t)$, then the value of the parameter γ_2 should be much higher than the value of the parameter γ_1. Then, the second part of the Eq. (8) provides the optimization, leading to the fact that the estimated values of the intensities $\hat{Q}(t)$ are close to the observed values of the traffic flows $Q(t)$. In this case, larger deviations between the estimated matrix $\widehat{OD}(t)$ and the prior $OD(t)$ matrix can be accepted. Therefore, the weights are strictly dependent on the available data and the modelling concept.

In the proposed approach, we do not have a prior $OD(t)$ matrix. However, it is needed to train the neural network. Therefore, an iterative method for estimating the reference OD matrix was developed. It assumes that the matrix of observed traffic volumes $Q(t)$ is the result of the assignment the prior $OD(t)$ matrix to the road network. Therefore, we are looking for such values of estimated traffic flows $\widehat{OD}(t)$ which, when assigned to the network, give minimal deviations between the estimated volumes of traffic flows $\hat{Q}(t)$ and the real ones $Q(t)$, i.e. recorded by the video-sensing detectors during the period t. This assumption can be formally expressed by taking the values $\gamma_1 = 0$ and $\gamma_2 = 1$, which reduces the objective function $F\left(\widehat{OD}(t), \hat{Q}(t)\right)$ to the form:

$$F\left(\widehat{OD}(t), \hat{Q}(t)\right) = f_2\left(Q(t), \hat{Q}(t)\right) \tag{9}$$

and formulates the optimization task as:

$$\widehat{OD}(t)^* = \arg\min_{(i,j)\in L} f_2\left(\left[q_{ij}(t)\right], \left[\hat{q}_{ij}(t)\right]\right) \quad \wedge$$
$$\left[\hat{q}_{ij}(t)\right] = \text{asssign}\left[\hat{x}^{od}(t)\right] \tag{10}$$

where the notation asssign $\left[\hat{x}^{od}(t)\right]$ denotes the assignment of the matrix $\widehat{OD}(t)$ to the road network. Formula (10) presents a general description of the optimization task, which should be interpreted as searching for such values of traffic flows, i.e., $\widehat{OD}(t)^*$, for which the function $f_2\left(\left[q_{ij}(t)\right], \left[\hat{q}_{ij}(t)\right]\right)$ is minimized.

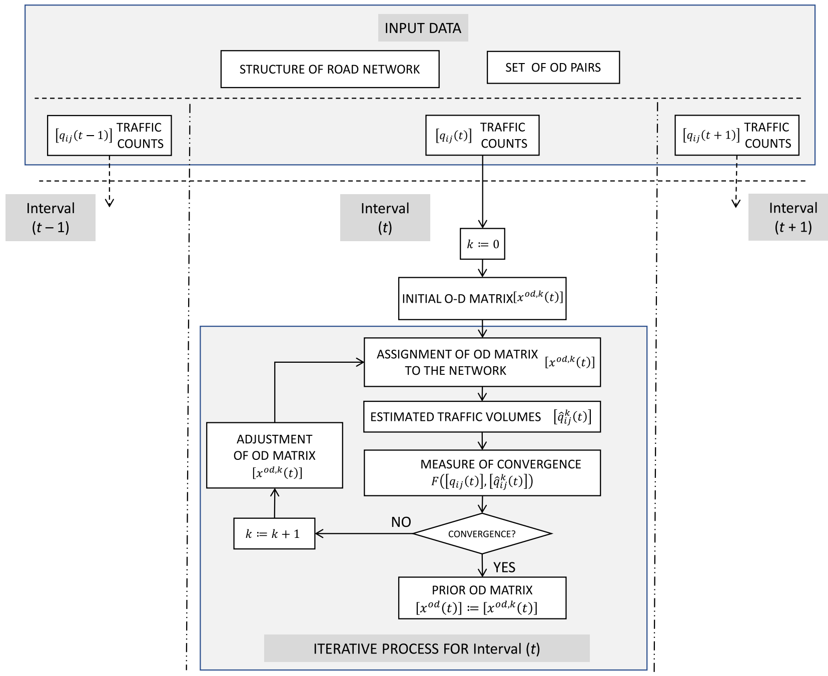


Fig. 2. General diagram of the iterative method of estimating the OD matrix based in traffic volume on sections of the road network for three consecutive intervals: (t − 1), t and (t + 1)

The minimization of the objective function is performed separately for each interval t. This means that the effect of this procedure is to obtain such a set of OD matrices (for each interval), the assignment of which on the road network leads to the best convergence with the observed results. These matrices constitute reference matrices for the neural network training process.

For each border node (i.e., an element of the NB set), it was assumed that the volume of the traffic flow entering the analysis area in the interval t is equal to the value of the inbound traffic intensity recorded by the detector. By compiling the data for all detectors located at the boundaries of the area, a vector $XO(t)$ was obtained, which contains the values of the traffic flows entering the area in the period t, which was written as:

$$XO(t) = \langle x^o(t) : \quad o \in NB \rangle \tag{11}$$

where o denotes the node located on the border of the analysed area, for which the intensities of the flows entering the area were recorded.

The volume of traffic leaving the analysis area was determined similarly, obtaining the vector:

$$XD(t) = \left\langle x^d(t) : \quad d \in NB \right\rangle \tag{12}$$

where d is the node located on the border of the analysed area, for which the intensities of the flows leaving the area were recorded.

These quantities constitute the boundary conditions of the OD matrix for the interval t, i.e.

$$x^o(t) = \sum_{d \in NB} x^{od}(t) \tag{13}$$

and

$$x^d(t) = \sum_{o \in NB} x^{od}(t) \tag{14}$$

where $x^{od}(t)$ corresponds to the value of a single cell of the OD matrix, i.e. volume of traffic flow in the OD pair (o, d) in the interval t.

It was assumed that the spatial scope of the analysis covers the area in which traffic flows can travel in particular OD pair in a time shorter than the analysis interval t. Moreover, due to the lack of congestion in the road network, it was assumed that for each OD pair, one path is defined in all intervals. Therefore, the overall relationship between link volumes and OD flows can be written as

$$q_{ij}(t) = \sum_{(o,d)} \alpha^{od}_{ij}(t) \cdot x^{od}(t) \tag{15}$$

where $q_{ij}(t)$ is registered traffic volumes on link $(i, j) \in L$ in period t, $\alpha^{od}_{ij}(t)$ denotes the fraction (share) of OD flows for the pair (o, d) that uses link $(i, j) \in L$ in period t, and $x^{od}(t)$ indicates the O–D flows for the pair (o, d) in period t.

The general scheme of the iterative method for estimating the OD matrix based in the volume of traffic on the sections of the road network is shown in Fig. 2. The final form of matrix (i.e., prior OD matrix) is a reference matrix for training the neural network.

After gathering data from detectors, for each interval, the initial OD matrix is built. It is denoted as $OD^0(t) \equiv \left[x^{od,0}(t)\right]$ (for $k = 0$). Due to the lack of information on the actual trip distribution, it is constructed as a proportional distribution. Then, after assigning it to the road network, the sums of the components of the traffic flow are calculated in each section of the network. Thus, the matrix $\hat{Q}^0(t) \equiv \left[\hat{q}^0_{ij}(t)\right]$ is obtained. In the next step of the method, for the sections for which traffic data from detectors are available (i.e., $(i, j) \in LD$), the convergence between the matrix $\hat{Q}^0(t)$ and the matrix of intensities recorded by the detectors, i.e., $Q(t)$, is estimated.

As a measure of convergence, the relationship based on the maximum value of the relative deviation of traffic volumes on road sections was adopted in all iterations. It is determined using the formula:

$$F\left(\left[q_{ij}(t)\right], \left[\hat{q}^k_{ij}(t)\right]\right) = 1 - \max_{(i,j)\in LD} \frac{q_{ij}(t)}{\hat{q}^k_{ij}(t)} \tag{16}$$

where $\hat{q}^k_{ij}(t)$ denotes the element of the matrix $\hat{Q}^k(t)$ with the interpretation of the intensity value on the section $(i, j) \in LD$, which is the result of the assignment of the matrix $OD^k(t) \equiv \left[x^{od,k}(t)\right]$ to the road network in the kth iteration ($k = 0, 1, \ldots$).

If there is no acceptable convergence level, the OD matrix is corrected and the next iteration proceeds. The process is repeated until the assumed number of iterations is achieved or the adopted level of convergence with the data from the detectors is obtained. Then the matrix values from the last iteration are taken as reference values to train the neural network.

