# SECOND MOMENT OF AREA CALCULATIONS

Book 9

Designer's Den

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# Steiner’s theorem

Steiner's theorem, also known as the parallel axis theorem, is a fundamental principle in mechanics that relates the moments of inertia of a body about two different axes. It provides a method to calculate the moment of inertia of a body about an axis parallel to a known axis.

According to Steiner's theorem, the moment of inertia of a body about an axis parallel to and at a distance (e) from a known axis can be determined by adding the moment of inertia about the known axis to the product of the body's mass and the square of the distance (e) between the two axes.

Mathematically, Steiner's theorem can be expressed as:

$I_{Parallel} = I_{Local} + A \cdot e^2$

Where:

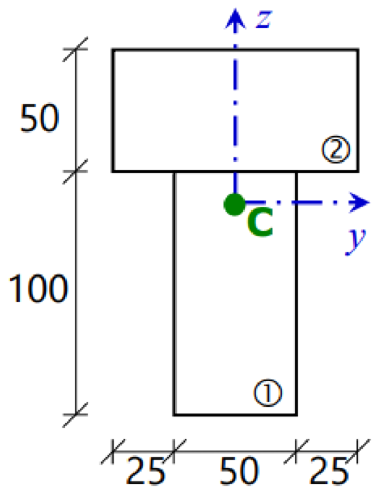
- $I_{Parallel}$ is the moment of inertia about the parallel axis.
- $I_{Local}$ is the moment of inertia about the local axis.
- A is the area of the body.
- e is the distance between the two axes.

Steiner's theorem is particularly useful when calculating the moment of inertia of complex shapes or composite bodies. Instead of deriving the moment of inertia directly about the desired axis, it allows us to determine it based on the moment of inertia about a simpler or more convenient axis.

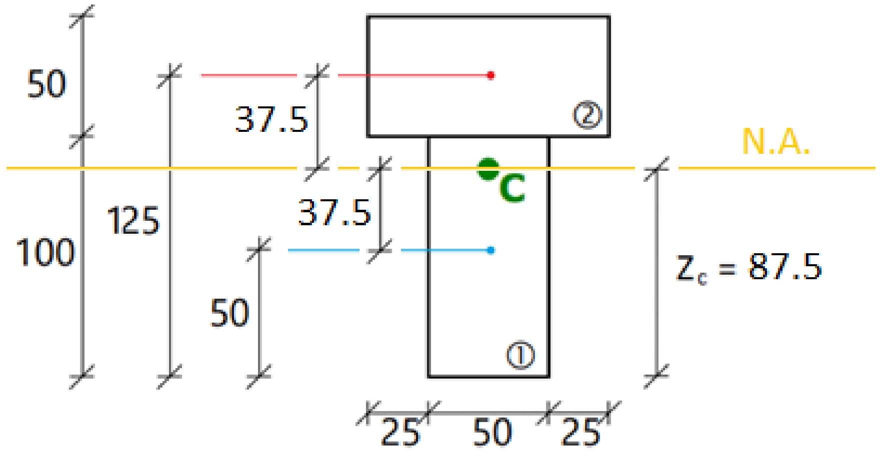
By applying Steiner's theorem, engineers and physicists can simplify calculations for objects with irregular shapes or distributed masses. It enables the determination of the moment of inertia about various parallel axes, facilitating accurate analysis of rotational motion, stability, and structural behavior.

# Examples

**Example:** Find the second moment of area for the following cross-section:



**Solution:**




*Analytical method:*

$$
\begin{aligned}
I_y &= \int_A z^2 dA \\
&= \int_{A_1} z^2 dA + \int_{A_2} z^2 dA \\
&= \int_{-25}^{25}\int_{-87.5}^{12.5} z^2 dz dy + \int_{-50}^{50}\int_{12.5}^{62.5} z^2 dz dy \\
&= \int_{-25}^{25}\left[\frac{1}{3}z^3\right]_{-87.5}^{12.5} dy + \int_{-50}^{50}\left[\frac{1}{3}z^3\right]_{12.5}^{62.5} dy \\
&= \int_{-25}^{25} 223{,}96\cdot 10^3 dy + \int_{50}^{50} 80{,}73\cdot 10^3 dy \\
&= 223{,}96\cdot 10^3\cdot[y]_{-25}^{25} + 80{,}73\cdot 10^3\cdot[y]_{-50}^{50} \\
&= 223{,}96\cdot 10^3\cdot 50 + 80{,}73\cdot 10^3\cdot 100 \\
&= 11{,}20\cdot 10^6 + 8{,}073\cdot 10^6 \\
&= \underline{19{,}27\cdot 10^6 \text{ mm}^4}
\end{aligned}
$$

*Steiner's theorem:*

$$
\begin{aligned}
I_y &= \sum I_i = I_1 + I_2 \\
&= (I_{L,1} + A_1\cdot e_1{}^2) + (I_{L,2} + A_2\cdot e_2{}^2) \\
&= \left(\frac{50\cdot 100^3}{12} + (50\cdot 100)\cdot 37.5^2\right) + \left(\frac{100\cdot 50^3}{12} + (100\cdot 50)\cdot 37.5^2\right) \\
&= 11{,}20\cdot 10^6 + 8{,}073\cdot 10^6 \\
&= \underline{19{,}27\cdot 10^6 \text{ mm}^4}
\end{aligned}
$$