Application of Optimal Mass Transportation to Medical Image Analysis

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Outline

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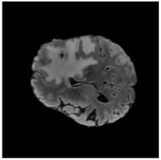
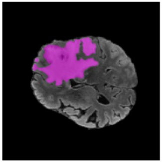
Introduction

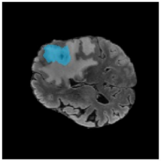
- RSNA-ASNR-MICCAI Brain Tumor Segmentation (BraTS)



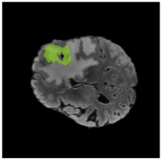
Flair



Whole Tumor



Tumor Core



Enhace Tumor

tumor	label
WT	1, 2, 4
TC	1, 4
ET	4

Table 1: The label of WT, TC, and ET.

Introduction

1. **Motivation:** reduce memory usage and keep global information
2. **Idea:** map irregular domain to regular domain
3. **Difficulty:** high conversion loss

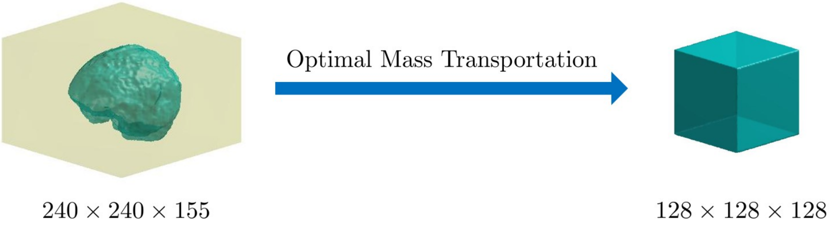


Figure 1: An illustration for the OMT

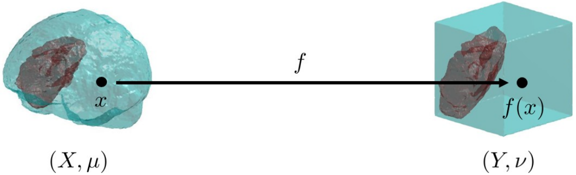
Outline

Optimal Mass Transportation

OMT Problem

Let $(X, \mu), (Y, \nu)$ be two measurable spaces which have the same total mass $\int_X 1 d\mu = \int_Y 1 d\nu$. Let $\mathcal{F}$ be the set of measure-preserving maps and $c : X \times Y \to [0, \infty]$ be a cost function of transportation. The OMT problem is to find a map $f^* \in \mathcal{F}$ that minimizes the transportation cost

$$f^* = \arg\min_{f \in \mathcal{F}} \int_X c(x, f(x)) d\mu.$$



Discrete OMT

Definition (Discrete OMT Problem)

The discrete OMT problem with respect to $\|\cdot\|_2$ can represent as

$$\mathbf{f}^* = \operatorname*{arg\,min}_{\mathbf{f}\in\mathbb{F}_{\mu_V}} \sum_{i=1}^{n_v} \|v_i - \mathbf{f}_i^*\|_2^2\, \mu_V(v_i)$$

where the local measure at the vertex v is

$$\mu_V(v) = \frac{1}{4} \sum_{v\subset\tau, \tau\in\mathcal{T}(\mathcal{B})} \mathrm{vol}(\tau)\cdot\rho(v)$$

and the space of mass-preserving map is

$$\mathbb{F}_{\mu_V} = \left\{ \mathbf{f}\in\mathbb{R}^{n_v\times 3} \;\middle|\; \begin{array}{l}\mathbf{f} \text{ is the inducing matrix for a mass-preserving} \\ \text{map } f:(\mathcal{B},\mu_V)\to(\mathcal{C},\mathrm{vol})\end{array} \right\}.$$

Discrete OMT

OMT steps

1. Solve the boundary map by projection gradient method.
2. Solve the interior map by homotopy method.

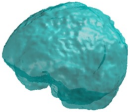
Solving the boundary map by projection gradient method

