G02_HW13

Group 02
HW 13
2019/12/10

ID	Name	Your works	Times you spend	Self score	TA
108202529	葉揚昀	Least squares Fit (linear & guess)	8hr	7	
108202009	田家瑋	Chi-square Fit-practice	5hr	4	
108202016	張家菖	Design coupled oscillator	3hr	7	

Least squares

1. Definition

Least-squares problems fall into two categories: linear or ordinary least squares (OLS) and nonlinear least squares. Either OLS or Non-linear least squares obtain parameter estimates that minimize the sum of squared residuals (SSR).

The residual, e_i, is the difference between the value of the dependent variable predicted by the model, $\hat{y}$, and the true value of the dependent variable, y_i :

$$e_i = y_i - \hat{y}_i \qquad (1)$$

Use eq.(1) the sum of squared residuals (SSR) expressed:

$$SSR = \sum_{i=1}^{n} e_i^2 \qquad (2)$$

The least-squares method minimize sum of squared residuals (SSR)

Least squares

Two categories of least-squares

Least-squares

- Ordinary least squares (OLS) (linear least squares)
- Non-linear least squares

2. Ordinary least squares (OLS)

ordinary least squares (OLS) is a type of linear least squares method for estimating the unknown parameters in a linear regression model.

Eg.

OLS

- Straight line: $y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \ \ i = 1, \ldots, n$
- Parabola: $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i, \ \ i = 1, \ldots, n$

(3)

In eq.(3) ϵ_i is an error term and the subscript i indexes a particular observation.

Least squares

A linear least squares example 1:

If:

- The data points are (0 , 2.1), (1 , 3.3), (2 , 3.9), (3 , 4.7).
- Assume the model function is $y_i = \beta_0 + \beta_1 x_i$

Use linear least squares to evaluate: $\beta_0 = 2.03, \quad \beta_1 = 0.98$

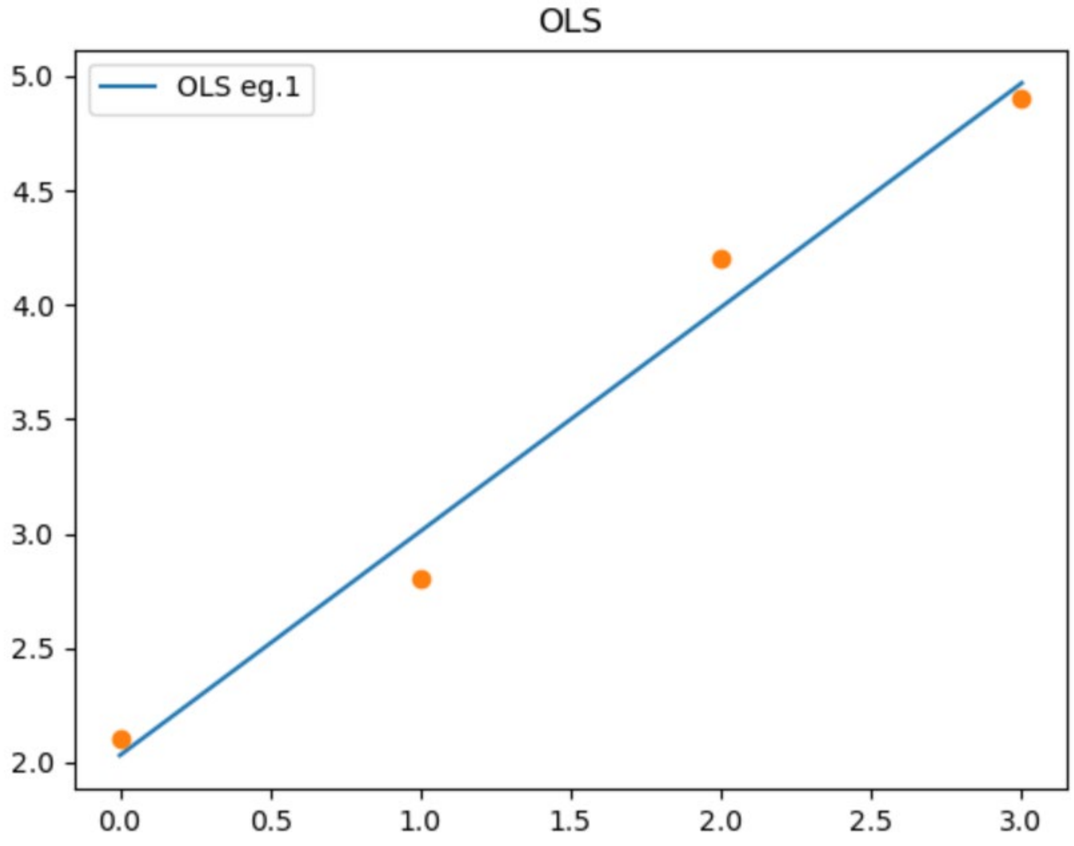


Fig.1
An linear least squares example 1

Least squares

A linear least squares example 2:

