NORMAL STRESS

Book 10

Designer's Den

Contents

Definiton

Stress refers to the internal resistance or force per unit area within a material when subjected to external loads or forces. It represents how a material reacts to applied forces and can affect its deformation, structural integrity, and failure behavior.

There are several types of stresses commonly encountered in engineering and mechanics:

1. **Normal Stress:** Normal stress acts perpendicular to the cross-sectional area of a material. Normal stress come from axial and bending forces. The axial forces cause tension or compression. Tensile stress occurs when the material is being pulled apart, while compressive stress occurs when the material is being compressed or squeezed together. Bending stress occurs in beams or structural members subjected to bending moments. It results from the distribution of forces and moments within the material, causing tension on one side (top) and compression on the other side (bottom) of the beam. *(In this course we are only dealing with normal stresses)*
2. **Shear Stress:** Shear stress arises when forces act parallel to the cross-sectional area of a material, causing adjacent layers to slide past each other. Shear stress plays a significant role in the behavior of materials under torsion or when subjected to cutting or sliding forces.

3. **Torsional Stress:** Torsional stress occurs in structures or components subjected to twisting or torsional loads. It is characterized by shear stresses acting on various planes within the material.
4. **Thermal Stress:** Thermal stress arises due to temperature variations within a material, leading to expansion or contraction. It can cause deformation and structural failure, particularly when materials with different coefficients of thermal expansion are combined.
5. **Residual Stress:** Residual stress exists within a material even in the absence of external forces or loads. It arises from manufacturing processes, such as welding, casting, or heat treatment, and can influence the material's behavior and stability.

Understanding and analyzing stresses is crucial in engineering design, structural analysis, and material selection. Engineers evaluate stress levels to ensure that they remain within acceptable limits for the material being used. By considering stresses, engineers can design structures and components that can withstand anticipated loads, maintain structural integrity, and avoid excessive deformation or failure.

Axial stress

Axial stress is a type of normal stress that acts perpendicular to the cross-sectional area of an object. It represents the force per unit area that acts within a material and can cause deformation or structural failure. There are two types of axial stresses: tensile stress and compressive stress.

Tensile Stress: Tensile stress occurs when forces act to elongate or stretch a material. It is considered positive when the material is being pulled apart. Tensile stress is calculated by dividing the applied force (F) by the cross-sectional area (A) perpendicular to the applied force. Mathematically, tensile stress (σ) is expressed as: $\sigma = \frac{F}{A}$

Compressive Stress: Compressive stress occurs when forces act to compress or squeeze a material. It is considered negative when the material is being pushed together. Compressive stress is also calculated by dividing the applied force (F) by the cross-sectional area (A). Mathematically, compressive stress (σ) is expressed as: $\sigma = -\frac{F}{A}$

Tensile stress is usually denoted with a positive sign (+) while compressive stress with a negative sign (-).

Bending stress

Normal stress due to bending, also known as bending stress, occurs in a beam or structural member when it is subjected to a bending moment. Bending stress results from the internal distribution of forces and moments within the material due to the applied bending moment.

In a beam, the top portion experiences tensile bending stress, while the bottom portion experiences compressive bending stress. At the neutral axis, the bending stress is zero.

The magnitude of bending stress varies along the cross-section of the beam. It is highest at the extreme fibers, farthest from the neutral axis. The bending stress can be calculated using the bending moment (M) and the distance from the neutral axis (z) to the fiber where the stress is being calculated. Mathematically, the bending stress (σ) is given by the formula: $\sigma = \frac{M}{I} \cdot z$

Where:

- M is the bending moment at that location,
- z is the distance from the neutral axis to the fiber where the stress is being calculated,
- I is the second moment of area (moment of inertia) of the cross-sectional shape.

Navier’s formula

Navier’s formula is the equation where the total stress (σ) is expressed as the sum of axial stress (σ_{Axial}) and bending stress ($\sigma_{Bending}$) is known as the Principle of Superposition or the Superposition Principle.

