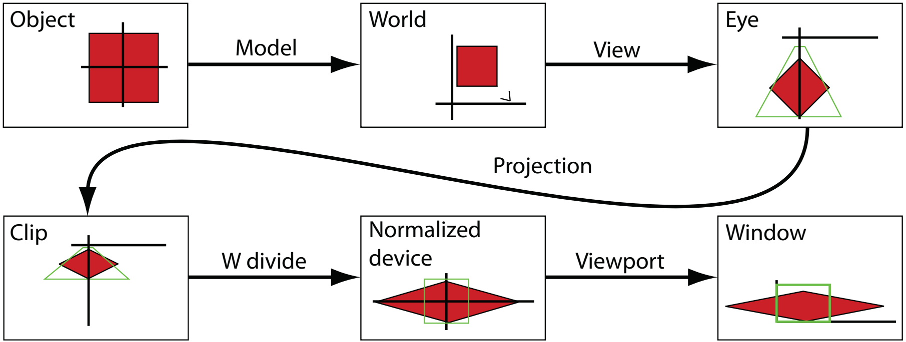
02561 Computer Graphics

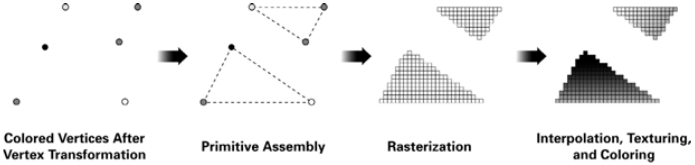
Model, view, projection

Jeppe Revall Frisvad

September 2024

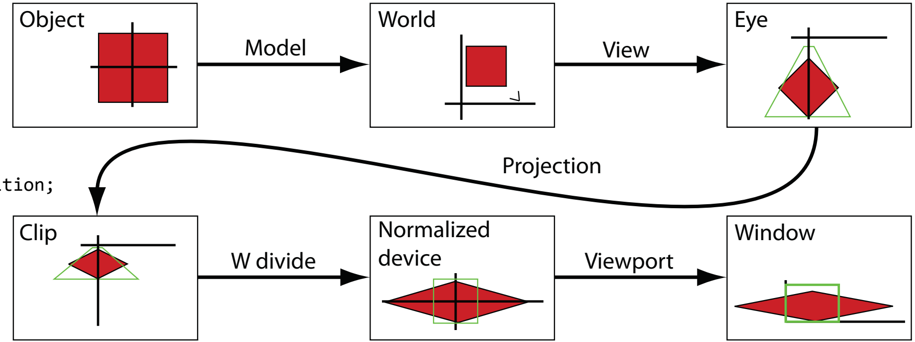
Rasterization pipeline





Rasterization pipeline

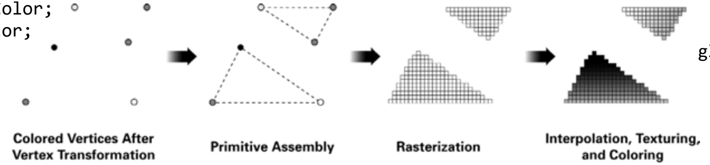
```
attribute vec4 vPosition;
```




```
gl_Position
= P*V*M*vPosition;
```

```
gl_FragCoord
```

```
attribute vec4 vColor;
varying vec4 fColor;
fColor = vColor;
```




```
varying vec4 fColor;
gl_FragColor = fColor;
```

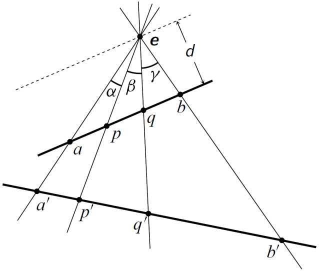
vertex shader

fragment shader

Why 4-vectors and what is w?