Definition (Spherical OMT Problem)

$$\mathbf{g}^* = \operatorname*{arg\,min}_{\mathbf{f} \in \mathbb{G}_{\mu_S}} \sum_{i=1}^{n_\mathrm{B}} \| v_i - \mathbf{g}_i^* \|_2^2 \, \mu_S(v_i)$$

where the space of mass-preserving map is

$$\mathbb{G}_{\mu_S} = \left\{ \mathbf{g} \in \mathbb{R}^{n_\mathrm{B} \times 3} \;\middle|\; \begin{array}{l} \mathbf{g} \text{ is the inducing matrix for a mass-preserving} \\ \text{map } g : (\partial \mathcal{B}, \mu_S) \to (\mathbb{S}^2, \text{area}) \end{array} \right\}$$



$(\partial \mathcal{B}, \mu_S)$ $\xrightarrow{\text{Spherical OMT}}$ $(\mathbb{S}^2, \text{area})$

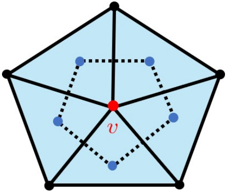
Solving the boundary map by projection gradient method

Definition (Spherical OMT Problem)

$$\mathbf{g}^* = \underset{\mathbf{f}\in\mathbb{G}_{\mu_S}}{\arg\min} \sum_{i=1}^{n_B} \|v_i - \mathbf{g}_i^*\|_2^2 \, \mu_S(v_i)$$

where the local measure at the vertex v as

$$\mu_S(v) = \frac{1}{3} \sum_{v\subset\tau, \tau\in\mathcal{F}(\partial\mathcal{B})} \mathrm{area}(\tau)\cdot\rho(v).$$



Solving the boundary map by projection gradient method

Let $\mathcal{P}_*$ be a projection operator. Then $\mathbf{g}^t$ can update by

$$\mathbf{g}^{t+1} = \mathcal{P}_{\mathbb{G}_{\mu_S}}(\mathbf{g}^t - \eta^t \nabla C(\mathbf{g}^t))$$

where the cost function $C(\mathbf{g}) = \sum_{i=1}^{n_\mathrm{B}} \|v_i - \mathbf{g}_i^*\|_2^2 \, \mu_S(v_i)$ and the learning rate η is chosen by line search.

Projection operator

1. Normalize to spherical $g(v) \leftarrow \frac{g(v)}{\|g(v)\|_2}$.
2. Compute the spherical mass-preserving parameterization with g as initial.
3. Adjust the optimal rotation by SVD.

Solving the boundary map by projection gradient method

1. Compute the spherical OMT map $g_1 : \partial\mathcal{B} \to \mathbb{S}^2$ with density ρ.
2. Compute the spherical OMT map $g_2 : \partial\mathcal{C} \to \mathbb{S}^2$ with area-preserving.
3. Compose the map by $g = g_2^{-1} \circ g_1$.

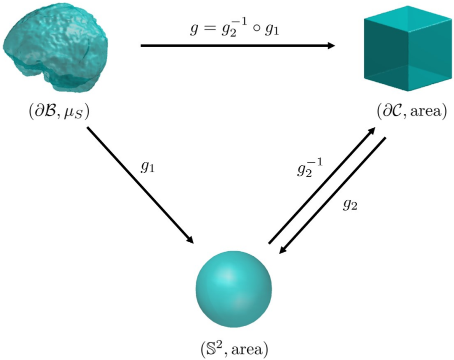


Figure 2: Compose the spherical OMT map

Solving the interior map by homotopy method

Let $0 = t_0 < t_1 < ... < t_p = 1$ be the p piece of $[0, 1]$ and

$$\mathbf{f}_{\mathrm{B}}^{(k)} = (1 - t_k)V_{\mathrm{B}} + t_k\mathbf{g}$$

be the homotopy of boundary map. We solve the interior map by

$$[L_V(f^{(k-1)})]_{\mathrm{I,I}}\mathbf{f}_{\mathrm{I}}^{(k)} = -[L_V(f^{(k-1)})]_{\mathrm{I,B}}\mathbf{f}_{\mathrm{B}}^{(k)}.$$