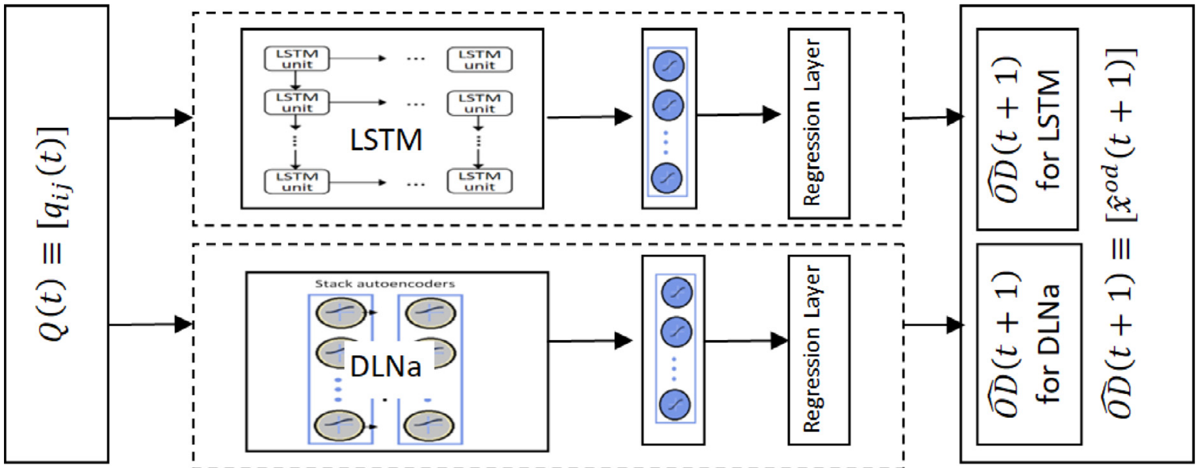


Fig. 3. Diagram of the neural network used in the OD matrix prediction model.

Table 3
Sets of nodes and links for the road network.

Elements of network	Set	Elements of the set
Nodes	NB	$\{P2, P3, P8, P10, P13, P15, P18\}$
	ND	$\{P4, P5, P6, P7, P9, P14, P19\}$
	NU	$\{X1, X2, X3\}$
Links	LD	$\{(P3, P4), (P4, P3), (P4, X1), (P2, X1), (P15, X1), (X1, P6), (P13, P14), (P14, P13), (P14, P5), (X2, P5), (P10, X2), (P19, P9), (P19, X3), (P9, X3), (P9, P18), (P18, P9), (P7, X3), (P7, P8), (P8, P7)\}$
	LU	$\{(X1, P4), (X1, P15), (X1, P2), (P5, X1), (P5, P14), (P5, P6), (P6, P5), (P6, X2), (X2, P19), (P19, X2), (X2, P10), (P9, P19), (X3, P19), (X3, P9), (X3, P7)\}$

3.4. Neural network structure

In the structure of the deep learning network, the LSTM layers or autoencoders layers the full connection (FC) layers and the regression layer (REG) were used to predict the OD matrix. LSTM layers are well suited for classifying, processing and forecasting from time series data. The LSTM layer learns the long-term relationship between time steps in a time series and sequence data. Relative insensitivity to gap length is an advantage of LSTM over RNN, hidden Markov models, and other sequence training methods in many applications. This property of the network was also demonstrated in this article by examining the impact of the forecast horizon on the accuracy of prediction. A regression layer computes the half-mean squared error loss for regression problems. For typical regression problems, a regression layer must follow the final fully connected layer.

The deep learning network diagram used in the OD matrix prediction model is shown in Fig. 3.

The network maps the function:

$$\widehat{OD}(t+1) = F(Q(t)) \tag{17}$$

During network training, traffic volume matrix sequences are provided for inputs: $Q(t)$, $Q(t-1)$, …, $Q(t-r)$, where $Q(t) \equiv [q_{ij}(t)]$, with $q_{ij}(t)$ interpreted as a traffic intensity in road section (i, j) in the interval t (15 min), $(r+1)$ — number of elements of training set.

The output data are the predicted OD matrices in the next interval, i.e., $\widehat{OD}(t+1), \widehat{OD}(t), \ldots, \widehat{OD}(t-r+1)$, where $\widehat{OD}(t+1) = [\hat{x}^{od}(t+1)]$, with $\hat{x}^{od}(t+1)$ interpreted as an element of the OD matrix in the interval $(t+1)$. Traffic flow sequences on the input of the network are time series. Information on the tested network configurations and the best configuration is described in Section 4.2.

4. Case study

4.1. Dataset

Research data were obtained from the Traffic Control Center in the medium-sized city of Poland, Gliwice. The analysis area and the traffic flow measurement points is shown in Fig. 4a. The border points (i.e. included in the NB set) are marked in red, and the places inside the area with the traffic intensities registered (i.e. included in the ND set) are marked in green.

The fixed shortest paths between the selected points are assumed. Paths were determined using the Dijkstra algorithm. On this basis, a schematic structure of the road network shown in Fig. 4b has been constructed. Sections with traffic flow data available (i.e. included in the LD set) are coloured green. The nodes of the network where the traffic volume are not recorded are marked in blue.

Table 3 presents the sets of nodes and links for the road network model.

The research was carried out using data from video-sensing detectors for a period of three months (May, June, July). Data were collected at 5-minute intervals, but for the study they were processed in 15 min.

Based on the selected locations of the measurement points and traffic data recorded by the video detectors, the OD flows were estimated and path matrices were determined. These matrices for the points shown in Fig. 4 had a dimension of 34 × 42 (i.e., 34 links and 42 OD pairs).

In the first stage of the research, several path matrices were determined, and the corresponding OD matrices were estimated by the iterative method. The number of iterations was chosen to obtain the best convergence with the measurement data.

After determining the number of iterations, the matrices covering OD flows between points corresponding to the border nodes for each interval were generated using an author's program (in C ++). The

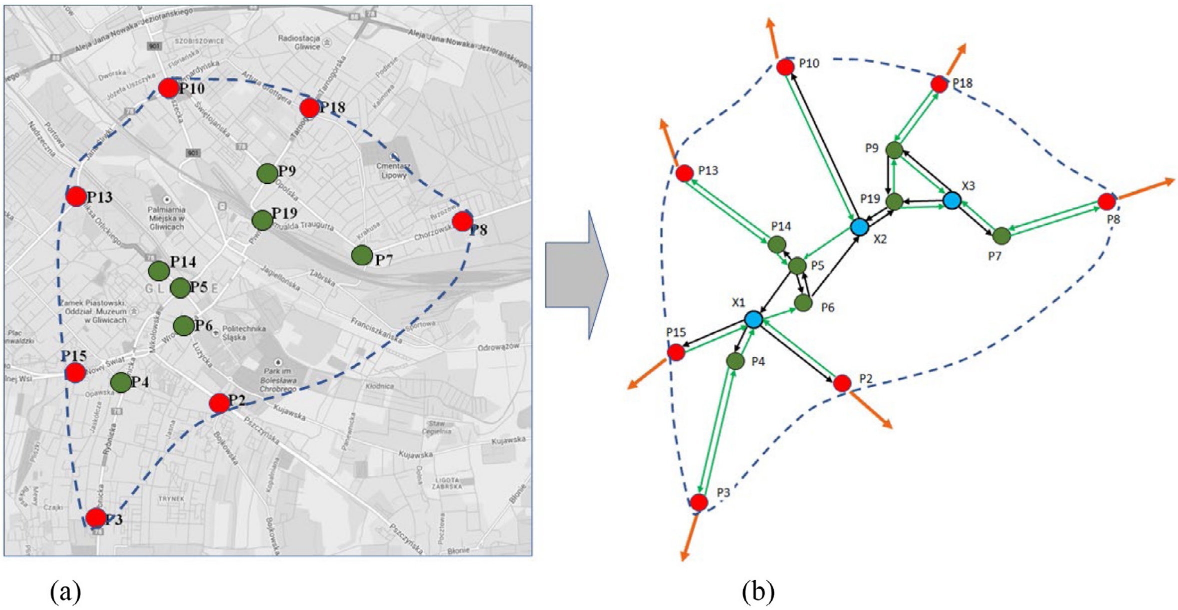


Fig. 4. Map, connections, location of detectors.

Table 4
Traffic intensities on 16.05.2016 (Monday) at interval 07:45 a.m.–08:00 a.m. [veh/15 min].

