Surface Morphing with Applications on Animation

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Abstract

In this report, we implement an approach to reconstruct a discrete manifold by mesh editing, and then using spline homotopy to morphing the horse walking. We uploaded the result to the youtube.

1 Triangular mesh structure

Let $\mathcal{M}$ be a discrete manifold which is composed by

- the vertex set $\mathcal{V}(\mathcal{M}) = \{v_i \mid v_i = (v_i^1, v_i^2, v_i^3) \in \mathbb{R}^3\}$.
- the edge set $\mathcal{E}(\mathcal{M}) = \{[v_i, v_j] \mid \{v_i, v_j\} \subseteq \mathcal{V}(\mathcal{M})\}$.
- the face set $\mathcal{F}(\mathcal{M}) = \{[v_i, v_j, v_k] \mid \{v_i, v_j, v_k\} \subseteq \mathcal{V}(\mathcal{M})\}$.

The following notation is used in this report. Other notation is clearly defined wherever it appears.

- $|V|$ be the number of vertex.
- $V \in \mathbb{R}^{|V| \times 3}$ is a vertex matrix and V_i is the i's row of V.

2 Mesh editing

Given the discrete manifold $\mathcal{M}$, an order index set of landmarks $\mathcal{L}$ and set of target points $\mathcal{T}$ by

$$\mathcal{L} = \{\ell_1, ..., \ell_k\}, \quad \mathcal{T} = \{t_1, ..., t_k\}.$$

Our goal is to adjust $v_{\ell_i} \in \mathcal{V}$ to $t_i \in \mathcal{T}$, then we will get the deformed manifold $\widetilde{\mathcal{M}}$.

For $W \in \mathbb{R}^{|V| \times 3}$, consider the energy functional

$$E(W) = \|L(W - V)\|^2 + \sum_{i=1}^{k} \|W_{\ell_i} - t_i\|^2 \tag{1}$$

where L is the Laplacian matrix given as

$$L_{i,j} = \begin{cases} -\dfrac{\left(\cot\theta_{i,j} + \cot\theta_{j,i}\right)}{2}, & \text{if } [v_i, v_j] \in \mathcal{E}(\mathcal{M}), \\ -\displaystyle\sum_{k \neq i} L_{i,k}, & \text{if } j = i, \\ 0, & \text{otherwise.} \end{cases}$$

Our goal is minimize the energy functional (1). For the convenience, we can reduce (1) to

$$E(W) = \left\| \begin{bmatrix} L \\ C \end{bmatrix} W - \begin{bmatrix} LV \\ T \end{bmatrix} \right\|^2 \tag{2}$$

where $C \in \mathbb{R}^{|\mathcal{L}| \times |V|}$ is a sparse matrix by

$$C_{i,j} = \begin{cases} 1, & i \text{ is the index of landmark and } j \text{ is the index of vertex with respect to landmark,} \\ 0, & \text{otherwise} \end{cases}$$

and

$$T = \begin{bmatrix} t_1 \\ \vdots \\ t_k \end{bmatrix}.$$

Next, taking gradient to (2) and letting $\nabla E(W) = 0$, then we have

$$\begin{bmatrix} L \\ C \end{bmatrix} W = \begin{bmatrix} LV \\ T \end{bmatrix} \tag{3}$$

We can get $\widetilde{V} = \arg\min E(W)$ by solving the linear system (3).

3 Surface morphing

Definition. (Homotopy)
Let $\mathcal{M}, \mathcal{N}$ be two topological spaces and f, g are continuous map from $\mathcal{M}$ to $\mathcal{N}$. f and g are said to be homotopy if there is a continuous map $\mathcal{H} : \mathcal{M} \times [0,1] \to \mathcal{N}$ such that

$$\mathcal{H}(x,0) = f(x) \quad \text{and} \quad \mathcal{H}(x,1) = g(x).$$

The mapping H is called the homotopy between f and g.

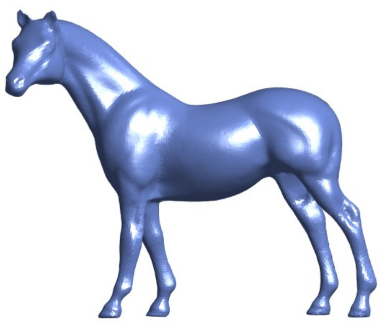
A smooth morphing sequence between discrete manifold $\mathcal{M}_0, ..., \mathcal{M}_T$ can be obtained by a suitable homotopy $\mathcal{H} : [0,T] \times \mathcal{M}_0 \to \mathbb{R}^3$ satisfying

$$\mathcal{H}(0,v) = v \text{ and } \mathcal{H}(t,v) = f_t(v), \text{ for } t = 1, ..., T.$$

where $f_t(v)$ is represent a vertex on deformed manifold $\mathcal{M}_t$.