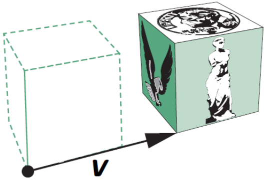
- As with curves (Week 3), we can include more advanced (rational) transformations in a matrix representation if we use homogeneous coordinates.
- Homogeneous coordinates: add a w-coordinate that we divide by in the end. In this projective space, we have vectors $(x, y, z, w) \mapsto (\frac{x}{w}, \frac{y}{w}, \frac{z}{w})$.
  - Position vectors: $(x, y, z, 1)$.
  - Direction vectors: $(x, y, z, 0)$ (at infinity and thus invariant under translation).
- Points along a straight line passing through the origin (the origin excluded) are equivalent in projective space (equivalence relation):

$$(x, y, z, 1) \sim (wx, wy, wz, w)\,, \quad \text{for } w \neq 0\,.$$



- Perspective is mapping of 3D shapes to a surface. This is what a camera does.
- If we move our virtual camera to the origin ($\boldsymbol{e}$) and rotate the coordinate system to the basis of the image plane, we can use w to do perspective projection (to d).

Transformation matrices



- Translation of a point $\boldsymbol{x}$ along a vector $\boldsymbol{v}$ to a point $\boldsymbol{x}'$:

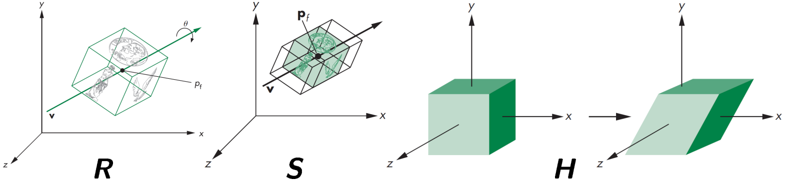
$$\begin{bmatrix} \boldsymbol{x}' \\ 1 \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} \boldsymbol{x} \\ 1 \end{bmatrix}, \quad \boldsymbol{T} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{v} \\ \boldsymbol{0} & 1 \end{bmatrix},$$

  $\boldsymbol{T} \in \mathbb{R}^{4\times 4}$ is a translation matrix and
  $\boldsymbol{I} \in \mathbb{R}^{3\times 3}$ is an identity matrix.

- Other transformations for which $\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x}$:

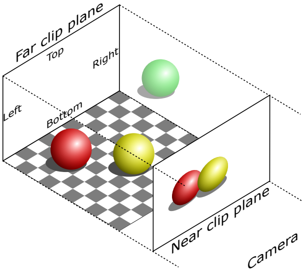
$$\begin{bmatrix} \boldsymbol{x}' \\ 1 \end{bmatrix} = \boldsymbol{B} \begin{bmatrix} \boldsymbol{x} \\ 1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & 1 \end{bmatrix},$$

- $\boldsymbol{A} \in \mathbb{R}^{3\times 3}$ is a rotation matrix ($\boldsymbol{R}$), scaling matrix ($\boldsymbol{S}$), or shearing matrix ($\boldsymbol{H}$).

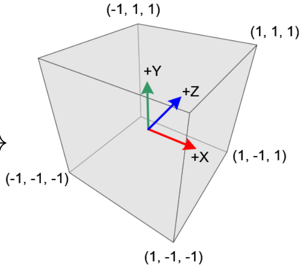


Normalized device coordinates (NDC)

- If we use identity matrices for all vertex shader transformations ($\boldsymbol{M} = \boldsymbol{V} = \boldsymbol{P} = \boldsymbol{I}$) and $w = 1$, we are effectively working in NDC space.