P2 in	P2 out	P3 in	P3 out	P4 in	P4 out	P5 out	P6 in	P7 in	P7 out	P8 in	P8 out	P9 in	P9 out
257	233	199	115	148	108	379	350	164	191	157	138	132	100
P10 in	P10 out	P13 in	P13 out	P14 in	P15 in	P15 out	P18 in	P18 out	P19 in	P19 out	P13 in	P13 out	P5 in
101	126	57	128	141	211	125	154	104	99	77	182	135	142

Table 5
OD matrix for 16.05.2016 (Monday) at interval 07:45 a.m.–08:00 a.m.

16.05.2016 08:00:00 Mon							
	P2	P3	P15	P13	P10	P18	P8
P2	–	0.183	0.275	0.194	0.217	0.181	0.140
P3	0.201	–	0.167	0.159	0.167	0.150	0.125
P15	0.257	0.150	–	0.170	0.187	0.157	0.127
P13	0.200	0.144	0.163	–	0.166	0.147	0.125
P10	0.214	0.132	0.156	0.148	–	0.083	0.083
P18	0.153	0.121	0.123	0.125	0.097	–	0.438
P8	0.208	0.160	0.166	0.169	0.147	0.127	–

three-month period from May to July was considered. After subtracting the days with missing data for the entire day and public holidays, 30 working days were used in the study, of which 25 days of data was the neural network basis for the training sequence, and the remaining 5 days from Monday to Friday were used for testing. The test data were not elements of the training set.

An interval of 15 min was taken into account when generating the OD matrix. A total of 2400 prior OD matrices were estimated for the training sequence and 480 (i.e., $5 * 96$) for the test sequence. The OD matrices were estimated for each of the five working days separately.

An exemplary input and output data (i.e., training data) of the neural network for the selected interval are presented in Tables 4 and 5. The values of the reference OD matrix are expressed in relative units.

Traffic data and OD flows are time series. Fig. 5a and b show traffic distributions in exemplary measurement points (i.e., P3 and P10) in both directions (noted as "in" and "out") for four Mondays, May 9, May 16, May 23, and June 6, 2016.

The graphs of traffic flows in Fig. 5 show that at the entry to point P3 the highest traffic intensities is in the morning rush hours (6:00 a.m.–8:00 a.m.) and at the exit — during the afternoon rush hours (3:00 p.m.–4:00 p.m.). On the other hand, at the entrance to point P10, the traffic volumes are more than twice as high as at the exit, which may indicate that more than half of the vehicles entering the city at point P10 leave it elsewhere or stay in the city.

Fig. 6a and b show the traffic distribution for all OD pairs starting from point P3 and from point P10, respectively. The highest values of traffic volumes for the point P3 occur for the pairs $(P3, P2)$ and $(P3, P15)$ and for the point 10 for the pairs $(P10, P15)$ and $(P10, P2)$. Analysis of the traffic distributions in intervals during the day for the OD pairs (Fig. 6a, b) may be useful in traffic control as well as in planning new or modernizing the existing road network infrastructure.

4.2. Neural network configuration

Fig. 3 shows the structure of a deep learning network for the prediction of the OD matrix. The network consists of an input layer, an LSTM or autoencoders layer, an FC layer, and a REG layer as the output layer. The traffic flow corresponding to successive intervals was given at the network input. The number of entries was 28 values of the traffic flows on links registered in the same interval.

The networks with one and two LSTM layers were examined. There were 100 to 200 units in each of them. The number of neurons in the FC layer was equal to the number of output data. In our case, it was 42 neurons for the OD matrix with dimension of 7×7 and disregarding the values on the main diagonal corresponding to intra-zonal flows.

For sequence-to-sequence regression networks, the loss function of the regression layers is the half-mean squared error of the predicted responses for each time step.

The best results were obtained for the following configuration: LSTM layer with 200 hidden units, followed by a fully connected layer of size 50 and a dropout layer with dropout probability 0.2.

The training options: 200 epochs with mini-batches of size 10 using the solver of stochastic gradient descent with momentum (SGDM) optimizer. Specify the learning rate as 0.005. To prevent the gradients from exploding, set the gradient threshold to 1.

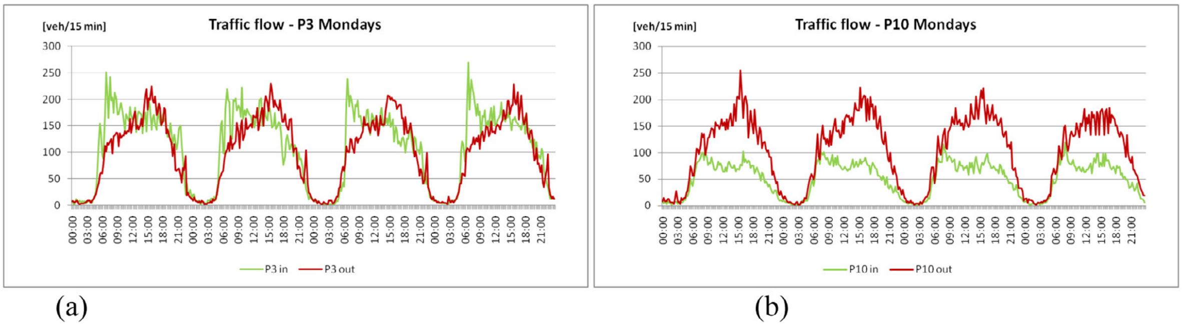


Fig. 5. Traffic distributions at the entry and exit of measuring points: (a) P3, (b) P10.

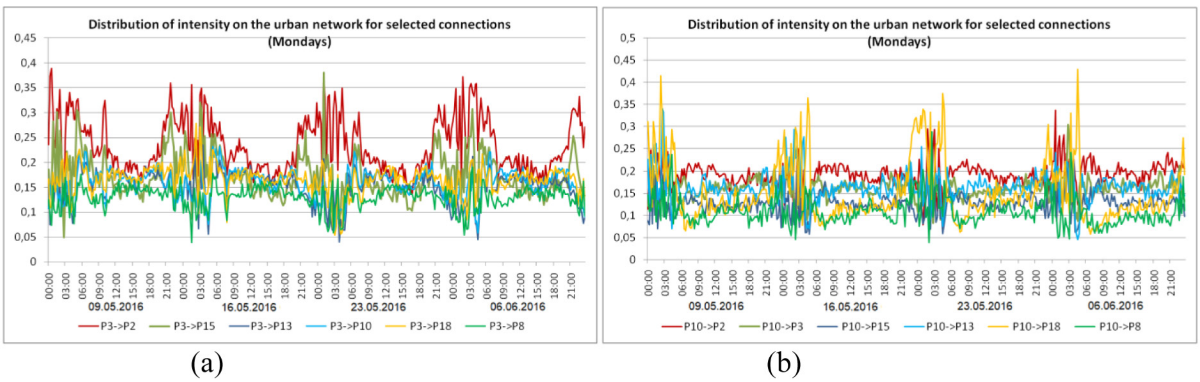


Fig. 6. Traffic distribution in intervals during the day for selected OD pairs, (a) starting from point P3, (b) starting from point P10.

The best network containing autoencoders consisted of a stack of two layers of autoencoders with 20 neurons in each layer. The number of inputs and outputs and the remaining network layers were the same as in the network with LSTM layers — Fig. 3. The maximum number of epochs for the final configuration was 100.

The MATLAB 2020a (Academic License) software and a laptop with an Intel (R) Core (TM) i5, 1.19 GHz processor, 8 GB RAM and 1TB SDD disk were used for network training. The training time depended mainly on the number of epochs, the number of layers, and LSTM memory units, as well as the mini-batch value and ranged from 5 min to 4 h.

After training the network, the input was the traffic flow in a determined interval, and the output were the forecasted OD matrix in the next interval.

5. Results and discussion

The developed model tested the data without elements of the training set. These data included traffic volumes in the points located in the urban road network. The working weeks, two in May, two in June, and one in July were selected for the tests. These data can be considered representative for the entire data set.

For the assessment of the results obtained, the MAPE and RMSE errors were calculated, which are the most often used to evaluate the prediction results. Prediction errors were calculated for the OD matrix:

- for working days from Monday to Friday and for different time horizons,
- for individual intervals,
- for the OD flows,

The obtained results were compared with the results obtained directly from the historical analysis.

