EQUATIONS OF EQUILIBRIUM

Book 4

Designer's Den

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Definition

In statics, the equations of equilibrium are mathematical expressions that describe the conditions for an object to be in a state of equilibrium. These equations relate the forces and moments acting on an object to ensure that it remains at rest or in a state of constant motion with no acceleration. The equations of equilibrium are derived from Newton's laws of motion and are essential for analyzing the forces and moments in a static system.

Translational Equilibrium

$\sum F_x = 0$: The sum of all the horizontal (x-direction) forces acting on the object is zero.

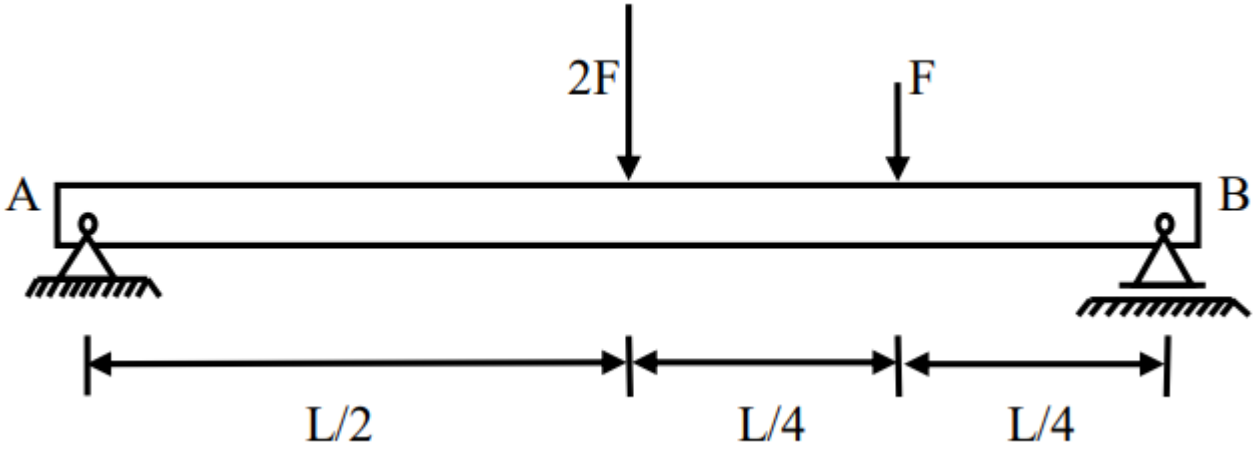
$\sum F_z = 0$: The sum of all the vertical (z-direction) forces acting on the object is zero.

Rotational Equilibrium

$\sum M = 0$: The sum of all the moments about a reference point or axis is zero. The moments are calculated by multiplying the force magnitude by the perpendicular distance from the reference point or axis.

Examples

Example 1: Calculate the reaction forces of the beam using the equations of equilibrium



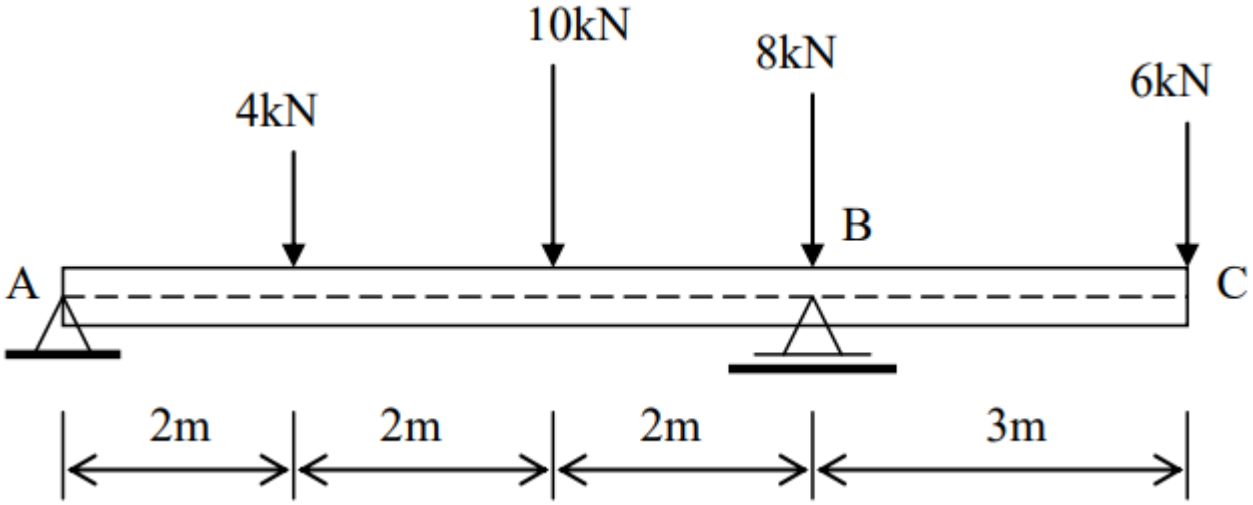
Solution:

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum M_A = 0 = 2F \cdot \frac{L}{2} + F \cdot \frac{3L}{4} - B_y \cdot L \Rightarrow B_y = \frac{7F}{4}$$

$$\sum F_y = 0 = A_y + B_y - 2F - F \Rightarrow A_y = \frac{5F}{4}$$

Example 2: Calculate the reaction forces of the beam using the equations of equilibrium