If:

- The data points are (-1 , 5.1), (0 , 1.8), (1 , 0.9), (2 , 2.1), (3 , 4.8).
- Assume the model function is $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2$

Use linear least squares to evaluate: $\beta_0 = 1.96$, $\beta_1 = -2.04$, $\beta_2 = 1.007$

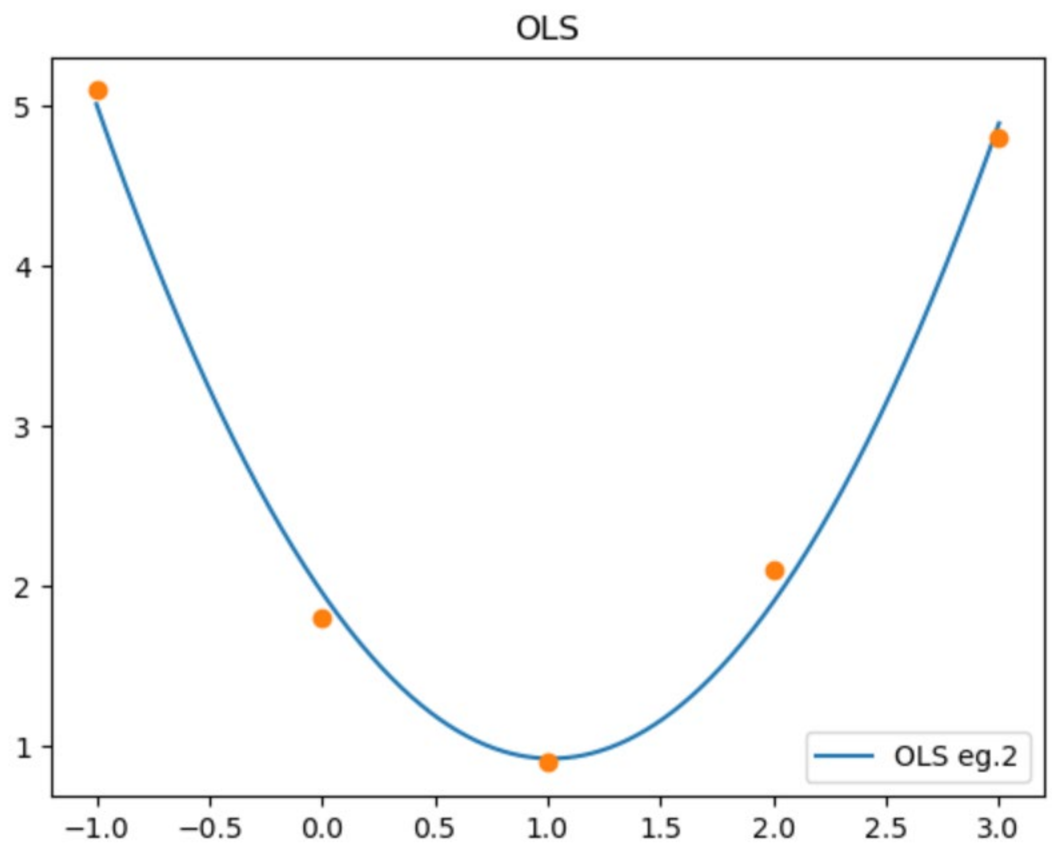


Fig.2
An linear least squares example 2

3. Non-linear least squares

Set m data points, $(x_1, y_1), (x_2, y_2), \ldots.(x_m, y_m)$, and a curve (model function)

$\hat{y}_i = f(x, \boldsymbol{\beta})$, $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots., \beta_n)$, with m ≧ n.

It is desired to find the vector $\boldsymbol{\beta}$ of parameters such that the curve fits best the given data in the least squares sense, that is, the SSR (eq.(4)) is minimized.

$$e_i = y_i - \hat{y}_i \rightarrow SSR = \sum_{i=1}^{m} e_i^2 = \sum_{i=1}^{m} (y_i - f(x_i, \boldsymbol{\beta}))^2, \qquad i = 1,2, \ldots.m \tag{4}$$

The minimum value of SSR occurs when the gradient is zero. Since the model contains *n* parameters there are n gradient equations (eq.(5)):

$$\frac{\partial SSR}{\partial \beta_j} = \frac{\partial}{\partial \beta_j} \sum_{i=1} e_i^2 = 2 \sum_{i=1} e_i \frac{\partial e_i}{\partial \beta_j} = 0, \qquad j = 1,2, \ldots., n \tag{5}$$

Least squares

A non-linear least squares example 2

If:

- $\hat{y}_i = f(x, \boldsymbol{\theta}) = \theta_1 \exp(\theta_2 x) \cos(\theta_3 x + \theta_4)$
- $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$

A nonlinear least squares problem with four variables θ1, θ2, θ3, θ4. Minimize:

$$SSR = \sum_{i=1}^{m} (y_i - \theta_1 e^{\theta_2 x_i} \cos(\theta_3 x_i + \theta_4)\ )^2$$

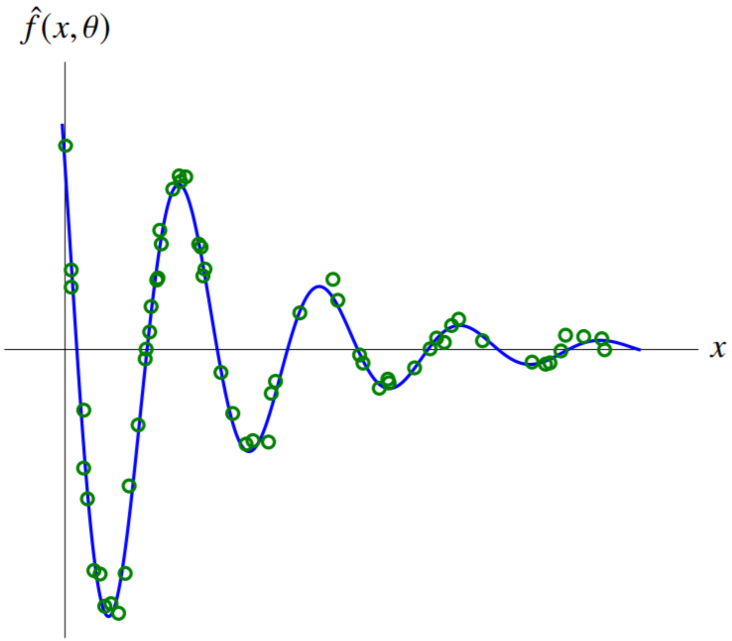


Fig.3
A non-linear least squares example 2.
Source:
http://www.seas.ucla.edu/~vandenbe/133A/lectures/nlls.pdf

Chi-square test

Chi-square test for goodness of fit _ preview

Chi-square test is a way to test between 2 population , one is the population derived from observation , the other is from the model we create .

To look deep inside the test , we need to understand some technical terms for a population :

1. Degree of freedom : This variable is how many number (at least) can “finish” a number or a formula . For example , Equation $x + y + z = 10$ have only 2 degree of freedom because it can be “ finished “ by giving 2 of variable and vary 3rd value . To speak more general , An equation involved n variable , it’s degree of freedom is $n - 1$.

2. Significant level : This value shows the accuracy between observed population and the “ true “ population . Therefore the number is given by the compared with the observation or some computation . For example , if a population is with a significant level 5% , it means the prediction of the “ true “ value can be in 5% of error by observed population .

Chi-square test for goodness of fit _ preview

3. Chi-square statistic : The value is the core of the test . E_i is expected value . O_i is the observation . The value is used for the test later .

$$X^2 = \sum \frac{(E_i - O_i)^2}{E_i}$$

Fig 1 : Chi-square statistic

Chi-square test for goodness of fit _ test

The basic process of the test is following below .

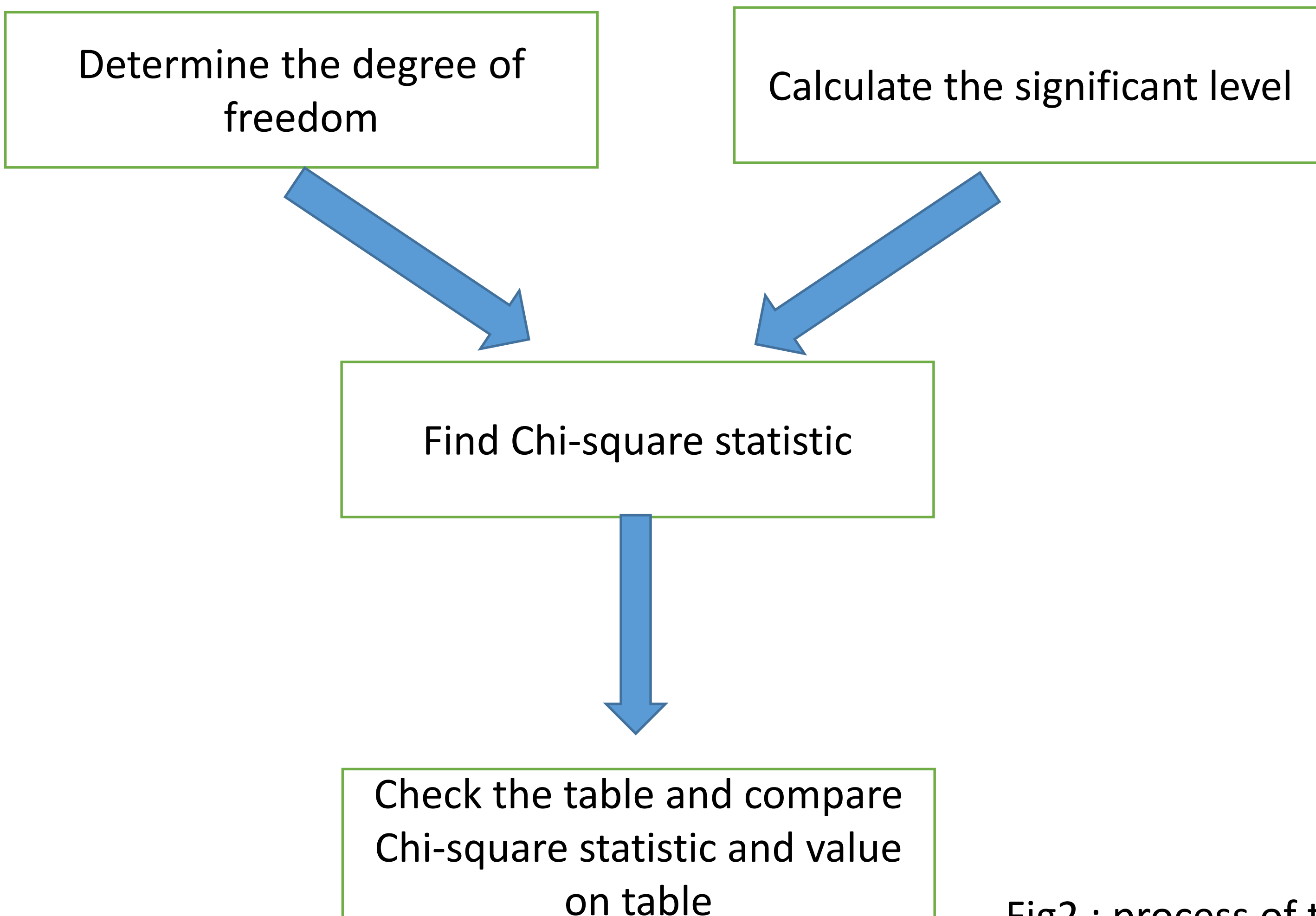


Fig2 : process of the test

Chi-square test for goodness of fit _ preview

For an example :

A shop have numbers of customers in a week as below (Fig 3 as observed population) . At the same time , we make a prediction that the percentage of incoming customers (Fig 4 as predicted) . And then we create the 3rd chart that include 2 kind of chart (Fig 5)

We can say that the significant level is 0.05 (It's still need to be checked) . And the degree of trust is 6 (Only 6 variable are real free while 7th variable is determined by others .) .

Day	Sun	Mon	Tue	wed	Thu	Fri	Sat
Pup	20	3	5	2	9	10	25

Fig 3:Observed

Day	Sun	Mon	Tue	wed	Thu	Fri	Sat
Pup	23%	10%	3%	9%	14%	16%	25%

Fig 4:Predicted

Day	Sun	Mon	Tue	wed	Thu	Fri	Sat	Total
Pup(O)	20	3	5	2	9	10	25	74
Pup(E)	14.8	7.4	2.22	1.48	6.66	7.4	18.5	74
CSS	1.827	2.616	3.481	0.183	1.675	0.914	2.284	12.98

Fig 5:combination of 2 charts

Chi-square test for goodness of fit _ preview

At the end , check the table . The x-axis is for significant levels , y-axis is for degree of freedom . In our case , we get 12.59 as our standard . Then our model don’t fit the our observed population (12.98(e) >12.59(t)).

$$CSS_e > CSS_t$$ The expected population is fit

$$CSS_e < CSS_t$$ The expected population is not fit

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72

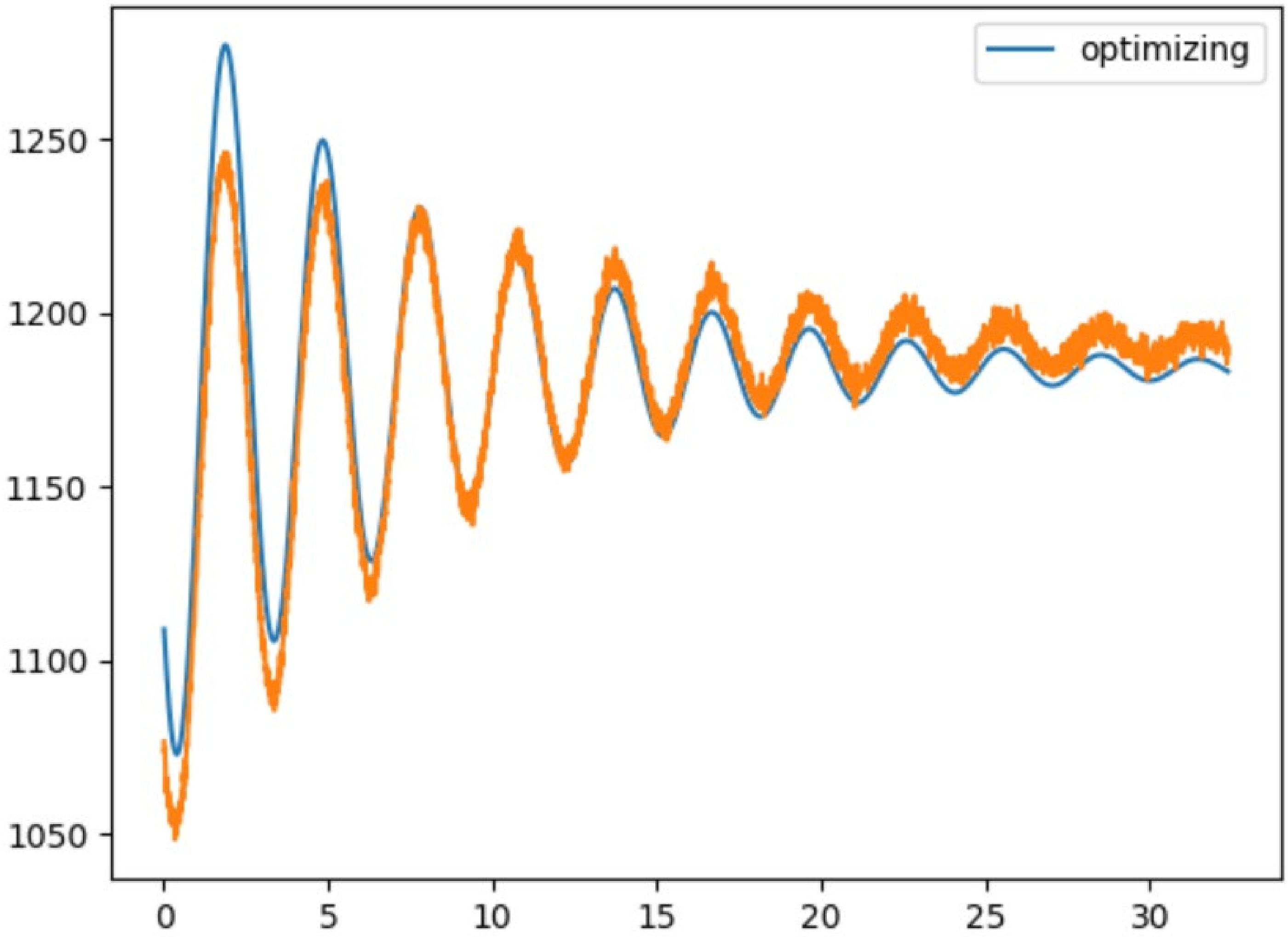
Fig 6 : table of CS test

Chi-square test for goodness of fit _ preview

The example can be generalized . For a normal distribution , divided X axis into pieces , and compared to distribution from prediction . The degree of freedom is $N - 1$ for N x axis pieces . Significant level is dependent on the distribution . We now have CSS , degree of freedom , and significant level . Then we can use Chi-square test for goodness of fit on a distribution .

Fit practice

Fit practice

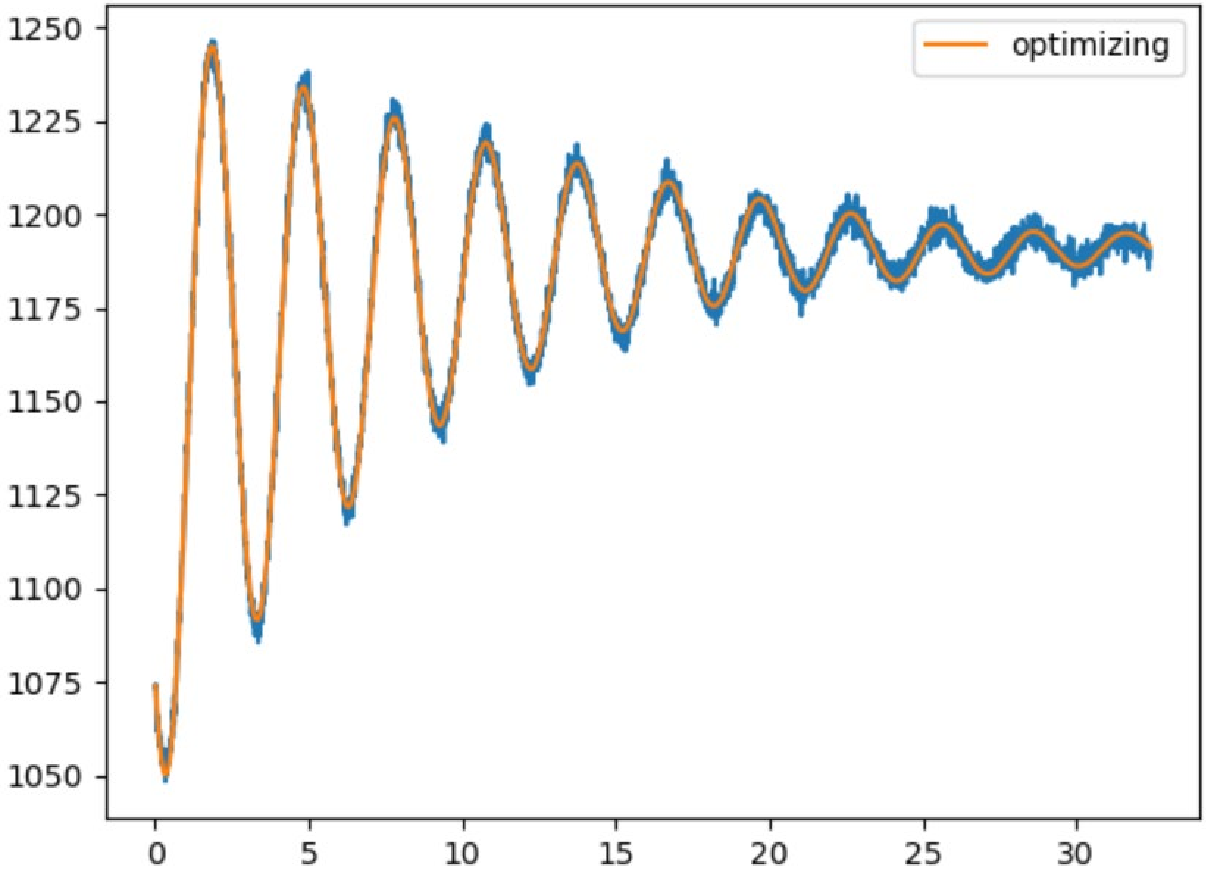


$$\gamma = 0.1771(\frac{1}{s})$$
$$A = -116.455(m)$$
$$\omega_d = -2.122(\frac{1}{s})$$
$$\varphi = 2.463$$
$$h = 1184(m)$$

$$y = e^{-\gamma t} * A * \sin(\omega_d t + \varphi) + h$$

Fig 1: data and fitting line

Fit practice



$a0 = 1149.06$
$a1 = 5.6543$
$a2 = -0.2496$
$a3 = 0.00357$
$a4 = -104.80$
$a5 = 0.105$
$a6 = 2.114$
$a7 = 0.783$

Fig 2: data and fitting line

$$y = a0 + a1 * t + a2 * t ** 2 + a3 * t ** 3 + a4 * np.exp(-a5 * t) * np.sin(a6 * t + a7)$$

Pic1. device

Schedule:
12/12- Bracket installation
12/19- breaking
12/26- do experiment
1~~~~~~~~report and breaking again.

Summary-葉揚昀 108202529

This week, I had learned three main kinds of things through myself, my groupmates, and the teaching assistants. There're including machining, Arduino (stepper motor, oscilloscope), and data processing of the program (VB & Python). First, in machining, when I used a M3 tapping device to tap on our PVC experiment material, I was so hurried that the screw thread in the PVC material was too not obvious to match the screw. However, it can be remedied by stick the screw in the hole of this PVC. I also realized how to cut the iron screw via a hacksaw (It was the teaching assistant teaching us). One tip is not exerting a downward force at first, as a result of it will be a little bit difficult to saw if we don't cut a groove on the workpiece at the beginning. Another tip is using one hand to saw and another hand to fix the screw. It will be a quicker method when you are cutting a screw.

Second, in Arduino, due to the infrared detector had outputted the oscillatory signals. Our teaching assistants used an oscilloscope to detect the signals of infrared detector and found that this phenomenon has a chance with the design of IC in the infrared detector. In stepper motor, we found several stepper motor in the laboratory and planned to use it. Nevertheless, we couldn't found an appropriate circuit board to match these four wires' stepper motors. What's worse, its datasheet (the teaching assistant recommended us searching) may be too unpopular to search. Accordingly, we hoped that the original, small stepper motor can afford this job.

Third, I took advantage of VB to transform the array into txt file and used a string format of array to read txt file, but my groupmate can operate Python to read txt file into float format and it's convenience to draw plot in Python. Based on the two reasons, we were determined to use Python.

Summary_田家瑋

- This week , I am trying to process our project . We were cutting the gear on bicycle . I asked Mr. Zhou to help us cut it . Mr. Zhou at first wanted to break the gear by understanding the structure and taking it into apart . But he failed . Though the structure might not be hard to figure , it’s had been rust to such an extent . He decided to cut it by hand grinder latefully. The hand grinder is so dangerous . The sparkle splash from the grinder and gear looked pretty harmful . Mr. Zhou did a good job . He looked experienced when cutting . Although some of sparkle hit his face , he didn’t show any fear . I could feel so much stress when I stood 5m away from the grinder , let alone Mr. Zhou . Give my all thankfulness to Mr. Zhou .
- After processing gear , I cut the brass cylinder by hydraulic band saw , The result was a complete disaster . We fixed the cylinder , and open the band saw . I still don’t get where is the “real” problem . The saw wasn’t cut through the cylinder . Instead , when the saw touch the cylinder , it was starting to roar . The cut on the cylinder looked messy and deviated . I supposed the problem was on the Clip . The friction on the saw and cylinder is numerous while the cylinder is also round .

Summary-張家菖

This week, I mostly spent my time in manufacturing our team’s experimental device. However, there are still some lessons I learned from the class. First, it is a kind of mathematic way called the least-squares method. The method of least squares is a standard approach in regression analysis to approximate the solution of overdetermined systems which sets of equations in which there are more equations than unknowns by minimizing the sum of the squares of the residuals made in the results of every single equation. The most important application is in data fitting. The best fit in the least-squares sense minimizes the sum of squared residuals which are the difference between an observed value, and the fitted value provided by a model. Secondly, our team used two ways to mimic a phenomenon about our experiment. One is a python. The other is visual basic. Everyone has their own way to write programs, so the programs written by different people will be very different, and you can learn a lot from them. Third, I knew more types of tools to manufacture our experimental device, including a grinder. And we used some informal methods to produce our experimental device, like brute force dismantling. Forth, I learned from my team member that there are some ways to approach to linear regression in Python. He introduced two ways. One is the code called “scipy.polyfit( ) or numpy.polyfit( )”. The other is the code called “Optimize.curve_fit( )”.