HUST-USYD Summer School on Parallel Programming Practice – Lecture 5

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Outline

- Synchronization
  - Race condition and mutual exclusion
- OpenMP synchronization constructs
  - Critical, atomic and barrier
- Data dependency
  - Instruction-level data dependency
  - Loop-carried data dependency
- Reduction operation
- OpenMP reduction clause
- Exercise: reduction operation
- Scan (or prefix) operation
- Homework 3

Synchronization

- In shared-memory machines global data are shared by multiple threads
- Synchronization must be used to
  - Protect access to shared data and so to prevent race conditions
  - Impose order constraints
- OpenMP supported synchronization constructs
  - Critical, atomic and barrier
  - The full OpenMP specification has several more

Synchronization Constructs

- OpenMP critical directive:
- #pragma omp critical
- Threads wait their turn – only one at a time can execute in the critical construct
- Critical directive must be inside a parallel region
- E.g., threads to increment count

```
#pragma omp parallel shared(count)
{
   ...
  #pragma omp critical
  count++;
   ...
}
```

Synchronization Constructs

- OpenMP atomic directive:
- #pragma omp atomic
- Also provides mutual exclusion, but only applies to the update of one memory location
- atomic directive applies only to the statement immediately following it
- E.g., accumulate partial results done by multiple threads

```
double sum = 0.0;
#pragma omp parallel shared(sum)
{
   double lsum;
   …
  #pragma omp atomic
  sum += lsum
   …
}
```

Synchronization Constructs

- OpenMP barrier directive:
- #pragma omp barrier
- barrier is a point in a program all threads must reach before any threads are allowed to proceed
- In OpenMP most constructs have an implied barrier at the end of the constructs
  - To remove that barrier, use onwait clause

```
#pragma omp parallel shared(sum)
{
   //all threads do something
  #pragma omp barrier //Threads wait until all threads hit the barrier
   //all threads do some other thing
}
```

Instruction-Level Data Dependency

- Data dependency at the instruction level – the dependency in a sequence of instructions:
  - Flow dependency (Read After Write or RAW)
    - E.g., a = b + c (Write b+c to a)
      d = a * e (Read a out and assign a*e to d)
  - Anti-dependency (WAR)
    - E.g., b = a + c (Read a out and assign a+c to b)
      a = d * e (Write d+e to a)
  - Output dependency (WAW)
    - E.g., a = b + c
      a = d * e

Instruction-Level Data Dependency

- Data dependency at the instruction level – the dependency in a sequence of instructions:
  - Flow dependency is true dependency
  - Anti-dependency and output dependency are not real (so called artificial) dependency and can be eliminated
    - E.g., output dependency in the previous example can be removed by writing d*e to another variable and then use the new variable (instead of a) in the future

Loop-Carried Data Dependency

- Can we parallelize the following for loop?

```
for (i=0; i<n; i++)
{
    tmp = a[i];
    a[i] = b[i];
    b[i] = tmp;
}
```

- Clearly there are data dependencies in the three instructions
- However, there are no data dependencies to be carried out to the next iteration
- Therefore, the loop can be safely parallelized
- In OpenMP we really need to check loop-carried dependency!

Loop-Carried Data Dependency

– Can we parallelize the following for loop?

```
for (i=0; i<n; i++)
{
    a[i] = d[i] +e[i];
    d[i] = e * a[i+1];
}
```

WAR dependency

```
a[0] = d[0]+e[0]
d[0] = e * a[1]
a[1] = d[1]+e[1]
d[1] = e * a[2]
a[2] = d[2]+e[2]
d[2] = 3 * a[3]
a[3] = d[3]+e[3]
d[3] = e * a[4]
   ⋮
```

– Not very clear?