5.1. Prediction results for working days and different time horizons

This analysis was to check for which day of the week the highest prediction errors were obtained. First, $MAPE^{wd}(t)$ and $RMSE^{wd}(t)$ were calculated for the OD matrix for each interval for a given working day of the week according to the formulas:

$$MAPE^{wd}(t) = \frac{1}{n_{od}} \sum_{\substack{o=1 \\ o\neq d}}^{n} \sum_{\substack{d=1 \\ d\neq o}}^{n} \left| \frac{x^{wd,od}(t) - \hat{x}^{wd,od}(t)}{x^{wd,od}(t)} \right| * 100\% \tag{18}$$

$$RMSE^{wd}(t) = \sqrt{\frac{1}{n_{od}} \sum_{\substack{o=1 \\ o\neq d}}^{n} \sum_{\substack{d=1 \\ d\neq o}}^{n} \left(x^{wd,od}(t) - \hat{x}^{wd,od}(t)\right)^2} \tag{19}$$

where $MAPE^{wd}(t)$ denotes the average of relative errors for all OD flows for a given interval t in working day wd and $RMSE^{wd}(t)$ is the standard deviation of the residuals (prediction errors) for all OD flows for a given interval t in working day wd. Real and predicted elements of OD matrix for a given interval t in working day wd are marked as $x^{wd,od}(t)$ and $\hat{x}^{wd,od}(t)$, respectively, and n_{od} is the number of elements of OD matrix $(7 \times 7 - 7)$.

Then the mean errors were calculated for the individual intervals in period 6:00 a.m.–10:00 p.m. in individual working day as:

$$MAPE^{wd} = \frac{1}{m} \sum_{t=1}^{m} MAPE^{wd}(t) \tag{20}$$

$$RMSE^{wd} = \frac{1}{m} \sum_{t=1}^{m} RMSE^{wd}(t) \tag{21}$$

where m is the number of all intervals in the period under study, i.e., $m = 65$, (for the whole day the number of intervals of 15 min is 96).

As a result of the research carried out with the use of the developed deep learning model, predicted OD matrices with an average error ranging from 6.97–7.31% for a time horizon of 15 min were obtained. For horizons of 30 and 45 min, the errors were slightly larger. Their average values for individual days of the week are presented in Table 6.

In the case of the model using autoencoders, the average error for working days ranged from 6.54% to 7.18%. As for the LSTM network, for the 30- and 45-min horizons the error was greater — Table 7.

Table 6
MAPE and RMSE prediction error values for working days at 6:00 a.m.–10:00 p.m.

Prediction horizon	15 min	15 min	30 min	30 min	45 min	45 min
	MAPE [%]	RMSE	MAPE [%]	RMSE	MAPE [%]	RMSE
Monday	6.966	0.0143	6.839	0.0145	7.228	0.0151
Tuesday	7.060	0.0145	7.369	0.0151	7.502	0.0154
Wednesday	7.128	0.0146	7.386	0.0152	7.416	0.0154
Thursday	7.426	0.0156	7.455	0.0156	7.695	0.0160
Friday	7.312	0.0153	7.542	0.0158	7.691	0.0161
Average	**7.178**	**0.0149**	**7.318**	**0.0150**	**7.506**	**0.0160**

Table 7
MAPE and RMSE prediction error values for working days at 6:00 a.m.–10:00 p.m.

Prediction horizon	15 min	15 min	30 min	30 min	45 min	45 min
	MAPE [%]	RMSE	MAPE [%]	RMSE	MAPE [%]	RMSE
Monday	6.54	0.0143	6.60	0.0146	6.70	0.0145
Tuesday	6.64	0.0137	6.73	0.0138	7.06	0.0146
Wednesday	6.76	0.0138	7.07	0.0146	7.18	0.0148
Thursday	6.86	0.0143	6.85	0.0143	7.32	0.0154
Friday	7.18	0.0151	7.30	0.0155	7.60	0.0159
Average	**6.80**	**0.0143**	**6.91**	**0.0144**	**7.23**	**0.0150**

Tables 6 and 7 shows the mean values of the MAPE and RMSE for the OD matrices for 5 days of the week. The OD matrices were predicted with a time horizon of 15, 30, and 45 min between 6:00 a.m.–10:00 p.m. and were calculated with a step of 15 min. For example, for a horizon of 45 min at 8:00 a.m., the OD matrix was forecasted for 8:45 a.m., and at 8:15 a.m., the OD matrix was forecasted for 9:00 a.m., etc. In total, 65 OD matrices were taken into account for each day.

5.2. Prediction errors for individual intervals

Tables 6 and 7 show the mean MAPE errors for each day of the week. Mean errors were computed from the MAPE errors for each time interval. Fig. 7a and b show the $MAPE^{wd}(t)$ errors for Monday and Thursday for the predicted OD matrices at 6:00 a.m.–10:00 p.m. (22:00) for the horizon of 15 min.

Fig. 7 show the $MAPE^{wd}(t)$ errors for all workdays for the predicted OD matrices using LSTM model (Fig. 7a) and DLNa model (Fig. 7b) at 6:00 a.m.–10:00 p.m. (22:00) for the horizon of 15 min.

In the graphs presented in Fig. 7a, it can be seen that large errors occur in the evening hours from 8:00 p.m.–10:00 p.m. This is related to a decrease in traffic during these hours, which increases the error value. At low traffic intensities, the difference of even one vehicle gives large relative errors. The MAPE errors in subsequent time intervals for the model with the LSTM network are clearly higher than for the model with autoencoder layers. In the paper (Dapeng Zhang et al., 2021), the value of the MAPE prediction error for the DNEAT method was 6.96%, but the input data set contained many parameters that are often difficult to access. Our average scores of 7.18% for deep learning network LSTM and 6,80% for network DLNa were for more accessible traffic data. Our model is much simpler and faster and requires much less computing power. Referring to Table 6 of this paper, we can conclude that our OD matrix prediction error is much smaller than the errors obtained using other methods.

5.3. Errors for the OD flows

Relative error values for the OD flows for Monday, June 20, 2016, are shown in Fig. 8. The matrices in upper part of the figure contain the relative errors calculated as follows:

$$E^{wd,od}(t) = \left| \frac{x^{wd,od}(t) - \hat{x}^{wd,od}(t)}{x^{wd,od}(t)} \right| * 100\% \tag{22}$$

where $x^{wd,od}(t)$ and $\hat{x}^{wd,od}(t)$ are real and predicted element of OD matrix for a given interval t in working day wd, respectively.

Table 8
Descriptive statistics for errors $MAPE^{wd,od}$ [%] for Monday, June 20, 2016, at 6:00 a.m.–10:00 p.m.

Statistics	LSTM model	DLNa model
Average value	6.97	6.54
Standard deviation	1.76	1.71
Coefficient of variation	25.21	26.23
Minimum value	4.57	4.14
Maximum value	11.68	11.90
Median	6.48	6.26

Table cells filled with yellow in Fig. 8 show relative error, expressed in [%], for the exemplary pair $(P2, P3)$ at 6:00 a.m., 6:15 a.m. and 10:00 p.m.

From the data in the tables in Fig. 8, the mean values were calculated for individual working day according to the formula:

$$MAPE^{wd,od} = \frac{1}{m} \sum_{t=1}^{m} E^{wd,od}(t) \tag{23}$$

where m — number of 15-min intervals for the period from 6:00 a.m. to 10:00 p.m. (22:00), m = 65.

Fig. 8 shows that for twelve OD pairs very good results, less than 6%, have been obtained. They constitute approximately 28.6% of all OD pairs under study. Only for four cells of OD matrix the errors exceed 10%. For the model with autoencoders (DLNa) the results are better. In the heatmap diagram on the right, it can be seen that only for two OD pairs the MAPE error exceeded 10%.

The basic descriptive statistics for errors $MAPE^{wd,od}$ for the LSTM model and the model with the autoencoder layer are presented in Table 8.

The average error values for the selected day range from 4.57 to 11.68 for the LSTM model and from 4.14 to 11.90 for the model with autoencoders. Therefore, the dispersion of the values of 〚 $MAPE^{wd,od}$ errors for both models is similar. Moreover, for the LSTM model over 66% of the error values are less than 7%, and for the DLNa model it is over 76%, which also proves better results for the model with an autoencoder layer. The values of the coefficient of variation range from 25.21% for the LSTM model to 26.23% for the DLNa model, which confirms the significance of the obtained results.