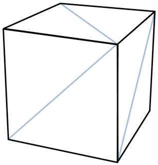
$\leftrightarrow$

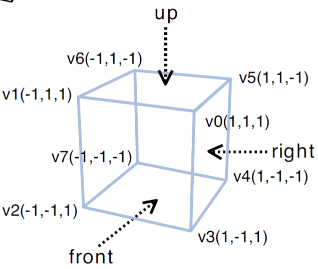


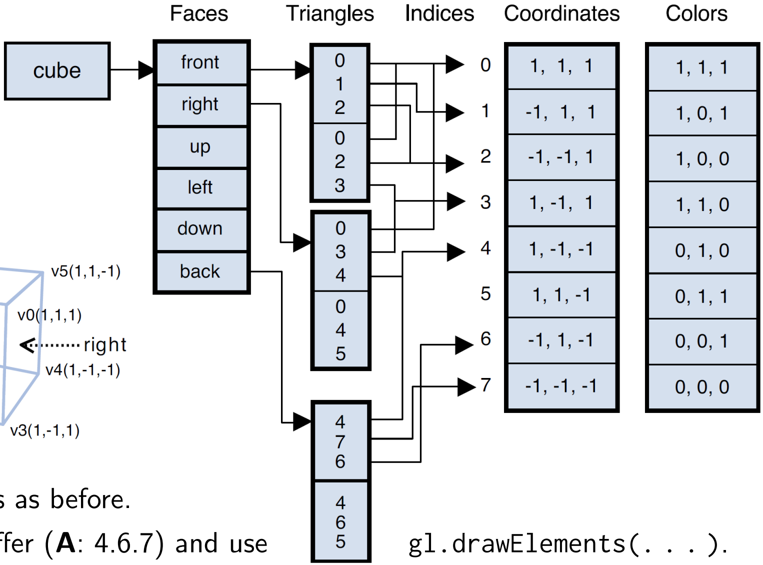
default
view volume
(NDC cube)

- We have so far used $\boldsymbol{M} = \boldsymbol{V} = \boldsymbol{P} = \boldsymbol{I}$ and $w = 1$ (no transformation).
- The image plane is then $z = -1$ and no geometry outside the NDC cube is drawn.
- Any standard projection matrix $\boldsymbol{P}$ (perspective or orthographic) flips the sign of the z-coordinate to have a right-handed coordinate system. The view direction in eye space is thus normally the negative z-axis.

Drawing a cube using an indexed face set





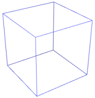


- Positions and colors as before.
- Create an index buffer (**A**: 4.6.7) and use `gl.drawElements(. . . )`.

The indexed face set of a cube in JavaScript code

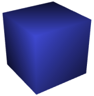
```
// Create a cube
//    v5----- v6
//   /|      /|
//  v1------v2|
//  | |     | |
//  | |v4---|-|v7
//  |/      |/
//  v0------v3

// Wireframe indices
var wire_indices = new Uint32Array([
  0, 1, 1, 2, 2, 3, 3, 0, // front
  2, 3, 3, 7, 7, 6, 6, 2, // right
  0, 3, 3, 7, 7, 4, 4, 0, // down
  1, 2, 2, 6, 6, 5, 5, 1, // up
  4, 5, 5, 6, 6, 7, 7, 4, // back
  0, 1, 1, 5, 5, 4, 4, 0  // left
]);
```



```
var vertices = [
  vec3(0.0, 0.0, 1.0),
  vec3(0.0, 1.0, 1.0),
  vec3(1.0, 1.0, 1.0),
  vec3(1.0, 0.0, 1.0),
  vec3(0.0, 0.0, 0.0),
  vec3(0.0, 1.0, 0.0),
  vec3(1.0, 1.0, 0.0),
  vec3(1.0, 0.0, 0.0),
];

// Triangle mesh indices
var indices = new Uint32Array([
  1, 0, 3, 3, 2, 1, // front
  2, 3, 7, 7, 6, 2, // right
  3, 0, 4, 4, 7, 3, // down
  6, 5, 1, 1, 2, 6, // up
  4, 5, 6, 6, 7, 4, // back
  5, 4, 0, 0, 1, 5  // left
]);
```



Setting up buffers and drawing an indexed face set (WebGL)

- Enabling use of 32-bit unsigned integers as WebGL indices:

```
var ext = gl.getExtension('OES_element_index_uint');
if (!ext) { console.log('Warning: Unable to use an extension'); }
```

- Setting up vertex position buffer and index buffer:

```
var iBuffer = gl.createBuffer();
gl.bindBuffer(gl.ELEMENT_ARRAY_BUFFER, iBuffer);
gl.bufferData(gl.ELEMENT_ARRAY_BUFFER, new Uint32Array(wire_indices), gl.STATIC_DRAW);

var vBuffer = gl.createBuffer();
gl.bindBuffer(gl.ARRAY_BUFFER, vBuffer);
gl.bufferData(gl.ARRAY_BUFFER, flatten(vertices), gl.STATIC_DRAW);

var vPosition = gl.getAttribLocation(program, "a_Position");
gl.vertexAttribPointer(vPosition, 3, gl.FLOAT, false, 0, 0);
gl.enableVertexAttribArray(vPosition);
```