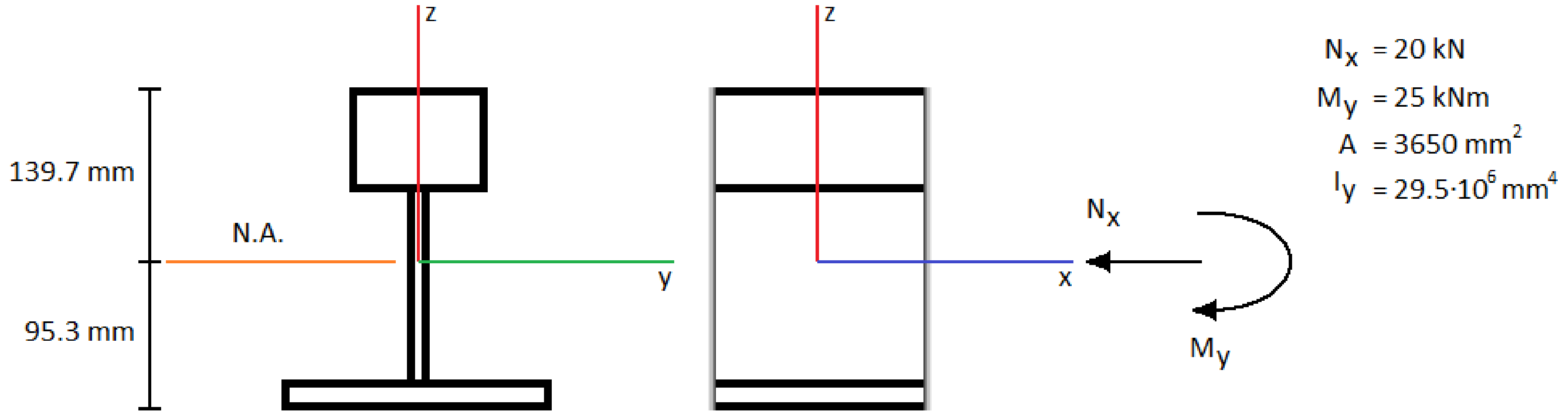
Mathematically, the equation can be written as: $\sigma = \sigma_N + \sigma_M = \frac{F}{A} + \frac{M}{I} \cdot z$

This equation is used when analyzing the combined effects of axial loading (such as tension or compression) and bending in a structural member, such as a beam or column. The axial stress accounts for the direct stress resulting from the axial force, while the bending stress represents the stress induced by the bending moment.

By considering the individual contributions of axial and bending stresses, engineers can assess the combined stress state and ensure that the total stress in the material remains within acceptable limits. This principle is commonly applied in structural analysis and design to evaluate the strength and structural integrity of components and systems subject to complex loading conditions.

Examples

Example: An arbitrary cross-section is experiencing the following conditions. What is the maximum compressive and tensile stresses that this element is undergoing?



Solution:

We recall that this element is undergoing both axial loading and bending. Therefore we will use Navier's formula which takes into account both of the stresses (axial and bending)

On top:

$$\sigma_{Top} = \sigma_N + \sigma_M = -\frac{F}{A} + \frac{M}{I} \cdot z = -\frac{20 \cdot 10^3\ \mathrm{N}}{3650\ \mathrm{mm}^2} + \frac{25 \cdot 10^6\ \mathrm{Nmm}}{29.5 \cdot 10^6\ \mathrm{mm}^4} \cdot 139.7\ \mathrm{mm} = -5.5\ \mathrm{N/mm}^2 + 118.4\ \mathrm{N/mm}^2$$

$\sigma_{Top} = 112.9\ \mathrm{N/mm}^2$ (Tension)

On bottom:

$$\sigma_{Bottom} = \sigma_N + \sigma_M = -\frac{F}{A} + \frac{M}{I} \cdot z = -\frac{20 \cdot 10^3\ \mathrm{N}}{3650\ \mathrm{mm}^2} + \frac{25 \cdot 10^6\ \mathrm{Nmm}}{29.5 \cdot 10^6\ \mathrm{mm}^4} \cdot (-95.3\ \mathrm{mm}) = -5.5\ \mathrm{N/mm}^2 - 80.7\ \mathrm{N/mm}^2$$

$\sigma_{Bottom} = -86.2\ \mathrm{N/mm}^2$ (Compression)

Visualization: