NTNU 影像處理 HW7,8

廖家緯

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1. Show $f(ax) \Longleftrightarrow \dfrac{1}{|a|}F\left(\dfrac{u}{a}\right)$ (Hint : let $y = ax$)

   Solution:

($\Longrightarrow$) Let $F(u) = \mathcal{F}\{f(x)\}$. WTS : $\mathcal{F}\{f(ax)\} = \dfrac{1}{|a|}F\left(\dfrac{u}{a}\right)$.

Note that: $F(u) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} f(x)e^{-iux}dx$.

Case 1: $a > 0$,

Let $y = ax \Longrightarrow dy = adx$. Then

$$\begin{aligned}
\mathcal{F}\{f(ax)\} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(ax)e^{-iux}dx \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(y)e^{-iu\frac{y}{a}}\left(\frac{1}{a}\right)dy \\
&= \frac{1}{a}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} f(y)e^{-i\frac{u}{a}y}dy\right) \\
&= \frac{1}{a}F\left(\frac{u}{a}\right).
\end{aligned}$$

Case 2: $a < 0$,

Let $y = ax \Longrightarrow dy = adx$. Then

$$\begin{aligned}
\mathcal{F}\{f(ax)\} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(ax)e^{-iux}dx \\
&= \frac{1}{2\pi}\int_{\infty}^{-\infty} f(y)e^{-iu\frac{y}{a}}\left(\frac{1}{a}\right)dy \\
&= \frac{1}{-a}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} f(y)e^{-i\frac{u}{a}y}dy\right) \\
&= \frac{1}{-a}F\left(\frac{u}{a}\right).
\end{aligned}$$

Hence $\mathcal{F}\{f(ax)\} = \frac{1}{|a|}F\left(\frac{u}{a}\right)$ with $a \neq 0$.

($\Longleftarrow$) Let $f(x) = \mathcal{F}^{-1}\{F(u)\}$. WTS : $\mathcal{F}^{-1}\{\frac{1}{|a|}F(\frac{u}{a})\} = f(ax)$.

Note that: $f(x) = \int_{-\infty}^{\infty} F(u)e^{iux}du$.

Case 1: $a > 0$,

Let $\frac{u}{a} = \omega \Longrightarrow \frac{1}{a}du = d\omega$. Then

$$\begin{aligned}\mathcal{F}^{-1}\left\{\frac{1}{|a|}F\left(\frac{u}{a}\right)\right\} &= \int_{-\infty}^{\infty} \frac{1}{a}F\left(\frac{u}{a}\right)e^{iux}du \\ &= \int_{-\infty}^{\infty} \frac{1}{a}F(\omega)e^{ia\omega x}ad\omega \\ &= \int_{-\infty}^{\infty} F(\omega)e^{i\omega(ax)}d\omega \\ &= f(ax)\end{aligned}$$

Case 2: $a < 0$,

Let $\frac{u}{a} = \omega \Longrightarrow \frac{1}{a}du = d\omega$. Then

$$\begin{aligned}\mathcal{F}^{-1}\left\{\frac{1}{|a|}F\left(\frac{u}{a}\right)\right\} &= \int_{-\infty}^{\infty} \frac{1}{-a}F\left(\frac{u}{a}\right)e^{iux}du \\ &= \int_{\infty}^{-\infty} \frac{1}{-a}F(\omega)e^{ia\omega x}ad\omega \\ &= \int_{-\infty}^{\infty} F(\omega)e^{i\omega(ax)}d\omega \\ &= f(ax)\end{aligned}$$

Hence $\mathcal{F}^{-1}\{\frac{1}{|a|}F(\frac{u}{a})\} = f(ax)$ with $a \neq 0$.

2. The FFT presented in the class is called the **successive doubling method (SDM)**. This method assumes that the sizes of images were power of two. However, it is common that the size of an image is not power of two? Address the idea how do you solve this problem using the SDM.

   Solution:

   根據之前的經驗，不足就補 0。這題可以也可以試看看在邊界補 0 至 2 的次方。