- Drawing with an indexed face set (`gl.LINES` for wireframe):

```
gl.drawElements(gl.LINES, wire_indices.length, gl.UNSIGNED_INT, 0);
```

- Use `gl.TRIANGLES` to draw full triangles:

```
gl.drawElements(gl.TRIANGLES, indices.length, gl.UNSIGNED_INT, 0);
```

Setting up buffers for an indexed face set (WebGPU)

```
var obj = new Object();

obj.vPositionBuffer = device.createBuffer({
  size: flatten(vertices).byteLength,
  usage: GPUBufferUsage.VERTEX | GPUBufferUsage.COPY_DST,
});
device.queue.writeBuffer(obj.vPositionBuffer, 0, flatten(vertices));

obj.vPositionBufferLayout = {
  arrayStride: sizeof["vec3"],
  attributes: [{
    format: "float32x3",
    offset: 0,
    shaderLocation: 0,
  }],
};

obj.indicesBuffer = device.createBuffer({
  size: wire_indices.byteLength,
  usage: GPUBufferUsage.INDEX | GPUBufferUsage.COPY_DST,
});
device.queue.writeBuffer(obj.indicesBuffer, 0, wire_indices);
```

- Note how the object (obj) is dynamically extended with new data fields.

Pipeline for wireframe drawing (WebGPU)

```
const pipeline = device.createRenderPipeline({
  layout: "auto",
  vertex: {
    module: wgsl,
    entryPoint: "main_vs",
    buffers: [obj.vPositionBufferLayout]
  },
  fragment: {
    module: wgsl,
    entryPoint: "main_fs",
    targets: [{ format: canvasFormat }]
  },
  primitive: {
    topology: "line-list",
    // GPUPrimitiveTopology { "point-list", "line-list", "line-strip", "triangle-list", "triangle-strip" };
  },
  multisample: {
    count: msaaCount,
  }
});
```

- Note that we ask for multi-sampling, which is enabled by default in WebGL.
- Set the variable `msaaCount` to 4. This is the value most commonly supported.

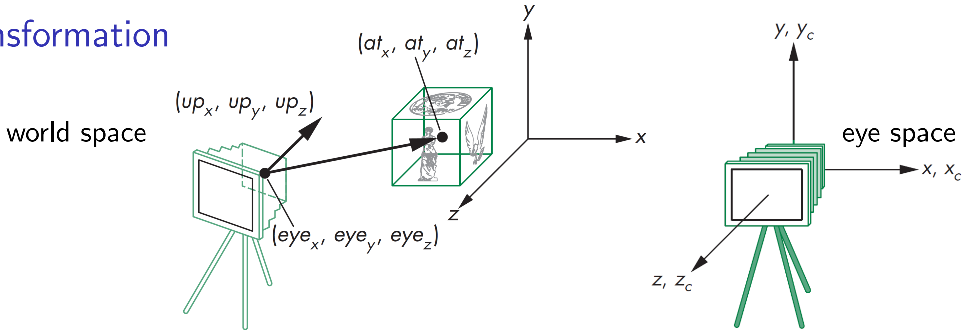
Drawing an indexed face set with multi-sampling (WebGPU)

```
const msaaTexture = device.createTexture({
  size: { width: canvas.width, height: canvas.height },
  format: canvasFormat,
  sampleCount: msaaCount,
  usage: GPUTextureUsage.RENDER_ATTACHMENT,
});

const encoder = device.createCommandEncoder();
const pass = encoder.beginRenderPass({
  colorAttachments: [{
     view: msaaTexture.createView(),
     resolveTarget: context.getCurrentTexture().createView(),
     loadOp: "clear",
     clearValue: { r: 1.0, g: 1.0, b: 1.0, a: 1.0 },
     storeOp: "store",
  }]
});
pass.setPipeline(pipeline);
pass.setVertexBuffer(0, obj.vPositionBuffer);
pass.setIndexBuffer(obj.indicesBuffer, 'uint32');
pass.setBindGroup(0, bindGroup);
pass.drawIndexed(wire_indices.length);
pass.end();
device.queue.submit([encoder.finish()]);
```

- Note that `pass.drawIndexed` is used instead of `pass.draw`.