– Let's expand the for loop starting from i=0

  – a[i] in red means assigned new value

  – a[i] in red means old value

Loop-Carried Data Dependency

- Now consider parallelize the for loop:

Thread 0:	Thread 1:	Thread 2:
a[0] = d[0]+e[0]	a[2] = d[2]+e[2]	a[4] = d[0]+e[0]
d[0] = e * a[1]	d[2] = 3 * a[3]	d[0] = e * a[5]
a[1] = d[1]+e[1]	a[3] = d[3]+e[3]	a[5] = d[1]+e[1]
d[1] = e * a[2]	d[3] = e * a[4]	d[1] = e * a[6]

Wrong values!

- Now it becomes clear – there are indeed loop-carried data dependencies
- Also potential data race
  - i.e., one thread write and another read on the same data
- How to deal with this problem?

Loop-Carried Data Dependency

```
for (i=0; i<n; i++)
{
    a[i] = d[i] +e[i];
    d[i] = e * a[i+1];
}
```

```
for (i=0; i<n; i++)
{
    d[i] = e * a[i+1];
    a[i] = d[i] +e[i];
}
```

```
for (i=0; i<n; i++)
   d[i] = e * a[i+1];
for (i=0; i<n; i++)
   a[i] = d[i] +e[i];
```

- In the same i iteration, the order of two instructions can be exchanged because there is no data dependency
- Also separate the instructions into two for loops

Loop-Carried Data Dependency

– Now parallelize two for loops one by one:

```
#pragma omp parallel for
for (i=0; i<n; i++)
    d[i] = e * a[i+1];
#pragma omp parallel for
for (i=0; i<n; i++)
    a[i] = d[i] +e[i];
```

Implied barrier here

Thread 0:	Thread 1:	Thread 3:
d[0] = e * a[1]	d[2] = 3 * a[3]	d[4] = e * a[5]
d[1] = e * a[2]	d[3] = e * a[4]	d[5] = e * a[6]
a[0] = d[0]+e[0]	a[2] = d[2]+e[2]	a[4] = d[0]+e[0]
a[1] = d[1]+e[1]	a[3] = d[3]+e[3]	a[5] = d[1]+e[1]

– We now obtain the correct results!

Loop-Carried Data Dependency

- Can we parallelize the following for loop?

```
for (i=0; i<n; i++)
{
    a[i+1] = d[i] +e[i];
    d[i] = e * a[i];
}
```

- A simple problem for you to work out at home

Loop-Carried Data Dependency

– In the previous lecture we also discussed how to deal with another type of loop-carried data dependency:

```
int i, j, A[MAX];
 j = 5;
 for (i=0;i< MAX; i++){
     j +=2;
     A[i] = big(j);
}
```

Note: loop index "i" is private by default

```
int i,  A[MAX];
#pragma omp parallel for
 for (i=0;i< MAX; i++) {
     int j = 5 + 2*(i+1);
     A[i] = big(j);
}
```

Remove loop carried dependence

– In general we need to pay attention on the following loop-carried data dependency:
  – index of an array data is not equal to current loop index
  – The value of a variable changes with the iterations

Reduction Operation

- Given associative operator $\bigoplus$
- Examples
  - Add (+)
  - Multiply (*)
  - And, Or (&&, ||)
  - max, min
- and also an array of elements $[a_0, a_1, a_2, \ldots, a_{n-1}]$
- Calculate
- S = $a_0 \bigoplus a_1 \bigoplus a_2 \bigoplus \ldots \bigoplus a_{n-1}$

Reduction Operation

- E.g., let $\oplus = +$, that is, summation of n numbers:
  - $S = a_0 + a_1 + a_2 + \ldots + a_{n-1} = \sum_{i=0}^{n-1} a_i$
- Use recursion:
  - $S_0 = a_0$
  - $S_i = S_{i-1} + a_i$ for $1 \le i < n$
- A sequential routine:

```
S = a[0];
for (i=1; i<n; i++)
   S = S + a[i];
```

- Can we simply use OpenMP for directive to parallelize this sequential routine?
  - No, it is recursive and the value of S changes with iterations
    - There are loop-carried data dependencies!

