Numerical Methods for Partial Differential Equations H.W.4

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- **Exercise**

  Given

$$\begin{cases} u'' = \sin \pi x, \\ u(0) = 0, \quad u(1) = 0 \end{cases}.$$

  Use method 1 and method 2 to compute U_j.

- **Code**

  1. **Thomas Algorithm function**

```
function x = Thomas(A, d, xn)
n = size(A,1);
a = [0; diag(A,-1)];
b = diag(A);
c = diag(A,1);
x = zeros(n,1);

for i = 2:n
    b(i) = b(i)-a(i)*c(i-1)/b(i-1);
    d(i) = d(i)-a(i)*d(i-1)/b(i-1);
end

if nargin ==3
    x(n) = xn;

else
    x(n) = d(n)/b(n);
end

for i = n-1:-1:1
    x(i) = (d(i)-c(i)*x(i+1))/b(i);
end
```

2. Finite Difference function

```
function U = FDM(m, x, f, alpha, beta, method)
h = (x(end)-x(1))/(m+1);

U = ones(m+2,1);
U(1) = alpha; U(end) = beta;

A = diag(-2*ones(m,1)) + diag(ones(m-1,1), 1) + diag(ones(m-1,1), -1);
Ah = A/h^2;

F = [0; f(x(2:end-1))'; 0];

% 4th order convergence
if nargin == 6
    F(2:end-1) = (F(1:end-2) + 10*F(2:end-1) + F(3:end))/12;
    F(2) = F(2)+alpha/h^2;
    F(end-1) = F(end-1)-beta/h^2;
end

F = F(2:end-1);
U(2:end-1) = Thomas(Ah, F);

end
```

3. PlotLogError

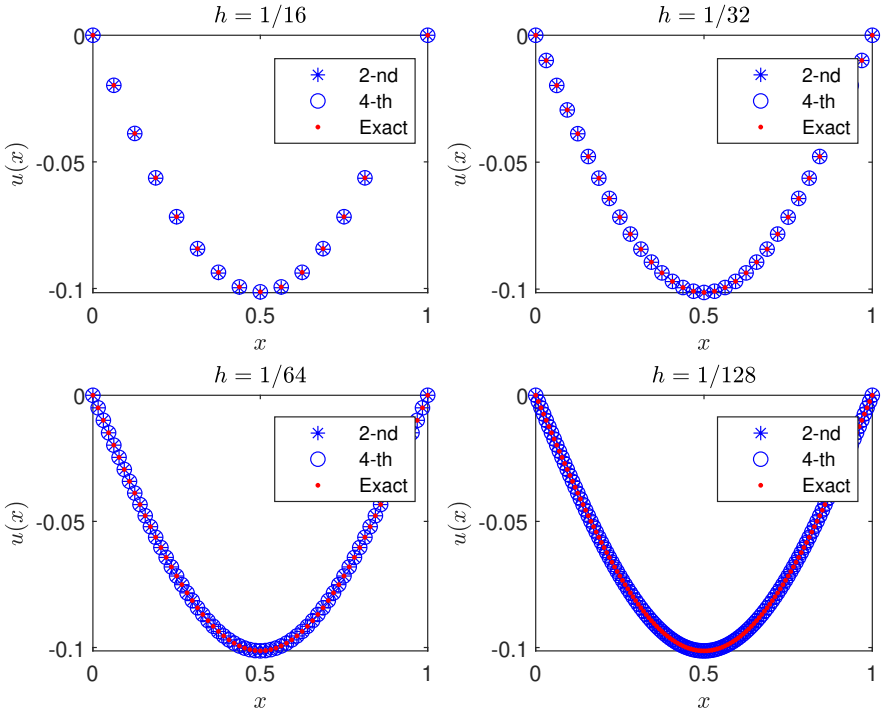
```
function PlotLogError(mList, ErrorList)
loglog(mList, ErrorList, '-ro', 'LineWidth', 1.2);
axis([min(mList), max(mList), ...
    min(ErrorList), max(ErrorList)]);
xlabel('$\log h $', 'interpreter', 'latex');
ylabel('$\log e$', 'interpreter', 'latex');

end
```

4. Main function

```
clc; clear; close all;
u = @(x) -sin(pi*x)/pi^2;
f = @(x) sin(pi*x);
x0 = 0;    xm_1 = 1;
alpha = 0; beta = 0;

%% FDM
mList = [15 ,31, 63, 127];
mListLength = length(mList);
ErrorList = zeros(mListLength,2);
RatioList = zeros(mListLength-1,2);

figure(1);
for i = 1:mListLength
    m = mList(i);
```

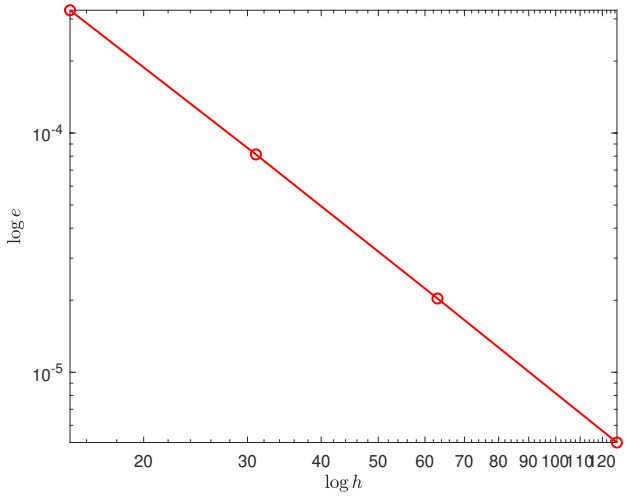
```
    h = (xm_1-x0)/(m+1); x = x0:h:xm_1;

    % Exact solution
    U_hat = u(x');

    % 2nd order convergence
    U1 = FDM(m, x, f, alpha, beta);

    % 4th order convergence
    U2 = FDM(m, x, f, alpha, beta, 4);

    % Compute error with infinity norm
    ErrorList(i,:) = max(abs([U1-U_hat, U2-U_hat]));

    % Plot the numerical solution and exact solution
    subplot(2,mListLength/2,i);
    plot(x, U1, 'b*',x , U2, 'bo', x, U_hat, 'r.');
    title(['$h=1/$', int2str(m+1)], 'interpreter', 'latex');
    xlabel('$x$', 'interpreter', 'latex');
    ylabel('$u(x)$', 'interpreter', 'latex');
    legend('2-nd', '4-th', 'Exact', 'Location', 'best');

    if i > 1
        RatioList(i,:) = log2(ErrorList(i-1,:)./ErrorList(i,:));
    end

end
hold off;

%% Plot error with loglog
figure(2);
subplot(121); PlotLogError(mList, ErrorList(:,1))
subplot(122); PlotLogError(mList, ErrorList(:,2))
```

- **Numerical result**



1. **Method 1 (2-nd convergence):**

h	Error: $\|U - \hat{U}\|_\infty$	$\log_2$ Ratio
$\frac{1}{16}$	3.2615e-05	
$\frac{1}{32}$	8.1419e-06	2.0021
$\frac{1}{64}$	2.0348e-06	2.0005
$\frac{1}{128}$	5.0864e-07	2.0001



2. **Method 2 (4-th convergence):**

h	Error: $\|U - \hat{U}\|_\infty$	$\log_2$ Ratio
$\frac{1}{16}$	6.2845e-07	
$\frac{1}{32}$	3.9233e-08	4.0017
$\frac{1}{64}$	2.4514e-09	4.0004
$\frac{1}{128}$	1.5320e-10	4.0001