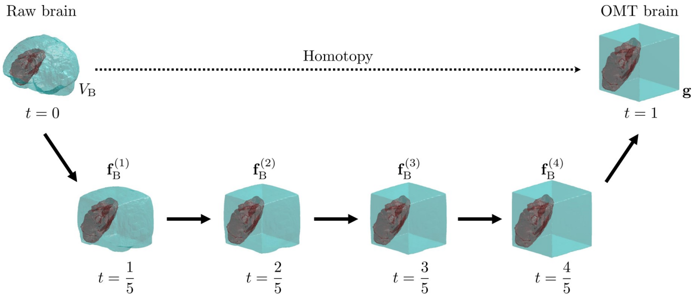


Figure 3: Homotopy flowchart

Outline

Two-Phase training process

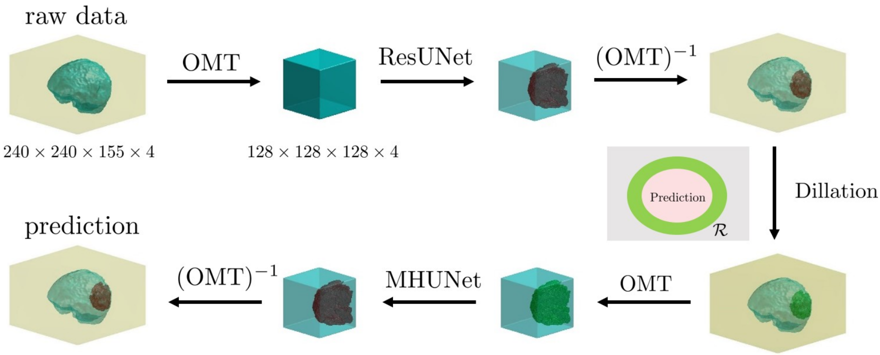


Figure 4: Two-Phase training flowchart

Density function

Let I be the grayscale of flair, HE be the histogram equalization.

1. **Phase 1 OMT density function:**

$$\rho_1(v) = \exp(\gamma \cdot \mathrm{HE}(I(v))), v \in \mathcal{B}$$

2. **Phase 2 OMT density function:**

$$\rho_2(v) = \begin{cases} \exp(\gamma \cdot \mathrm{HE}(I(v))), & \text{if } v \in \mathcal{R} \\ 1.0, & \text{if } v \in \mathcal{B} \setminus \mathcal{R} \end{cases}$$

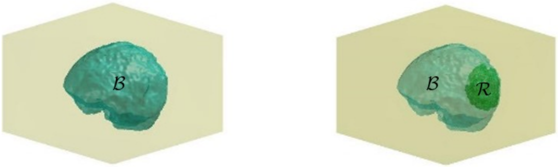


Figure 5: Region of density

Model architecture

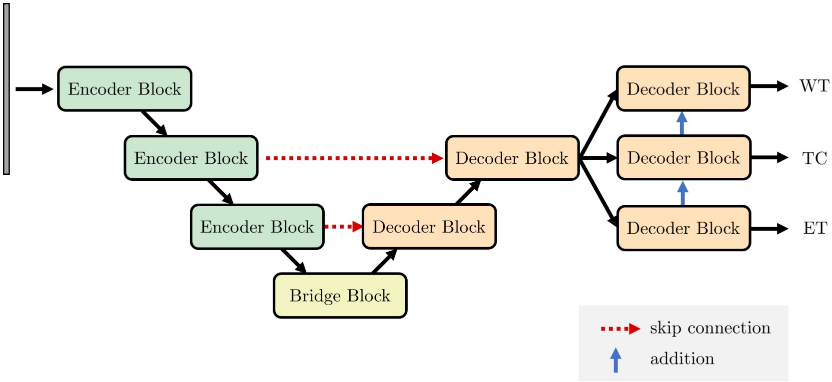


Figure 6: Multi-Head UNet (MHUNet) architecture

Outline

Conversion loss

Let Y be the ground truth, $T : \mathcal{B} \to \mathcal{C}$ be the OMT map, and $T^{-1} : \mathcal{C} \to \mathcal{B}$ be the inverse OMT map. We defined the conversion loss