View transformation



- Eye space has the camera at the origin looking down the negative z-axis.
- The view transformation performs a change of coordinates.
- Given eye point $\boldsymbol{e}$, look-at point $\boldsymbol{a}$, and up vector $\boldsymbol{u}$ in world space, the basis vectors of eye space in world space coordinates are

$$\vec{b}_1 = \frac{\boldsymbol{u} \times \vec{b}_3}{\|\boldsymbol{u} \times \vec{b}_3\|}, \qquad \vec{b}_2 = \vec{b}_3 \times \vec{b}_1, \qquad \vec{b}_3 = \frac{\boldsymbol{e} - \boldsymbol{a}}{\|\boldsymbol{e} - \boldsymbol{a}\|}.$$

Change of coordinates

- Given basis vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3 \in \mathbb{R}^{3\times 1}$ of space a in coordinates of space b, the change of basis matrix ${}_b\boldsymbol{M}_a \in \mathbb{R}^{3\times 3}$ from a to b is

$$ {}_b\boldsymbol{M}_a = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix}. $$

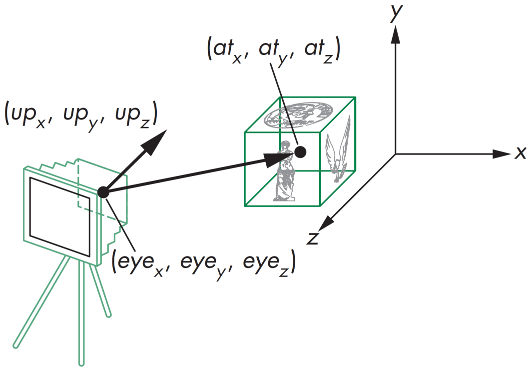
- If the basis is orthonormal: ${}_a\boldsymbol{M}_b = {}_b\boldsymbol{M}_a^{-1} = {}_b\boldsymbol{M}_a^T$.
- Change of coordinates is translation and rotation: $\boldsymbol{V} = \boldsymbol{RT} = \begin{bmatrix} \vec{b}_1^T & -\boldsymbol{e}\cdot\vec{b}_1 \\ \vec{b}_2^T & -\boldsymbol{e}\cdot\vec{b}_2 \\ \vec{b}_3^T & -\boldsymbol{e}\cdot\vec{b}_3 \\ \boldsymbol{0} & 1 \end{bmatrix}.$
  - Translation to displace geometry so that it is positioned relative to the origin as it was previously positioned relative to the camera:

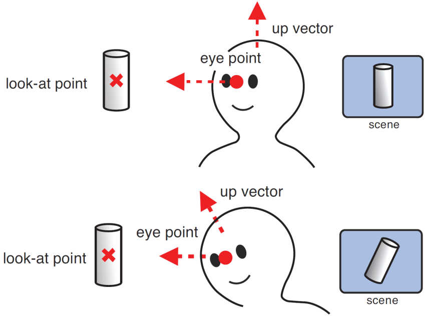
$$ \boldsymbol{T} = \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{e} \\ \boldsymbol{0} & 1 \end{bmatrix}. $$

  - Rotation to perform change of basis from world space to eye space:

$$ \boldsymbol{R} = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix}^T. $$

The look-at function





- To configure camera extrinsics (position and orientation), we use an eye point $\boldsymbol{e}$, a look-at point $\boldsymbol{a}$, and an up vector $\boldsymbol{u}$:

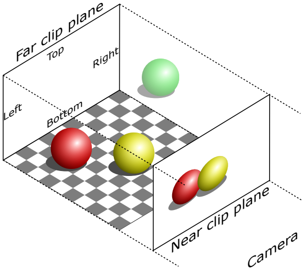
```
var V = lookAt(eye, at, up);
```

- Send to shader using

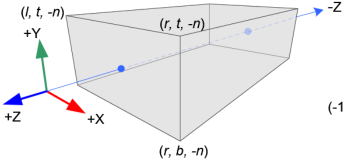
```
gl.uniformMatrix4fv(VLoc, false, flatten(V));
```