The average MAPE error for all OD pairs (for all working days) is 7.18% for the LSTM model and 6.80% for the DLNa model. With reference to data from the literature, for example (Dapeng Zhang et al., 2021), it can be said that these are very good results.

5.4. Comparison of results with a method based directly on historical data

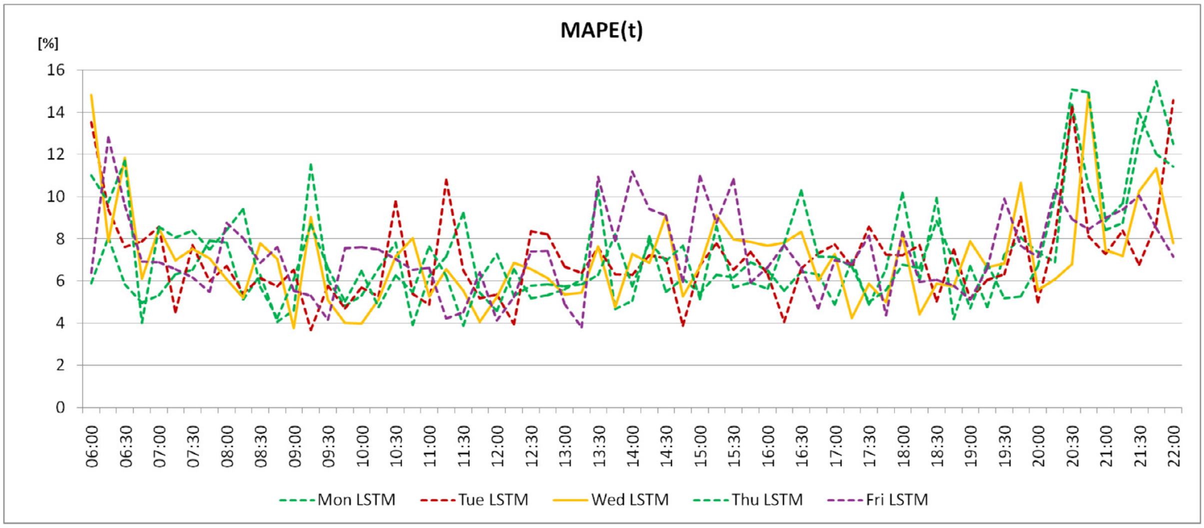
The results obtained from the prediction models proposed by various authors depend on a large extent on the available data set used for training and verification of these models. Taking into account the data from the literature, our results can only be roughly compared. Therefore, we propose to compare our results with another method to predict the OD matrix using the same data set.

In order to compare the prediction results, future OD matrices were calculated using the method based on the historical data. Since none of the classical methods allows for the computation of the OD matrix directly from the traffic data, the future OD matrix was determined in two stages. The first stage consisted of predicting the traffic volume for each individual road section $(i, j) \in LD$ for individual working day according to the following formula:

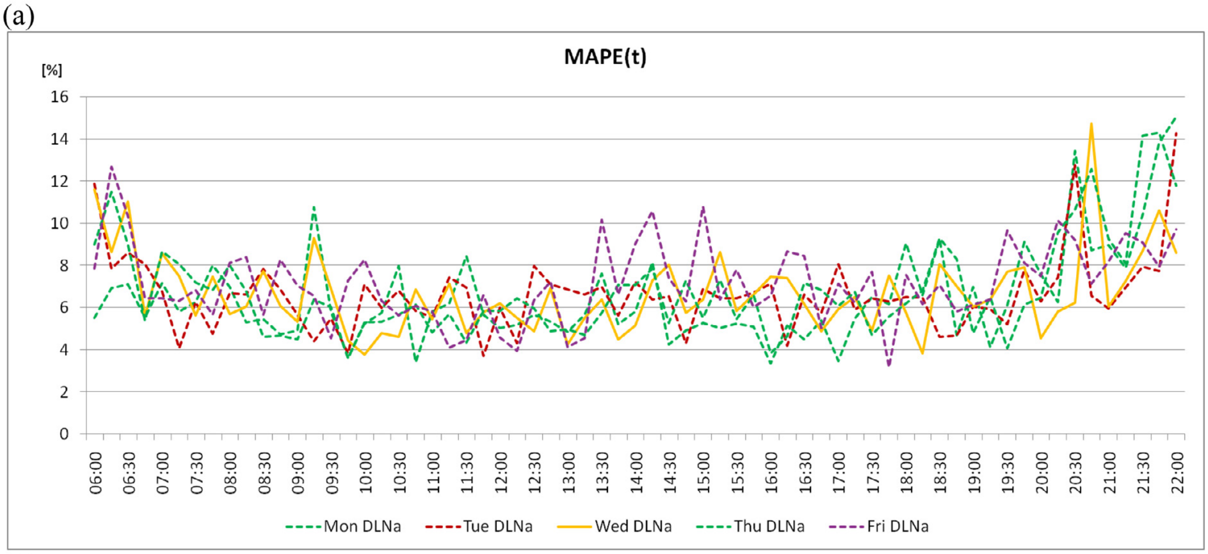
$$\hat{q}_{ij}^{wd}(t+1) = 0.5 q_{ij}^{wd,\text{avg}}(t+1) + 0.5 q_{ij}^{wd}(t) \tag{24}$$

where $q_{ij}^{wd,\text{avg}}(t+1)$ is the average of historical data for the same working day. The value is calculated as:

$$q_{ij}^{wd,\text{avg}}(t+1) = \frac{1}{3}\left(q_{ij}^{wd1}(t+1) + q_{ij}^{wd2}(t+1) + q_{ij}^{wd3}(t+1)\right) \tag{25}$$



(a)



(b)

Fig. 7. Distributions of the MAPE (t) errors in the predicted intervals during the analysis period; (a) for LSTM model, (b) for DLNa model.

where $wd1$, $wd2$ and $wd3$ refer to the same working day (i.e., wd) being one week, two weeks and three weeks earlier, respectively. In this way, the prediction takes into account the historical values in a wider range, which minimizes the impact of random errors.

The values of the predicted intensities for all sections of the road network can be summarized in the form of a matrix as:

$$\hat{Q}^{wd}(t+1) \equiv \left[\hat{q}_{ij}^{wd}(t+1)\right] \tag{26}$$

where $\hat{Q}^{wd}(t+1)$ is the matrix of predicting OD flows in the successive interval $(t+1)$, which can be 15, 30, and 45 min long for working day wd.

In the second stage, the iterative method using of the author's program estimated OD flows for each interval in period 6:00 a.m.–10:00 p.m. MAPE and RMSE errors were calculated according to formulas (20) and (21).

The results of the prediction made this way are shown in Table 9.

The biggest errors were obtained for Friday. The reason for such an error is the large divergence of traffic volumes on these days. The use of the deep learning network in this case gave much more accurate forecasts than the historical method.

Fig. 9 summarizes the MAPE (t) error values for the OD matrix for the 15-min forecasts made for the test working days using the DLNa and LSTM method and the historical method in the period from 6:00 a.m. to 10:00 p.m. (22:00).

Table 9
OD matrix prediction errors using a method based on historical data.

Prediction horizon	15 min	15 min	30 min	30 min	45 min	45 min
	MAPE [%]	RMSE	MAPE [%]	RMSE	MAPE [%]	RMSE
Monday	8.029	0.1696	8.156	0.0172	8.386	0.0177
Tuesday	7.321	0.0153	7.586	0.0158	7.870	0.0168
Wednesday	7.721	0.0165	7.969	0.0169	8.210	0.0174
Thursday	7.391	0.0155	7.639	0.0159	7.826	0.0162
Friday	9.076	0.0192	9.029	0.0190	9.167	0.0192
Average	**7.908**	**0.0470**	**8.076**	**0.0170**	**8.292**	**0.0170**

For most of the intervals in Fig. 9, the historical forecast is less accurate than the forecast using the deep learning network. This proves the good accuracy of the proposed OD matrix prediction model. The advantage of the proposed method is most noticeable on Friday. When comparing the results in Tables 6, 7 and 9, the mean MAPE error for that day is 1.764% (i.e., 9.076%–7.312%) lower in absolute terms for the 15-min interval.