4 Numerical experience

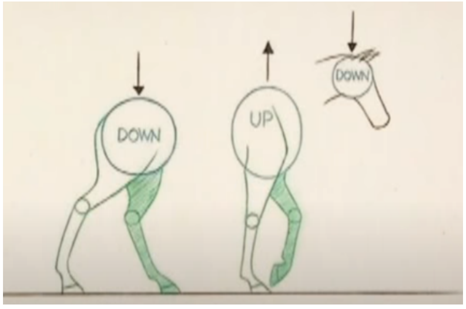
Initially, we have horse surface (which contains 21013 vertices and 42022 faces) looks like this

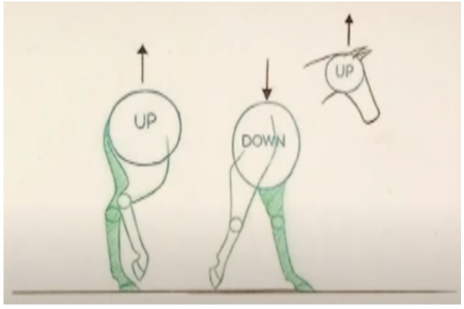


Our goal is to generate a video that the horse is "walking". [4] and [5] are taken as our reference.

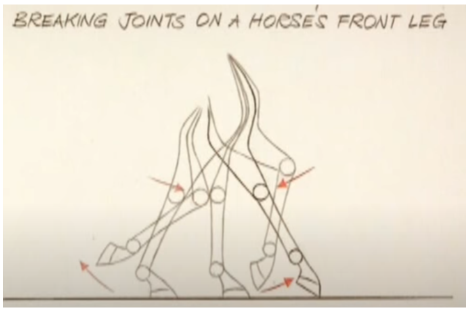
Because there are so many details in the first youtube link, we only catch a few features from it.

1. Maybe we can catch some landmark of the horse's back, and try to adjust it's height.

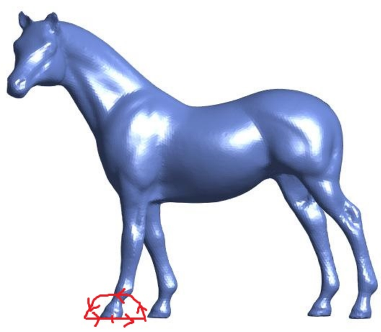




2. The motion of horse's leg requires a lot of landmark to control (see the figure below)

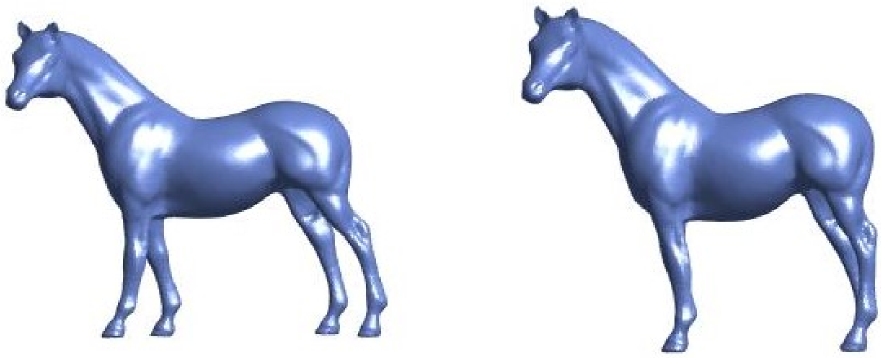


So, we only use an easy way to approach. (As a result, the leg's part looks strange)

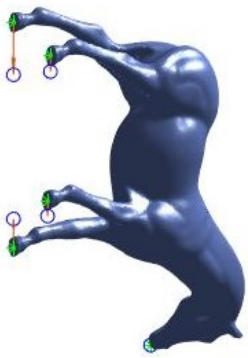


We catch some landmark from the horse's leg, and try to control its leg by using an angle θ.

For convenience (since we want to control its leg by an angle parameter), we first adjust perform mesh editing to get the following result



We only choose 5 landmark in the mesh editing above (see figure below)



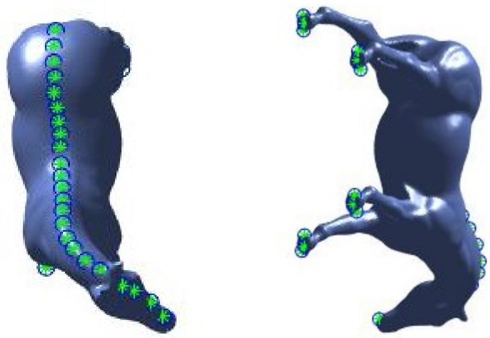
We will use the adjusted mesh data to generate the video.
In order to generate the video, it has 2 main steps