Orthographic projection

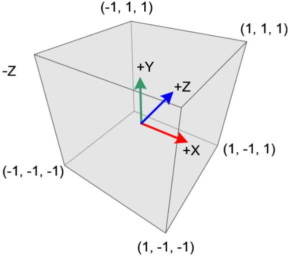
- Selecting an arbitrary view volume



orthographic projection



view volume



NDC cube

- The ortho function creates a projection matrix transforming from an orthographic view volume to the NDC cube.

  var P = ortho(left, right, bottom, top, near, far);

  $\boldsymbol{P}$ $\quad l \quad r \quad b \quad t \quad n \quad f$

$$\boldsymbol{P} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Send to shader using

```
gl.uniformMatrix4fv(PLoc, false, flatten(P));
```

Sending a uniform matrix to the GPU using WebGPU

```javascript
// NDC coordinates in WebGPU are in [-1,1]x[-1,1]x[0,1]
const projection = mat4(1.0, 0.0,  0.0, 0.0,
                        0.0, 1.0,  0.0, 0.0,
                        0.0, 0.0, -0.5, 0.5,
                        0.0, 0.0,  0.0, 1.0);
const view = lookAt(eye, lookat, up);
const mvp = mult(projection, view);

const uniformBuffer = device.createBuffer({
  size: sizeof['mat4'],
  usage: GPUBufferUsage.UNIFORM | GPUBufferUsage.COPY_DST,
});
const bindGroup = device.createBindGroup({
  layout: pipeline.getBindGroupLayout(0),
  entries: [{
    binding: 0,
    resource: { buffer: uniformBuffer }
  }],
});
device.queue.writeBuffer(uniformBuffer, 0, flatten(mvp));
```

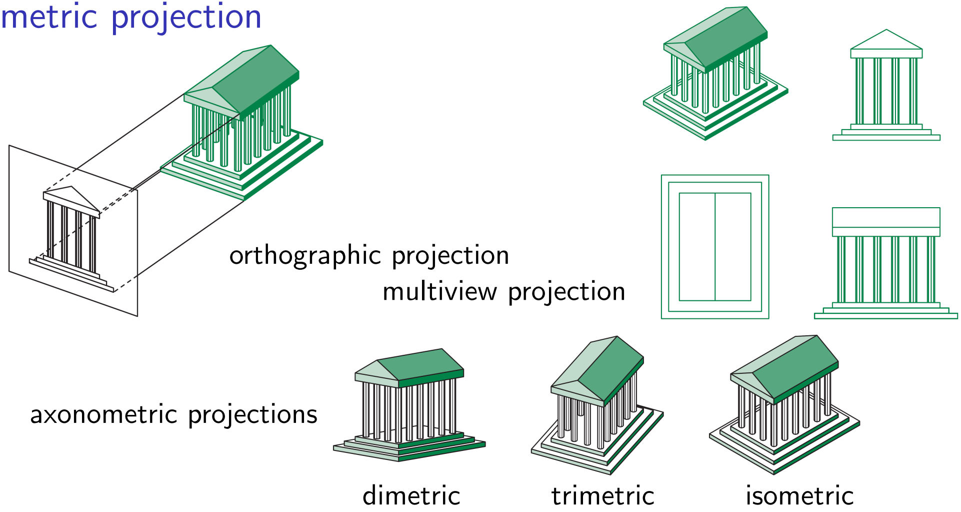
JavaScript

```wgsl
struct Uniforms {
  mvp: mat4x4f,
};
@group(0) @binding(0)
var<uniform> uniforms: Uniforms;

@vertex
fn main_vs(@location(0) inPos: vec4f)
  -> @builtin(position) vec4f
{
  return uniforms.mvp*inPos;
}
```

WGSL

Axonometric projection



- Axonometric views are about symmetry and foreshortening of distances in the three principal directions around a corner.
  A cube in isometric view is seen with three edges of equal length meeting at a corner.

