NTNU 影像處理 HW13

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1. (a) Apply wavelet transform to the following data sequence

$$[71 \quad 67 \quad 24 \quad 26 \quad 36 \quad 32 \quad 14 \quad 18]$$

using average ($s = \frac{a+b}{2}$) as the low pass filtering and difference ($d = a–s$) as the high pass filtering.

Solution.

Note that

$$s = \frac{a+b}{2}$$

$$d = a - s = a - \frac{a+b}{2} = \frac{a-b}{2}$$

$[71 \quad 67 \quad 24 \quad 26 \quad 36 \quad 32 \quad 14 \quad 18]$

$s_1 = \left[\left(\frac{71+67}{2}\right) \quad \left(\frac{24+26}{2}\right) \quad \left(\frac{36+32}{2}\right) \quad \left(\frac{14+18}{2}\right)\right] = [69 \quad 25 \quad 34 \quad 16]$

$d_1 = \left[\left(\frac{71-67}{2}\right) \quad \left(\frac{24-26}{2}\right) \quad \left(\frac{36-32}{2}\right) \quad \left(\frac{14-18}{2}\right)\right] = [2 \quad -1 \quad 2 \quad -2]$

$v_1 = [69 \quad 25 \quad 34 \quad 16 \quad 2 \quad -1 \quad 2 \quad -2]$

$s_2 = \left[\left(\frac{69+25}{2}\right) \quad \left(\frac{34+16}{2}\right)\right] = [47 \quad 25]$

$d_2 = \left[\left(\frac{69-25}{2}\right) \quad \left(\frac{34-16}{2}\right)\right] = [22 \quad 9]$

$v_2 = [47 \quad 25 \quad 22 \quad 9]$

$s_3 = \left[\left(\frac{47+25}{2}\right)\right] = [36]$

$d_3 = \left[\left(\frac{47-25}{2}\right)\right] = [11]$

$v_3 = [36 \quad 11]$

(b) Recover the input sequence from the result of wavelet transform in (a).

Solution.

$$[s3 \quad d3 \quad d2 \quad d1] = [36 \quad 11 \quad 22 \quad 9 \quad 2 \quad -1 \quad 2 \quad -2]$$

$$\Longrightarrow [(36+11) \quad (36-11) \quad 22 \quad 9 \quad 2 \quad -1 \quad 2 \quad -2]$$

$$= [47 \quad 25 \quad 22 \quad 9 \quad 2 \quad -1 \quad 2 \quad -2]$$

$$\Longrightarrow [([47 \quad 25] + [22 \quad 9]) \quad ([47 \quad 25] - [22 \quad 9]) \quad 2 \quad -1 \quad 2 \quad -2]$$

$$= [[69 \quad 34] \quad [25 \quad 16] \quad 2 \quad -1 \quad 2 \quad -2]$$

$$= [69 \quad 25 \quad 34 \quad 16 \quad 2 \quad -1 \quad 2 \quad -2]$$

$$\Longrightarrow [([69 \quad 25 \quad 34 \quad 16] + [2 \quad -1 \quad 2 \quad -2]) \quad ([69 \quad 25 \quad 34 \quad 16] - [2 \quad -1 \quad 2 \quad -2])]$$

$$= [[71 \quad 24 \quad 36 \quad 14] \quad [67 \quad 26 \quad 32 \quad 18]]$$

$$= [71 \quad 67 \quad 24 \quad 26 \quad 36 \quad 32 \quad 14 \quad 18]$$

2. Repeat 1. for the following data sequence

$$[18 \quad 14 \quad 32 \quad 36 \quad 26 \quad 24 \quad 67 \quad 71].$$

Solution.

Part 1.

$$[18 \quad 14 \quad 32 \quad 36 \quad 26 \quad 24 \quad 67 \quad 71]$$

$$s_1 = \left[\left(\tfrac{18+14}{2}\right) \quad \left(\tfrac{32+36}{2}\right) \quad \left(\tfrac{26+24}{2}\right) \quad \left(\tfrac{67+71}{2}\right)\right] = [16 \quad 34 \quad 25 \quad 69]$$

$$d_1 = \left[\left(\tfrac{18-14}{2}\right) \quad \left(\tfrac{32-36}{2}\right) \quad \left(\tfrac{26-24}{2}\right) \quad \left(\tfrac{67-71}{2}\right)\right] = [2 \quad -2 \quad 1 \quad -2]$$

$$v_1 = [16 \quad 34 \quad 25 \quad 69 \quad 2 \quad -2 \quad 1 \quad -2]$$

$$s_2 = \left[\left(\tfrac{16+34}{2}\right) \quad \left(\tfrac{25+69}{2}\right)\right] = [25 \quad 47]$$

$$d_2 = \left[\left(\tfrac{16-34}{2}\right) \quad \left(\tfrac{25-69}{2}\right)\right] = [-9 \quad -22]$$

$$v_2 = [25 \quad 47 \quad -9 \quad -22]$$

$$s_3 = \left[\left(\tfrac{25+47}{2}\right)\right] = [36]$$

$$d_3 = \left[\left(\tfrac{25-47}{2}\right)\right] = [-11]$$

$$v_3 = [36 \quad -11]$$

Part 2.

$$[s3 \quad d3 \quad d2 \quad d1] = [36 \quad -11 \quad -9 \quad -22 \quad 2 \quad -2 \quad 1 \quad -2]$$

$$\implies [(36 + (-11)) \quad (36 - (-11)) \quad -9 \quad -22 \quad 2 \quad -2 \quad 1 \quad -2]$$

$$= [25 \quad 47 \quad -9 \quad -22 \quad 2 \quad -2 \quad 1 \quad -2]$$

$$\implies [([25 \quad 47] + [-9 \quad -22]) \quad ([25 \quad 47] - [-9 \quad -22]) \quad 2 \quad -2 \quad 1 \quad -2]$$

$$= [[16 \quad 25] \quad [34 \quad 69] \quad 2 \quad -2 \quad 1 \quad -2]$$

$$= [16 \quad 34 \quad 25 \quad 69 \quad 2 \quad -2 \quad 1 \quad -2]$$

$$\implies [([16 \quad 34 \quad 25 \quad 69] + [2 \quad -2 \quad 1 \quad -2]) \quad ([16 \quad 34 \quad 25 \quad 69] - [2 \quad -2 \quad 1 \quad -2])]$$

$$= [[18 \quad 32 \quad 26 \quad 67] \quad [14 \quad 36 \quad 24 \quad 71]]$$

$$= [18 \quad 14 \quad 32 \quad 36 \quad 26 \quad 24 \quad 67 \quad 71]$$