SECOND MOMENT OF AREA

Book 8

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  (Find a demonstration in the Playroom)

Definition

The second moment of area, also known as the moment of inertia, is a property used in structural mechanics to quantify how a cross-sectional shape of an object is distributed around a particular axis. It provides information about the object's resistance to bending and torsional deformations.

The second moment of area is denoted by the symbol I and is specific to the axis of rotation or bending being considered. It is calculated by summing the products of infinitesimally small areas within the cross section and the square of their distances from the axis of rotation. Mathematically, the formula for calculating the second moment of area is:

$$I = \int z^2 \, dA$$

Where:

- I is the second moment of area or moment of inertia.

- z is the perpendicular distance between an infinitesimal area element (dA) and the axis of rotation.

- $\int$ represents integration, summing up all the infinitesimal areas over the entire cross section.

The second moment of area provides information about the cross-sectional shape's resistance to bending. The larger the value of the second moment of area, the more resistant the object is to bending deformations. It quantifies the distribution of material around the axis of rotation and plays a crucial role in determining the deflection, stress distribution, and overall structural behavior of beams, columns, and other structural elements.

The second moment of area varies depending on the shape and geometry of the cross section. For common shapes, such as rectangles, circles, and triangles, there are specific formulas to calculate the second moment of area. These formulas take into account the dimensions and geometry of the shape.

The second moment of area is an important parameter used in various engineering calculations, such as calculating bending stresses, deflections, and natural frequencies of structural elements. It is also used in beam theory equations, such as the Euler-Bernoulli beam theory and the Timoshenko beam theory, to analyze the behavior of beams under bending loads.

Understanding the second moment of area is crucial in structural design and analysis as it helps engineers optimize the shape and dimensions of structural elements to ensure they meet the required strength, stiffness, and performance criteria.

The second moment of area (moment of inertia, I) and the resistance to bending or deformation of a cross-section tend to increase when there is a greater distribution of area away from the neutral axis.
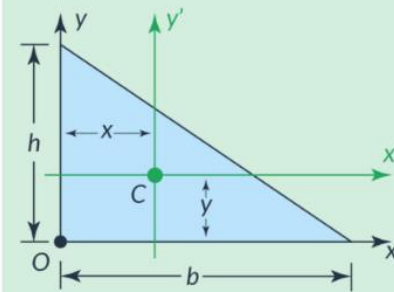
Common geometrical shapes

Rectangle
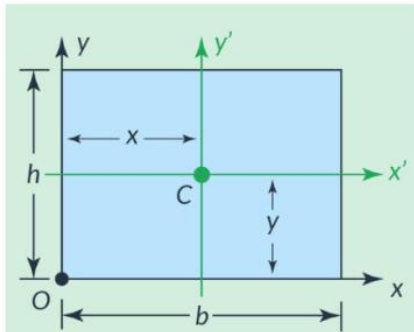
$$I_{x'} = \frac{1}{12}bh^3$$

$$I_{y'} = \frac{1}{12}b^3h$$

$$I_x = \frac{1}{3}bh^3$$

$$I_y = \frac{1}{3}b^3h$$

Triangle

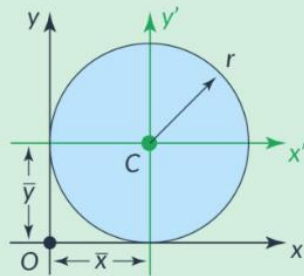

$$I_{x'} = \frac{1}{36}bh^3$$

$$I_{y'} = \frac{1}{36}b^3h$$

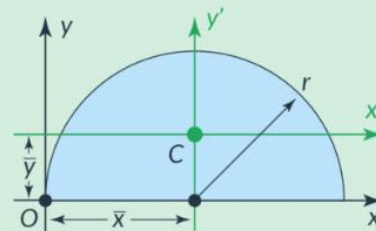$$I_x = \frac{1}{12}bh^3$$

$$I_y = \frac{1}{12}b^3h$$

Circle

$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{\pi}{4}r^4$$

$$I_{x'} = I_{y'} = \frac{5\pi}{4}r^4$$

Half Circle

$$\bar{I}_{x'} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$\bar{I}_{y'} = \frac{\pi}{8}r^4$$

$$I_x = I_y = \frac{\pi}{8}r^4$$