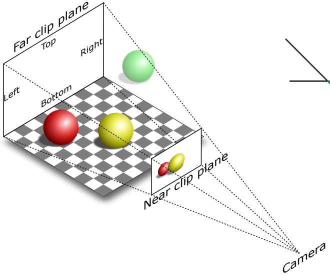
Exercise: isometric view of a wireframe cube (W03P1)

- Draw a cube using an indexed face set (**A**: 4.6.1 and 4.6.7).
- Functions are available for building transformation matrices (**A**: 4.11.2–4.11.3):

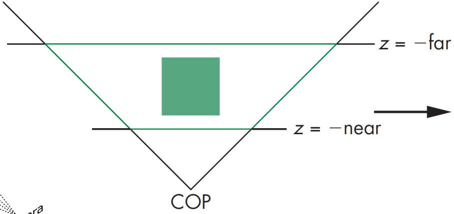
```
var I = mat4();      // identity matrix
var R = rotate(angle, direction);
var Rx = rotateX(angle);
var Ry = rotateY(angle);
var Rz = rotateZ(angle);
var S = scalem(s_x, s_y, s_z);
var T = translate(t_x, t_y, t_z);
var c = mult(a, b);  // c = a*b
```

- Use a model matrix to scale and translate the cube (**A**: 4.9) so that its diagonal is from $(0,0,0)$ to $(1,1,1)$, or set the coordinates of the vertex positions.
- Draw a wireframe cube by using different indices for the indexed face set and using the draw mode `gl.LINES` instead of `gl.TRIANGLES`.
- Use the `lookAt` function to construct a view matrix (**A**: 5.3.3) so that the wireframe cube is rendered in isometric view.

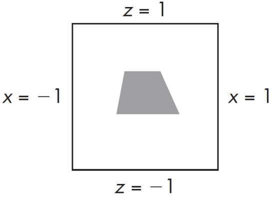
Perspective projection



perspective projection



clip space



NDC cube

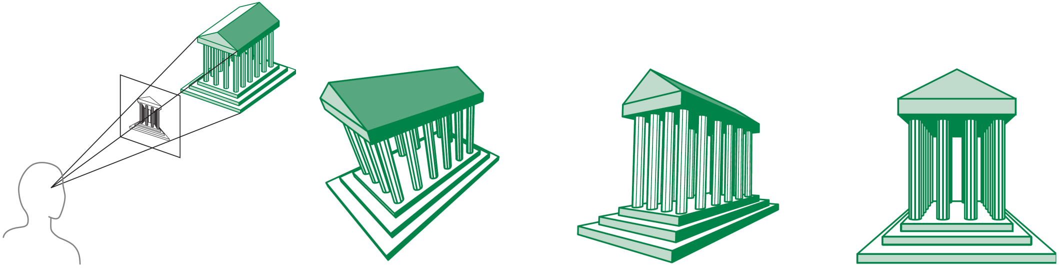
- The perspective function creates a projection matrix transforming the view frustum to clip space. The frustum becomes the NDC cube after w-divide.

$$\begin{array}{llllll} \textcolor{blue}{\text{var}}\ \text{P} = \text{perspective(fovy,} & \text{aspect,} & \text{near,} & \text{far);} \\ \quad \boldsymbol{P} \qquad\qquad\qquad \alpha & A & n & f \end{array}$$

$$\boldsymbol{P} = \begin{bmatrix} \frac{1}{A}\cot\frac{\alpha}{2} & 0 & 0 & 0 \\ 0 & \cot\frac{\alpha}{2} & 0 & 0 \\ 0 & 0 & \frac{n+f}{n-f} & \frac{2nf}{n-f} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- α is the vertical field of view (angle in degrees)
- $A = \frac{w}{h}$ is the aspect ratio of the canvas.

Classical perspective views



perspective view: three-point two-point one-point

- Classical perspective views are about the number of principal directions in the image with vanishing points.
  - 1-point: one vanishing point, two principal directions parallel to the image plane.
  - 2-point: two vanishing points, one principal direction parallel to the image plane.
  - 3-point: three vanishing points, no principal directions parallel to the image plane.
- When drawing a cube, look for edges that are parallel in the image.