$$1 - \frac{2|Y \cap (T^{-1} \circ T)(Y)|}{|Y| + |(T^{-1} \circ T)(Y)|}$$

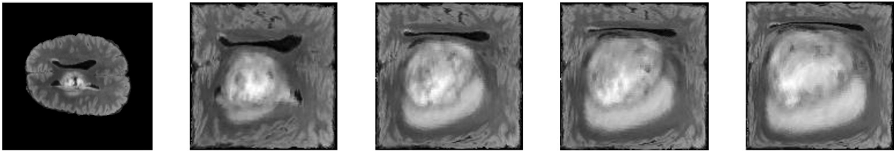
	Phase 1			Phase 2		
Grid size	WT	TC	ET	WT	TC	ET
112^3	0.97%	1.10%	2.70%	0.13%	0.12%	0.33%
128^3	0.34%	0.38%	0.96%	0.02%	0.02%	0.05%

Table 2: Conversion loss of data.

OMT data visualize

data	WT	TC	ET
raw data	6.49%	2.42%	1.45%
128^3 cube with $\gamma = 1.0$	13.47%	5.03%	3.04%
128^3 cube with $\gamma = 1.5$	18.27%	6.84%	4.14%
128^3 cube with $\gamma = 1.75$	20.93%	7.84%	4.75%
128^3 cube with $\gamma = 2.0$	23.72%	8.90%	5.40%

Table 3: Proportion of tumor in brain



raw data | $\gamma = 1.0$ | $\gamma = 1.5$ | $\gamma = 1.75$ | $\gamma = 2.0$

Figure 7: Case BraTS2021_00003

Validation result

Let P, G be a prediction and ground truth. The dice similarity coefficient (DSC) is defined as

$$\mathrm{DSC}(P,G) = \frac{2|P \cap G|}{|P| + |G|}$$

Model	WT	TC	ET
VNet	0.9318	0.9005	0.8691
UNet	0.9305	0.9023	0.8646
ResUNet	0.9307	0.9101	0.8602
MHUNet	**0.9321**	**0.9146**	**0.8683**

Table 4: DSC of validation dataset

Test result

Model	WT	TC	ET
MDUNet	0.9196	0.8644	0.8277
MDUNet+TTA	**0.9201**	**0.8686**	**0.8308**

Table 5: DSC of testing dataset

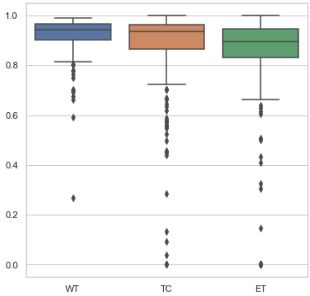


Figure 8: Box plot of DSC of testing dataset

Outline

Conclusion

1. OMT map an object from an irregular domain to a regular domain, which reduce memory usage and keep global information.
2. OMT can generate data augmentation by using the different parameters of density.
3. We propose the Two-Phase training process.

Reference

1. M.-H. Yueh, T.-M. Huang, T. Li, W.-W. Lin, and S.-T. Yau, "Projected gradient method combined with homotopy techniques for volume-measure-preserving optimal mass trans-portation problems," J. Sci. Comput., no. 64, 2021.
2. W.-W. Lin, C. Juang, M.-H. Yueh, T.-M. Huang, T. Li, S. Wang, and S.-T. Yau, 3D Brain Tumor Segmentation Using a Two-Stage Optimal Mass Transport Algorithm, Scientific Reports, 11, 14686, 2021. https://doi.org/10.1038/s41598-021-94071-1
3. O. Ronneberger, P. Fischer, and T. Brox, "U-net: Convolutional networks for biomedical image segmentation," in International Conference on Medical image computing and computer-assisted intervention. Springer, 2015, pp. 234–241.

THE END

Thanks for listening!