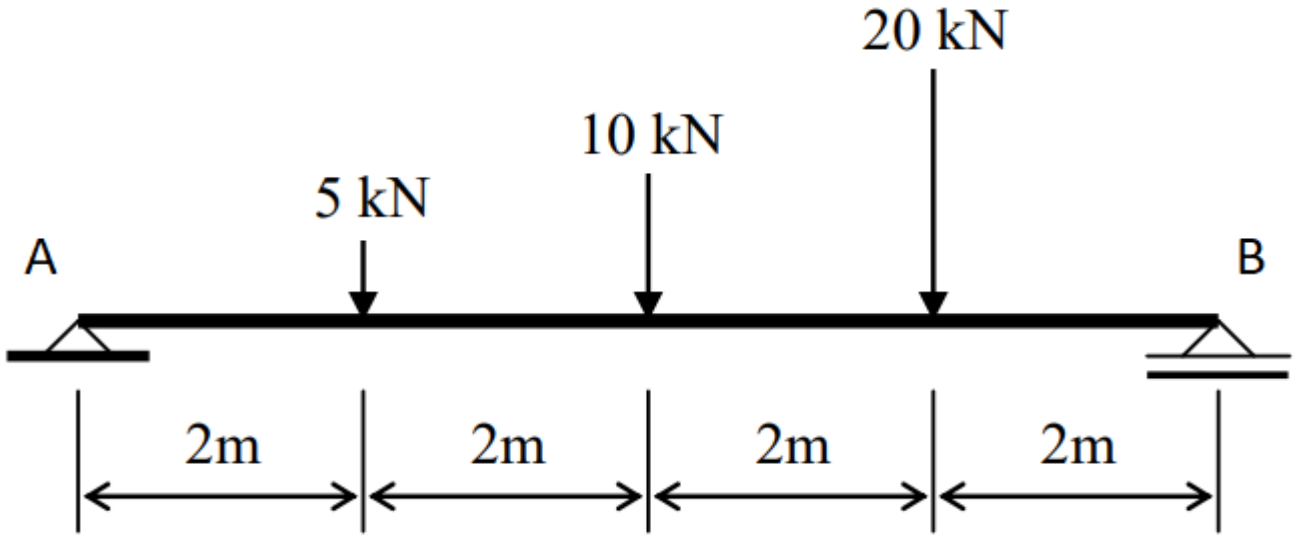
Solution:

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma M_A = 0: \quad B_y \cdot 6 - 4 \cdot 2 - 10 \cdot 4 - 8 \cdot 6 - 6 \cdot 9 = 0 \Rightarrow B_y = (8 + 40 + 48 + 54)/6 = 25 \text{ kN}$$

$$\Sigma F_y = 0: \quad A_y + B_y - 4 - 10 - 8 - 6 = 0 \Rightarrow A_y = 4 + 10 + 8 + 6 - 25 = 3 \text{ kN}$$

Example 3: Calculate the reaction forces of the beam using the equations of equilibrium



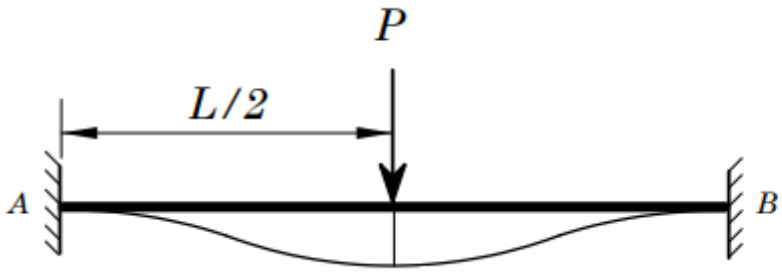
Solution:

$$\Sigma M_A = 0: \quad B_y \cdot 8 - 5 \cdot 2 - 10 \cdot 4 - 20 \cdot 6 = 0 \quad \Rightarrow \quad B_y = 21{,}25 \text{ kN}$$

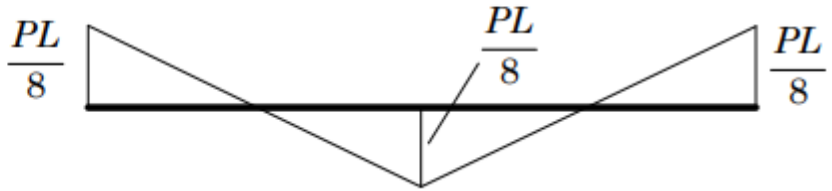
$$\Sigma F_y = 0: \quad A_y = 5 + 10 + 20 - 21{,}25 = 13{,}75 \text{ kN}$$

$$\Sigma F_x = 0: \quad A_x = 0$$

Example 4: Calculate the reaction forces of the beam using the equations of equilibrium given that the moments at the fixed supports A and B are $M_A = M_B = \frac{PL}{8}$



Moment diagram:



Solution:

$$\Sigma F_x = 0: \quad A_x = B_x = 0$$

$$\Sigma F_y = 0: \quad A_y + B_y = P \Rightarrow A_y = B_y = \frac{P}{2}$$

Alternatively:

$$\Sigma M_A = 0: \; -M_A + M_B + P \cdot \frac{L}{2} - B_y \cdot L = 0$$

$$-\frac{PL}{8} + \frac{PL}{8} + P \cdot \frac{L}{2} - B_y \cdot L = 0 \Rightarrow B_y = \frac{P}{2}$$

(Same process from point B for A_y*)*