Model, view, and projection matrices

- Recommended mode of operation:
  - The projection matrix $\boldsymbol{P}$ depends on camera intrinsics and is set during initialization.
  - The view matrix $\boldsymbol{V}$ depends on camera extrinsics and is set during animation.
  - the model matrix $\boldsymbol{M}$ places objects in the scene and is set during rendering.
- We can use the same vertex buffer to draw multiple instances of an object.
- This simply requires a different model matrix for each instance.
- Before making a draw call, we set the model matrix of the instance to be rendered.
- The order of multiplication of matrices is important (matrix multiplication is not commutative):

$$
\begin{aligned}
\boldsymbol{x}_{\mathrm{world}} &= \boldsymbol{M}\,\boldsymbol{x}_{\mathrm{model}} \\
\boldsymbol{x}_{\mathrm{view}} &= \boldsymbol{V}\,\boldsymbol{x}_{\mathrm{world}} \\
\boldsymbol{x}_{\mathrm{clip}} &= \boldsymbol{P}\,\boldsymbol{x}_{\mathrm{view}} \\
\boldsymbol{x}_{\mathrm{clip}} &= \boldsymbol{PVM}\,\boldsymbol{x}_{\mathrm{model}}\,,
\end{aligned}
$$

Drawing multiple instances of the same object

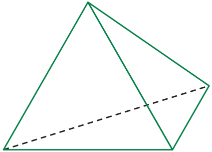
- Since WebGL is a state machine, the work flow after setting projection and view matrices is to repeatedly
  - Set the model matrix and send it to the GPU as a uniform variable.
  - Call a draw function (e.g. `drawElements`) to draw an instance.
- WebGPU is not a state machine. The work flow is instead
  - Define the model matrices for the different instances.
  - Set up a uniform buffer and a bind group for each instance.
  - In the render pass, write the uniform buffer to the GPU, set the bind group, and call a draw function (e.g. `drawIndexed`) for each instance.
- Switching uniform buffer and bind group is inefficient.
- We can make an `array<mat4x4f,`N`>` of uniform model matrices and use *instancing*.
- A second argument in the draw call tells the GPU how many instances we would like to draw: `pass.drawIndexed(wire_indices.length,`N`);`
- A second argument in the vertex shader tells us what model matrix to use:

```
@vertex fn main_vs(@location(0) inPos: vec4f, @builtin(instance_index) instance: u32) -> @builtin(position) vec4f { … }
```

Exercise: perspective views of three wireframe cubes (W03P2)

- Return to your program that draws a wireframe cube in isometric view.
- Use the `perspective` function to construct a projection matrix (**A**: 5.6.1) so that the cube is in a view frustum with a 45° vertical field of view.
- Use the `lookAt` function to construct a view matrix (**A**: 5.3.3) so that the cube is rendered in one-point perspective.
- Use rotation and translation matrices to construct model matrices for rendering three instances of the cube in one-, two-, and three-point perspectives.

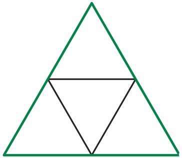
Drawing a sphere using subdivision (**A**: 6.6)



- Take a tetrahedron (3D simplex) with vertex positions:

  $$(0,0,1),\ (0,\tfrac{2\sqrt{2}}{3},-\tfrac{1}{3}),\ (-\tfrac{\sqrt{6}}{3},-\tfrac{\sqrt{2}}{3},-\tfrac{1}{3}),\ (\tfrac{\sqrt{6}}{3},-\tfrac{\sqrt{2}}{3},-\tfrac{1}{3}).$$

- These are points on the unit sphere with center at the origin.
- Do Loop subdivision of the triangles:
  For two vertices $\boldsymbol{a}$ and $\boldsymbol{b}$ the edge midpoint is $\boldsymbol{c}' = \dfrac{\boldsymbol{a}+\boldsymbol{b}}{2}$.



- Normalize the new vertex positions
  to push them back onto the unit sphere: $\boldsymbol{c}' = \dfrac{\boldsymbol{a}+\boldsymbol{b}}{\|\boldsymbol{a}+\boldsymbol{b}\|}$.
- Subdivide each triangle while pushing all vertex positions into an array.
- Do n recursions.