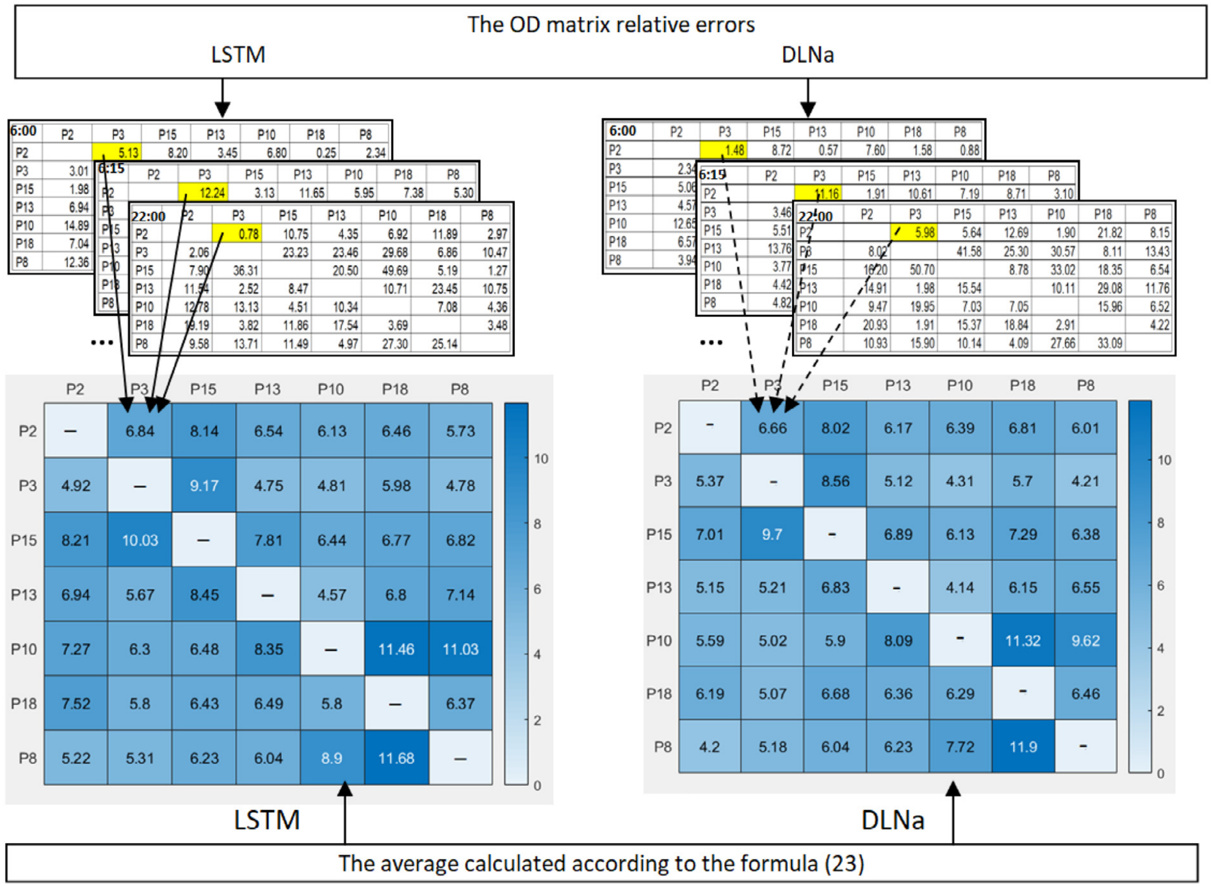


Fig. 8. The method of determining the value of $MAPE^{wd,od}$ error values for OD matrix from 6.00 a.m. to 10:00 p.m. (22:00) for the proposed deep learning models.

6. Conclusions

The article presents a new model of OD matrix prediction using deep learning based on traffic volumes at the entry and exit points of the city centre. When developing the model, the spatial distribution of the origins and destinations of the trips, as the well as data on the structure of the road network, was taken into account. The proposed method of predicting the OD matrix does not require household survey or detailed sociological and demographic information. Therefore, it is not as expensive and time consuming as the methods based on these data.

The developed model gives good results compared to others (Dapeng Zhang et al., 2021). A mean MAPE prediction error of 7.18% was achieved for LSTM networks and 6.80% for DLNa networks. The network with the use of autoencoder layers turned out to be slightly better. Comparative analysis of the results with the historical method showed that the historical method is not as flexible as the deep learning network method, which gives much better results when data are irregular. Also, the proposed method enables the prediction of the OD matrix in the case of missing data, which is often a big problem.

Although the accuracy of our model has been confirmed for traffic data in a specific city, it may also apply to other cities in other countries. The ability to work in real time allows the model to be used in urban traffic control systems. The developed model can also be useful in planning the development of urban transport and its infrastructure.

Future work will involve the use of other data, such as vehicle license plate data, in the prediction of the OD matrix. The video detectors located in the city of Gliwice are currently equipped with the ANPR system, so we will be able to use the data on vehicle registration plates. Other data, such as demographics or household survey data, can also be optionally included.

CRediT authorship contribution statement

Teresa Pamuła: Conceptualization, Software, Validation, Formal analysis, Investigation, Resources, Writing – original draft, Visualization, Supervision, Project administration. **Renata Żochowska:** Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Supervision, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

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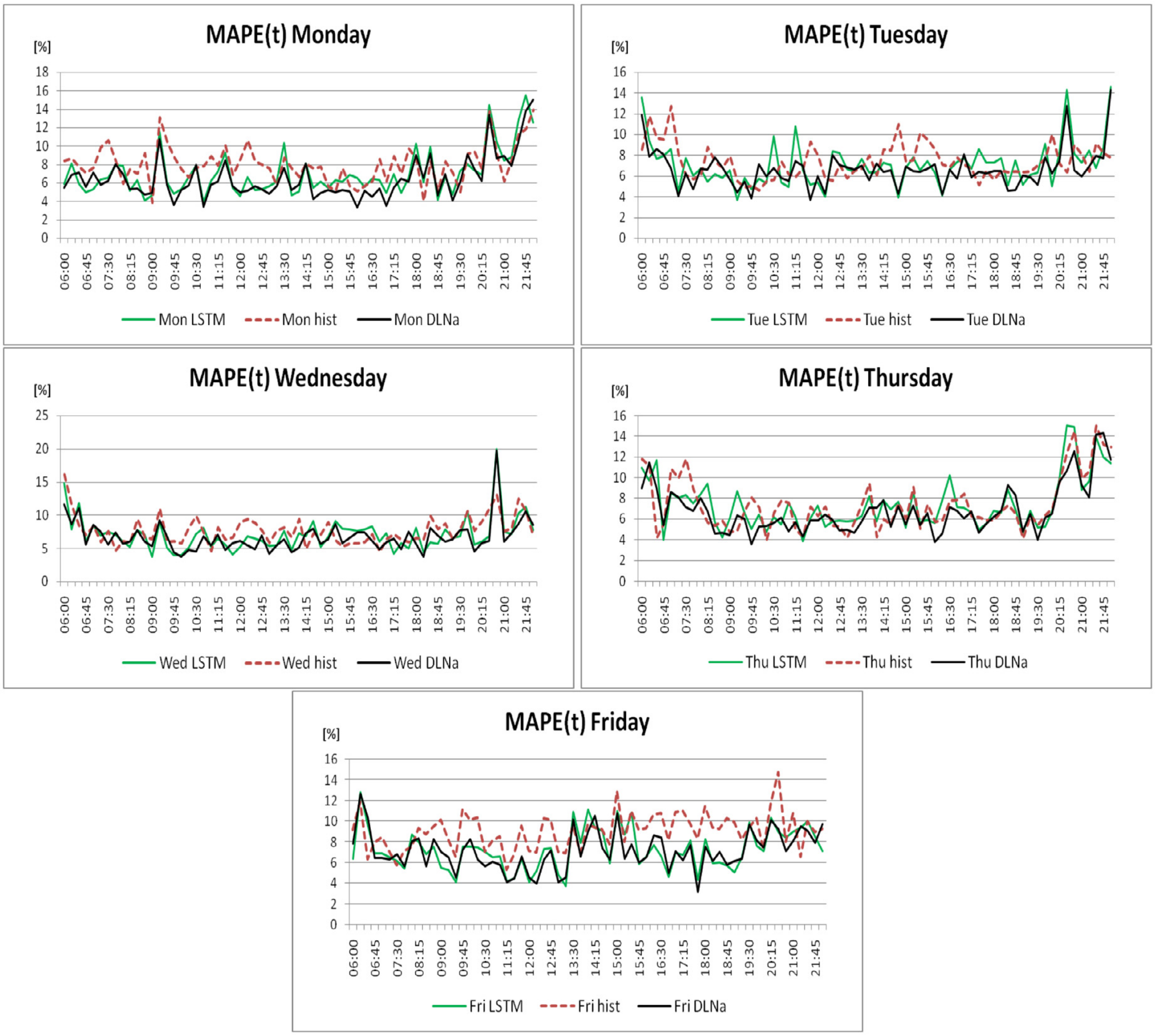

Fig. 9. Distributions of MAPE (t) error in intervals during the analysis period for LSTM and DLNa network forecasts and the historical method for the test days.