Reduction Operation

- E.g., let $\bigoplus = +$, that is, summation of n numbers:
  - $S = a_0 + a_1 + a_2 + \ldots + a_{n-1} = \sum_{i=0}^{n-1} a_i$

- However, addition is associative and commutative
  - We can add the numbers in any order
  - E.g., $S = \sum_{i=0}^{k-1} a_i + \sum_{i=k}^{2k-1} a_i + \sum_{i=2k}^{n-1} a_i$
    $= S_0 + S_1 + S_2$
- Then we can use the OpenMP directive to parallelize the for loop for each thread to compute a partial sum and then accumulate them to obtain the final result

Reduction Operation

- E.g., let $\oplus = +$, that is, summation of n numbers:
  - $S = a_0 + a_1 + a_2 + \ldots + a_{n-1} = \sum_{i=0}^{n-1} a_i$

```
#pragma omp parallel shared(a,S) private(i)
{
  double lsum = 0.0; //each thread has a local lsum variable
  #pragma omp for nowait
  for (i=0; i<n; i++) //the number of iterations distributed
      lsum +=  a[i];    //each thread calculate a partial sum
  ··· //accumulate partial sums
}
```

- How to accumulate the partial sums?

OpenMP Reduction Clause

- OpenMP supports a reduction clause:
  - reduction (op : list)
- E.g.,

```
double S=0;
#pragma omp parallel for shared(S, a), reduction(+:S)
for (int i=0; i<n; ++)
   S += a[i];
```

- When reduction clause is added
  - A local copy of each list variable is made and initialized, depending on the “op” (e.g. 0 for “+”)
  - Updates occur on the local copy
  - Local copies are then reduced into a single value
  - The variables in “list” must be shared in the enclosing parallel region

OpenMP Reduction Clause

– Many different associative operands can be used
– Initial values are the ones that make sense mathematically

Operator	Initial value	Operator	Initial value
+	0	&&	1
*	1	||	0
-	0	&	~0
min	Largest pos. number	|	0
max	Most neg. number	^	0

Lab Exercise: Reduction Operation

- Write an OpenMP program to compute summation of n numbers
- You need to write two routines:
  - One uses OpenMP reduction clause
  - The other is to let each thread compute a partial sum and then accumulate the partial sums
- Run your program to compare the performance of these two routines

Scan Operation

- Let $A = [a_0, a_1, a_2, \ldots, a_{n-1}]$ be an array of elements and $\oplus$ an associative operator
- Scan (or prefix) operation produces another array C:

  $C = [a_0, (a_0 \oplus a_1), (a_0 \oplus a_1 \oplus a_2), \cdots, (a_0 \oplus a_1 \oplus a_2 \oplus \cdots \oplus a_{n-1})]$

- E.g., $\mathrm{A} = [1,2,3,4,5,6]$, and after add scan of A we obtain $C = [1,3,6,10,15,21]$
- It can be seen that there are many redundancy in calculating c_i
  - Use recursion to remove redundant operations
- A sequential routine:

```
c[0] = a[0];
for (i=1; i<n; i++)
    c[i] = c[i-1] ⊕ a[i];
```

- It takes only $n - 1$ $\oplus$ operations

Parallel Algorithm for Scan Operation

- The sequential routine is very simple, but strictly sequential in nature
  - True data dependency and cannot directly use OpenMP for directive to parallelize the for loop
- Need to design new parallel algorithms
- Many parallel algorithms for scan operation have been developed based on the fact that $\oplus$ is an associative operator
  - Key is to minimize the amount of operations
- In the following we discuss one parallel algorithm which is efficient for parallel computation of scan operation on shared-memory machines

Parallel Algorithm for Scan Operation

– Rewrite the c_i:

$$
\begin{aligned}
c_0 &= a_0 \\
c_1 &= a_0 \oplus a_1 \\
c_2 &= a_0 \oplus a_1 \oplus a_2 \\
c_3 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \\
c_4 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \\
c_5 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \\
c_6 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \\
c_7 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \\
c_8 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \oplus a_8 \\
c_9 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \oplus a_8 \oplus a_9 \\
c_{10} &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \oplus a_8 \oplus a_9 \oplus a_{10} \\
c_{11} &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \oplus a_8 \oplus a_9 \oplus a_{10} \oplus a_{11}
\end{aligned}
$$

