NTNU 影像處理 HW11

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1. Prove $\overline{A \oplus B} = \overline{A} \Theta \widehat{B}$.

   Proof:

$$\overline{A \oplus B} = \overline{\{x | (\widehat{B})_x \cap A \neq \varnothing\}} \text{ (by definition)}$$

$$= \{x | (\widehat{B})_x \cap A = \varnothing\}$$

$$= \{x | (\widehat{B})_x \subseteq \overline{A}\}$$

$$= \overline{A} \Theta \widehat{B} \text{ (by definition)}$$

2. Prove $\overline{A \circ B} = \overline{A} \bullet \widehat{B}$.

   Proof:

$$\overline{A \circ B} = \overline{(A \Theta B) \oplus B} \text{ (by definition)}$$

$$= \overline{(A \Theta B)} \Theta \widehat{B} \text{ (by 1.)}$$

$$= (\overline{A} \oplus \widehat{B}) \Theta \widehat{B} \text{ (by property)}$$

$$= \overline{A} \bullet \widehat{B} \text{ (by definition)}$$

3. Prove if $A \subseteq C$, then $(A \circ B) \subseteq (C \circ B)$.

   Proof:

   Let $x \in (A \circ B)$. By definition, $x \in (A \Theta B) \oplus B$.

   Then $x = p + q$ for some $p \in (A \Theta B)$ and $q \in B$

   Note that

   $\because p \in (A \Theta B) \quad \therefore$ by definition, $p \in A$ and $B_p \subseteq A$

   Moreover, $\because A \subseteq C \quad \therefore p \in C$ and $B_p \subseteq C \Longrightarrow p \in (C \Theta B)$

   Hence $x = p + q$ for some $p \in (C \Theta B)$ and $q \in B$

   $\Longrightarrow x \in (C \Theta B) \oplus B$ (by definition)

   $\Longrightarrow x \in (C \circ B)$ (by definition)

   Therefore, $(A \circ B) \subseteq (C \circ B)$.

4. Prove if $A \subseteq C$, then $(A \bullet B) \subseteq (C \bullet B)$.

   Lemma.

   If $A \subseteq C$, then $(A \oplus B) \subseteq (C \oplus B)$.

   Proof:

   Let $x \in (A \oplus B)$. Then $x = p + q$ for some $p \in A$ and $q \in B$

   $\because A \subseteq C \quad \therefore p \in C$ and $q \in B \quad \Longrightarrow x = p + q$ for some $p \in C$ and $q \in B$.

   Hence $x \in (C \oplus B)$. Therefore, $(A \oplus B) \subseteq (C \oplus B)$

   Proof:

   Let $x \in (A \bullet B)$. By definition, $x \in (A \oplus B) \Theta B$.

   Then $x \in (A \oplus B)$ and $B_x \subseteq (A \oplus B)$

   Moreover, $\because A \subseteq C \quad \therefore$ by Lemma, $x \in (C \oplus B)$ and $B_x \subseteq (C \oplus B)$

   $\Longrightarrow x \in (C \oplus B) \Theta B$. (by definition)

   $\Longrightarrow x \in (C \bullet B)$ (by definition)

   Therefore, $(A \bullet B) \subseteq (C \bullet B)$.