References

Alexander, L., Jiang, S., Murga, M., González, M.C., 2015. Origin–destination trips by purpose and time of day inferred from mobile phone data. Transp. Res. C 58 (Part B), 240–250. http://dx.doi.org/10.1016/j.trc.2015.02.018.

Antoniou, C., Ben-Akiva, M., Koutsopoulos, H.N., 2006. Dynamic traffic demand prediction using conventional and emerging data sources. IEEE Proc. - Intell. Transp. Syst. 153 (1), 97–104. http://dx.doi.org/10.1049/ip-its:20055006.

Ashok, K., 1996. Estimation and Prediction of Time-Dependent Origin-Destination Flows. (Ph.D. Thesis). MIT.

Ashok, K., Ben-Akiva, M., 1993. Dynamic Origin–Destination Matrix Estimation and Prediction for Real-Time Traffic Management Systems. Transportation and Traffic Theory, Proceedings of the 12th ISTTT. Elsevier, Amsterdam.

Ashok, K., Ben-Akiva, M., 2000. Alternative approaches for real-time estimation and prediction of time-dependent origin–destination flows. Transp. Sci. 34, 21–36. http://dx.doi.org/10.1287/trsc.34.1.21.12282.

Ashok, K., Ben-Akiva, M., 2002. Estimation and prediction of time-dependent origin–destination flows with a stochastic mapping to path flows and link flows. Transp. Sci. 36, 184–198. http://dx.doi.org/10.1287/trsc.36.2.184.563.

Barcelo, J., Montero, L., Bullejos, M., Serch, O., Carmona, C., 2013. A Kalman filter approach for exploiting bluetooth traffic data when estimating time-dependent OD matrices. J. Intell. Transport. Syst.: Technol. Plann. Oper. 17, 123–141. http://dx.doi.org/10.1080/15472450.2013.764793.

Barcelo, J., Montero, L., Marques, L., Carmona, C., 2010. Travel time forecasting and dynamic origin–destination estimation for freeways based on bluetooth traffic monitoring. Transport. Res. Rec.: J. Transport. Res. Board 2175, 19–27. http://dx.doi.org/10.3141/2175-03.

Bell, M.G.H., Shield, C., 1995. A log-linear model for path flow estimation. In: Proceedings of 4th International Conference on the Applications of Advanced Technologies in Transportation Engineering, Capri 1995. Italy, pp. 695–699.

Brzeziński, A., Dybicz, T., 2021. Possibility of big data application for OD-matrix callibration in transport demand models. Arch. Civ. Eng. Polish Acad. Sci. 67 (1), 215–232. http://dx.doi.org/10.24425/ace.2021.136470.

Caggiani, L., Dell'Orco, M., Marinelli, M., Ottomanelli, M., 2012. A metaheuristic dynamic traffic assignment model for O-D matrix estimation using aggregate data. In: EWGT 2012-15th Meeting of the EURO Working Group on Transportation. In: Procedia - Social and Behavioral Sciences, vol. 54, pp. 685–695. http://dx.doi.org/10.1016/j.sbspro.2012.09.786.

Cascetta, E., Inaudi, D., Marquis, G., 1993. Dynamic estimators of origin–destination matrices using traffic counts. Transp. Sci. 27, 363–373. http://dx.doi.org/10.1287/trsc.27.4.363.

Ceccato, R., Gecchele, G., Rossi, R., Gastaldi, M., 2022. Cost-effectiveness analysis of origin-destination matrices estimation using floating car data. Experimental results from two real cases. Transp. Res. Procedia 62, 541–548. http://dx.doi.org/10.1016/j.trpro.2022.02.067.

Chen, A., Chootinan, P., Recker, W., 2005. Examining the quality of synthetic origin–destination trip table estimated by path flow estimator. J. Transp. Eng. 131 (7), 506–513. http://dx.doi.org/10.1061/(ASCE)0733-947X(2005)131:7(506).

Chen, A., Chootinan, P., Recker, W., Zhang, H.M., 2004. Development of a Path Flow Estimator for Deriving Steady-State and Time-Dependent Origin-Destination Trip Tables. California PATH Research Report UCB-ITS-PRR-2004-29.

Cheng, S., Zhu, Z., Chu, J., Cheng, 2014. A bayesian network model for origin–destination matrices estimation using prior and some observed link flows. Discrete Dyn. Nat. Soc. http://dx.doi.org/10.1155/2014/192470.

Cipriani, E., Nigro, M., Fusco, G., Colombaroni, C., 2014. Effectiveness of link and path information on simultaneous adjustment of dynamic O-D demand matrix. Eur. Transp. Res. Rev. 6, 139–148. http://dx.doi.org/10.1007/s12544-013-0115-z.

Dapeng Zhang, Xiao, Feng, Shen, Minyu, Zhong, Shaopeng, 2021. DNEAT: A novel dynamic node-edge attention network for origin–destination demand prediction. Transp. Res. C 122, 102851. http://dx.doi.org/10.1016/j.trc.2020.102851.

Djukic, T., Flotterod, G., Van Lint, H., Hoogendoorn, S., 2021. Efficient real time od matrix estimation based on principal component analysis. In: IEEE Conf. Intell. Transport. Syst., Proc., ITSC (2012). pp. 115–121. http://dx.doi.org/10.1109/ITSC.2012.6338720.

Djukic, T., van Lint, J.W.C., Hoogendoorn, S.P., 2012. Application of principal component analysis to predict dynamic origin–destination matrices. Transp. Res. Rec. 81–89, http://dx.doi.org/10.3141/2283-09.

Fisk, C.S., 1988. On combining maximum entropy trip matrix estimation with user-optimal assignmen. Transp. Res. 22B, 69–73. http://dx.doi.org/10.1016/0191-2615(88)90035-5.

Ge, Q., Fukuda, D., 2016. Updating origin–destination matrices with aggregated data of GPS traces. Transport. Res. C 69, 291–312. http://dx.doi.org/10.1016/j.trc.2016.06.002.

Hazelton, M.L., 2000. Estimation of origin–destination matrices from link flows on uncongested networks. Transp. Res. B 34, 549–566. http://dx.doi.org/10.1016/S0191-2615(99)00037-5.

He, Y., Zhao, Y., Tsui, K.L., 2022. Short-term forecasting of origin–destination matrix in transit system via a deep learning approach. Transportmetrica A: Transp. Sci. 1–28. http://dx.doi.org/10.1080/23249935.2022.2033348, Ahead-Of-Print.

Hualan, Wang, Jiaying, Xu, Zeyu, Wang, 2020. An estimation method of OD matrix in non-congestion ring expressway. Transp. Res. Procedia 48, 883–889. http://dx.doi.org/10.1016/j.trpro.2020.08.103.

Hussain, E., Bhaskar, A., Chung, E., 2021. Transit OD matrix estimation using smartcard data: Recent developments and future research challenges. Transp. Res. C 125, 103044. http://dx.doi.org/10.1016/j.trc.2021.103044.

Jing Liu, Zheng, Fangfang, van Zuylen, Henk J., Li, Jie, 2020. A dynamic OD prediction approach for urban networks based on automatic number plate recognition data. Transp. Res. Procedia 47, 601–608. http://dx.doi.org/10.1016/j.trpro.2020.03.137.

Jinlei Zhang, Che, Hongshu, Chen, Feng, Ma, Wei, He, Zhengbing, 2021. Short-term origin–destination demand prediction in urban rail transit systems: A channel-wise attentive split-convolutional neural network method. Transp. Res. C 124, 102928. http://dx.doi.org/10.1016/j.trc.2020.102928.

Jintao Ke, Qin, Xiaoran, Yang, Hai, Zheng, Zhengfei, Zhu, Zheng, Ye, Jieping, 2021. Predicting origin–destination ride-sourcing demand with a spatio-temporal encoder–decoder residual multi-graph convolutional network. Transp. Res. C 122, 102858. http://dx.doi.org/10.1016/j.trc.2020.102858.