Parallel Algorithm for Scan Operation

– Reduce redundant calculation:

$$
\begin{aligned}
c_0 &= a_0 \\
c_1 &= a_0 \oplus a_1 \\
c_2 &= a_0 \oplus a_1 \oplus a_2 \\
c_3 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \\
c_4 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \\
c_5 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \\
c_6 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \\
c_7 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \\
c_8 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \oplus a_8 \\
c_9 &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \oplus a_8 \oplus a_9 \\
c_{10} &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \oplus a_8 \oplus a_9 \oplus a_{10} \\
c_{11} &= a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_5 \oplus a_6 \oplus a_7 \oplus a_8 \oplus a_9 \oplus a_{10} \oplus a_{11}
\end{aligned}
$$

Divide the computation into blocks

Parallel Algorithm for Scan Operation

- Reduce redundant calculation:

$$c_{i-j} = a_i \oplus a_{i+1} \cdots \oplus a_j$$

e.g., $c_{3-5} = a_3 \oplus a_4 \oplus a_5$

$$c_{i-k} \oplus c_{(k+1)-j} = c_{i-j}$$

e.g., $c_{0-2} \oplus c_{3-5} = c_{0-5}$

$$
\begin{aligned}
c_0 &= a_0 \\
c_1 &= a_0 \oplus a_1 \\
c_2 &= a_0 \oplus a_1 \oplus a_2 \\
c_3 &= c_{0-2} \oplus a_3 \\
c_4 &= c_{0-2} \oplus a_3 \oplus a_4 \\
c_5 &= c_{0-2} \oplus a_3 \oplus a_4 \oplus a_5 \\
c_6 &= c_{0-2} \oplus c_{3-5} \oplus a_6 \\
c_7 &= c_{0-2} \oplus c_{3-5} \oplus a_6 \oplus a_7 \\
c_8 &= c_{0-2} \oplus c_{3-5} \oplus a_6 \oplus a_7 \oplus a_8 \\
c_9 &= c_{0-2} \oplus c_{3-5} \oplus c_{6-8} \oplus a_9 \\
c_{10} &= c_{0-2} \oplus c_{3-5} \oplus c_{6-8} \oplus a_9 \oplus a_{10} \\
c_{11} &= c_{0-2} \oplus c_{3-5} \oplus c_{6-8} \oplus a_9 \oplus a_{10} \oplus a_{11}
\end{aligned}
$$

Parallel Algorithm for Scan Operation

- Parallel algorithm:
- Stage 1: Each thread gets a subset of n/t elements (block partitioning), performs local scan operation on assigned elements, and then thread i (except the last one) stores its last local c element to a working array at $w[i+1]$

$[a_0, a_1, a_2]$	$[a_3, a_4, a_5]$	$[a_6, a_7, a_8]$
$c_0 = a_0$	$c_3 = a_3$	$c_6 = a_6$
$c_{0-1} = c_0 \oplus a_1$	$c_{3-4} = c_3 \oplus a_4$	$c_{6-7} = c_6 \oplus a_7$
$c_{0-2} = c_{0-1} \oplus a_2$	$c_{3-5} = c_{3-4} \oplus a_5$	$c_{6-8} = c_{6-7} \oplus a_8$

$$W: [0, c_{0-2}, c_{3-5}]$$

Sync must be applied (implicitly or explicitly) btw stages!