1. Generate 8 mesh data (8 different timestamp) by using mesh editing

$$\{(t_0 = 0, \mathcal{M}_0), (t_1, \mathcal{M}_1), ..., (t_7, \mathcal{M}_7)\}$$

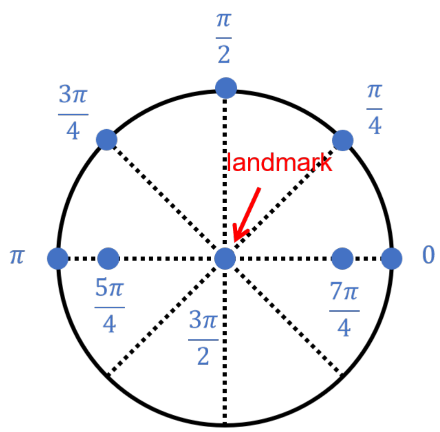
2. Generate the frames by using spline

Step1: As we mention above (some features of horse walking), we choose some landmarks (see the figure below)

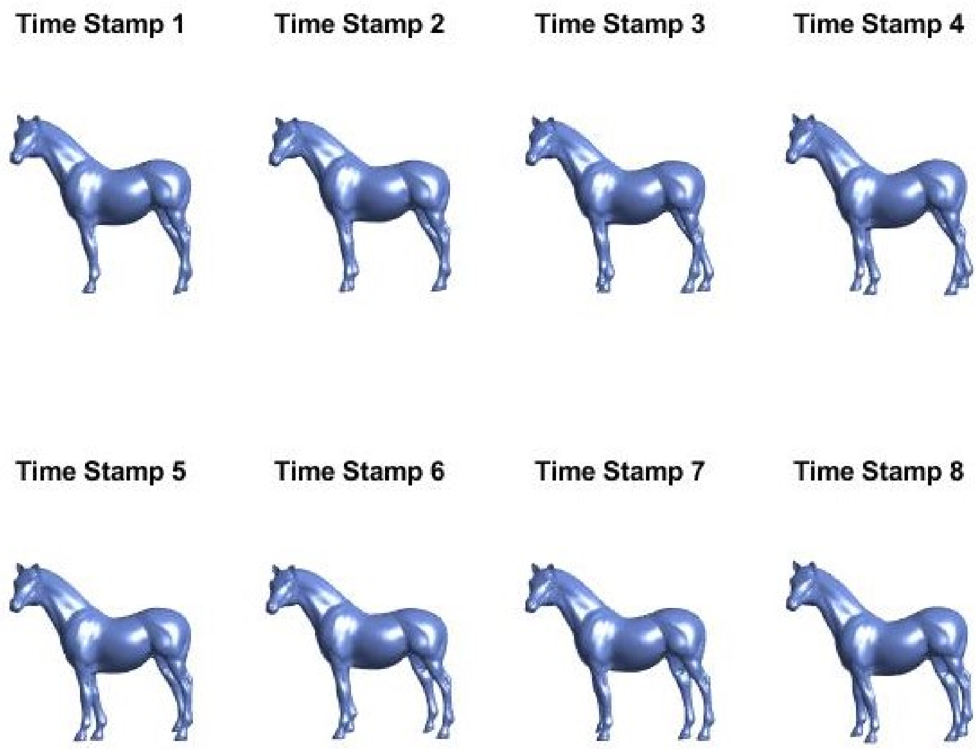


24 landmarks at the horse's back, and 12 landmarks at the bottom of horse's legs.

- **Horse's back part (24 landmarks):**
  By using the idea from figure 2, we pick 3 of the most important landmarks (head, front leg, back leg) from there. Control their displacement (height) by hand, and the rest by linear-interpolation.
- **Horse's leg part (12 landmarks):**
  In order to generate 8 mesh data and control the legs by using a parameter (angle), we may divide 2π into 8 pieces. We update the landmark location by using the following figure



By using these, we generate 8 mesh data



Step2: We generate the frame by using spline and those 8 frames above, the result as follows https://youtu.be/ulX4y9-ar8A.

References

[1] Mei-Heng Yueh, Hsiao-Han Huang, Tiexiang Li, Wen-Wei Lin, and Shing-Tung Yau, Optimized surface parameterizations with applications to Chinese virtual broadcasting (2020).

[2] Mei-Heng Yueh, Xianfeng David Gu, Wen-Wei Lin, Chin-Tien Wu, and Shing-Tung Yau, Conformal Surface Morphing with Applications on Facial Expressions (2015).

[3] Gu Xianfeng David, Yau Shing-Tung, Computational conformal geometry (2008).

[4] https://www.youtube.com/watch?v=INQx-Lzs8mU&t=233s&ab_channel=ChrisMALUKAIChrisMALUKAI

[5] https://www.youtube.com/watch?v=tKpWFly0zGM&ab_channel=DmitryMelchenkoDmitryMelchenko