Junyi Li, Guo, Fangce, Sivakumar, Aruna, Dong, Yanjie, Krishnan, Rajesh, 2021. Transferability improvement in short-term traffic prediction using stacked LSTM network. Transp. Res. C 124, 102977. http://dx.doi.org/10.1016/j.trc.2021.102977.

Kim, H., Baek, S., Lim, Y., 2001. Origin-Destination Matrices Estimated with a Genetic Algorithm from Link Traffic Counts. Transportation Research Record 1771, TRB, National Research Council, Washington D.C, http://dx.doi.org/10.3141/1771-20.

Krishnakumari, P., van Lint, H., Djukic, T., Cats, O., 2020. A data driven method for OD matrix estimation. Transp. Res. C 113, 38–56. http://dx.doi.org/10.1016/j.trc.2019.05.014.

Liu, Z., Fu, X., 2021. Dynamic origin-destination flow prediction using spatial-temporal graph convolution network with mobile phone data. IEEE Intell. Transp. Syst. Mag. http://dx.doi.org/10.1109/MITS.2021.3082397.

Lu, Z., Rao, W., Wu, Y.J., Guo, L., Xia, J., 2015. A Kalman filter approach to dynamic od flow estimation for urban road networks using multi-sensor data. J. Adv. Transp. 49, 210–227. http://dx.doi.org/10.1002/atr.1292.

Maher, M., Zhang, X., Van Vliet, D., 2001. A bi-level programming approach for trip matrix estimation and traffic control problems with stochastic user equilibrium link flows. Transp. Res. 35B, 23–40. http://dx.doi.org/10.1016/S0191-2615(00)00017-5.

Nigro, M., Cipriani, E., del Giudice, A., 2018. Exploiting floating car data for time-dependent origin–destination matrices estimation. J. Intell. Transport. Syst.: Technol. Plan. Oper. 22, 159–174. http://dx.doi.org/10.1080/15472450.2017.1421462.

Okutani, I., 1987. The Kalman filtering approaches in some transportation and traffic problems. In: N.H. Gartner, N.H.M. Wilson (Ed.), Proceedings of the Tenth International Symposium on Transportation and Traffic Theory. Elsevier, New-York, pp. 397–416, July 8-10 1987.

Pamula, T., 2018. Road traffic conditions classification based on multilevel filtering of image content using convolutional neural networks. IEEE Intell. Transp. Syst. Mag. 10 (3), 11–21. http://dx.doi.org/10.1109/MITS.2018.2842040.

Pamuła, T., 2019. Impact of data loss for prediction of traffic flow on an urban road using neural networks. IEEE Trans. Intell. Transp. Syst. 20 (3), 1000–1009. http://dx.doi.org/10.1109/TITS.2018.2836141.

Ros-Roca, X., Montero, L., Barceló, J., Nökel, K., Gentile, G., 2022. A practical approach to assignment-free dynamic origin–destination matrix estimation problem. Transp. Res. C 134, 103477. http://dx.doi.org/10.1016/j.trc.2021.103477.

Sánchez-Cambronero, S., Álvarez-Bazo, F., Rivas, I., 2021. Dynamic route flow estimation in road networks using data from automatic number of plate recognition sensors. Sustainability 13, 4430. http://dx.doi.org/10.3390/su13084430.

Savrasovs, M., Pticina, I., 2017. Methodology of OD matrix estimation based on video recordings and traffic counts. In: 16thConference on Reliability and Statistics in Transportation and Communication, RelStat'2016, 19-22 2016, Riga, Latvia. In: Procedia Engineering, vol. 178, pp. 289–297. http://dx.doi.org/10.1016/j.proeng.2017.01.116.

Sherali, H.D., Narayanan, A., Sivanandan, R., 2003. Estimation of origin–destination trip-tables based on a partial set of traffic link volumes. Transp. Res. 37B, 815–836. http://dx.doi.org/10.1016/S0191-2615(02)00073-5.

Sherali, H.D., Park, T., 2001. Estimation of dynamic origin–destination trip tables for a general network. Transp. Res. 35B, 217–235. http://dx.doi.org/10.1016/S0191-2615(99)00048-X.

Sherali, H.D., Sivanandan, R., Hobeika, A.G., 1994. A linear programming approach for synthesizing origin destination (o-d) trip-tables from link traffic volumes. Transp. Res. 28B, 213–233. http://dx.doi.org/10.1016/0191-2615(94)90008-6.

Shuai, Ch, Shan, J., Bai, J., Lee, J., He, M., Ouyang, X., 2022. Relationship analysis of short-term origin–destination prediction performance and spatiotemporal characteristics in urban rail transit. Transp. Res. 164A, 206–223. http://dx.doi.org/10.1016/j.tra.2022.08.006.

Spiess, H., 1990. A Descent Based Approach for the OD Matrix Adjustment Problem. Publication (693) at Centre de recherchesur les transports, Universite de Montreal, Canada.

Tolouei, R., Psarras, S., Prince, R., 2017. Origin–destination trip matrix development: conventional methods versus mobile phone data. Transport. Res. Proc. 39–52. http://dx.doi.org/10.1016/j.trpro.2017.07.007.

Van Zuylen, H.J., Willumsen, L.G., 1980. The most likely trip matrix estimated from traffic counts. Transp. Res. 14B, 281–293. http://dx.doi.org/10.1016/0191-2615(80)90008-9.

Vogt, S., Fourati, W., Schendzielorz, T., Friedrich, B., 2018. Estimation of origin–destination matrices by fusing detector data and floating car data. In: 21st EURO Working Group on Transportation Meeting, EWGT 2018, 17-19, (2018) Braunschweig, Germany. In: Transportation Research Procedia, vol. 37, pp. 473–480. http://dx.doi.org/10.1016/j.trpro.2018.12.216, 2019.

Wang, Tong, Hussain, Azhar, Sun, Qi, Li, Shengbo Eben, JiaHua, Cao, 2021. Congestion prediction of urban road traffic by using deep stacked LSTM network. IEEE Intell. Transp. Syst. Mag. http://dx.doi.org/10.1109/MITS.2021.3049383, Early Access FEB.

Willumsen, L.G., 1984. Estimating time-dependent trip matrices from traffic counts. In: Proceedings of the 9th International Symposium on Transportation and Traffic Theory. Delft University, The Netherlands.

Yang, X., Lu, Y., Hao, W., 2017. Origin–destination estimation using probe vehicle trajectory and link counts. J. Adv. Transport. 2017, http://dx.doi.org/10.1155/2017/4341532.

Yin, X., Wu, G., Wei, J., Shen, Y., Qi, H., Yin, B., 2021. Deep learning on traffic prediction: Methods, analysis and future directions. IEEE Trans. Intell. Transp. Syst. http://dx.doi.org/10.1109/TITS.2021.3054840.

Yuan Wang, Zhang, Dongxiang, Liu, Ying, Dai, Bo, Lee, Loo Hay, 2019. Enhancing transportation systems via deep learning: A survey. Transp. Res. C 99, 144–163. http://dx.doi.org/10.1016/j.trc.2018.12.004.

Yun, I., Park, B.B., 2005. Estimation of dynamic origin destination matrix: A genetic algorithm approach. In: Proceedings of the 8th International IEEE Conference on Intelligent Transportation Systems, Vienna, Austria, September 13-16.

Zhou, X., Mahmassani, H.S., 2006. Dynamic origin–destination demand estimation using automatic vehicle identification data. IEEE Trans. Intell. Transp. Syst. 7, 105–114. http://dx.doi.org/10.1109/TITS.2006.869629.

Zhou, X., Mahmassani, H.S., 2007. A structural state space model for real-time traffic origin–destination demand estimation and prediction in a day-to-day learning framework. Transport. Res. B 41, 823–840. http://dx.doi.org/10.1016/j.trb.2007.02.004.

Zin, T.A., Kyaing, K., Lwin, K., Sekimoto, Y., 2018. Estimation of originating-destination trips in yangon by using big data source. J. Disast. Res. 13, 6–13.

Żochowska, R., 2012. Dynamic approach to the origin-destination matrix estimation in dense street networks. In: The Archives of Transport, Polish Academy of Sciences, Warsaw 2012, Vol. 24/2012. pp. 389–413. http://dx.doi.org/10.2478/v10174-012-0025-1, Issue 3.

Żochowska, R., 2014. Selected issues in modelling of traffic flows in congested urban networks. Arch. Transp. 29 (1), http://dx.doi.org/10.5604/08669546.1146971.