- Stage 2: A single thread performs scan operation on W

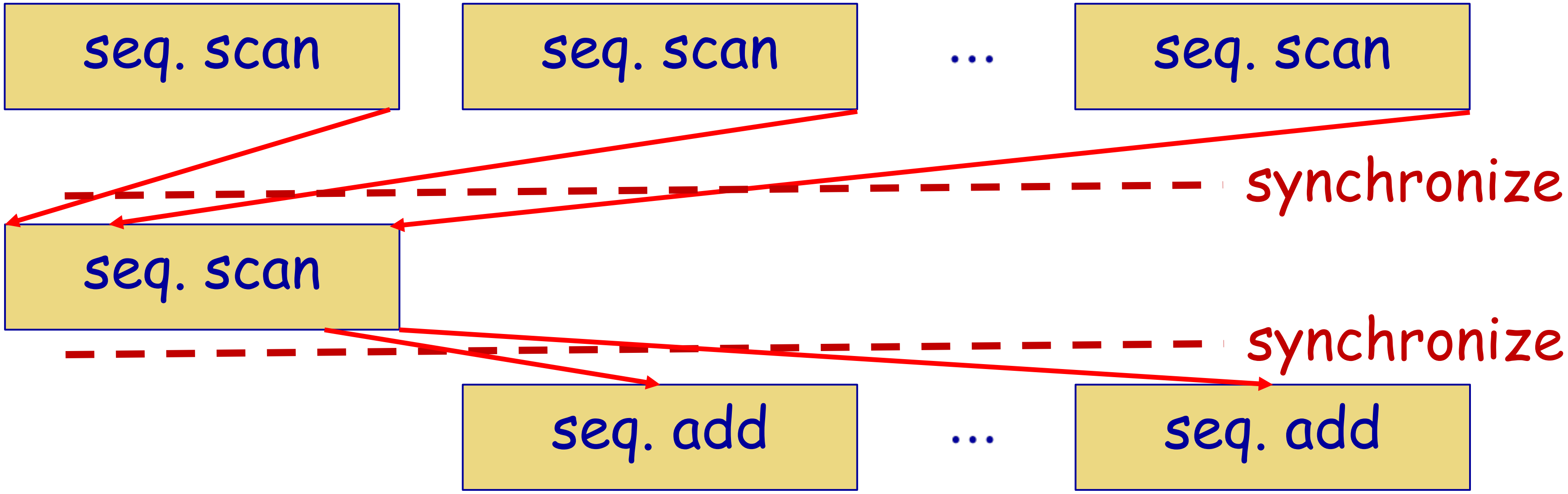
$$W: [0, c_{0-2}, c_{0-5}]$$

- Stage 3: Each thread $\oplus\, w[i]$ to every partial scan element

$c_0 = c_0 \oplus 0$	$c_3 = c_3 \oplus c_{0-2}$	$c_6 = c_6 \oplus c_{0-5}$
$c_1 = c_{0-1} \oplus 0$	$c_4 = c_{3-4} \oplus c_{0-2}$	$c_7 = c_{6-7} \oplus c_{0-5}$
$c_2 = c_{0-2} \oplus 0$	$c_5 = c_{3-5} \oplus c_{0-2}$	$c_8 = c_{6-8} \oplus c_{0-5}$

Parallel Algorithm for Scan Operation

- To summarize, this parallel algorithm consists of three parallel stages
  - Synchronization between stages
  - In 2nd stage only one thread is active
    - Purely sequential, but overhead not heavy as SM machines are usually small



Parallel Algorithm for Scan Operation

- Total amount of operations:
  - Stage 1: each thread performs local scan on n/t elements
    - $(n/t - 1) * t = n - t$
  - Stage 2: a single thread performs scan on W
    - $t - 1$ (the size of W is t)
  - Stage 3: each thread add w_i to the partially scanned elements
    - $n/t * t = n$
- Thus the total amount is
  - $n - t + t - 1 + n = 2n - 1$
- The total amount of operations in sequential computation is $n - 1$
- Thus the parallel algorithm takes less amount of time to complete the computation if the number of threads > 2

Homework 3: Parallel Scan Operation

- Implement the parallel algorithm for scan operation using OpenMP
- Compare the performance with the sequential algorithm

- In a parallel region, if we want part of code to be done by a single (any) thread, we can use single directive

#pragma omp single

- All other threads will skip the single region and stop at the barrier at the end of the single construct until all threads have reached the barrier

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