Volatility Spillover Between UK Exchange Market and Stock Market by Wavelet Analysis

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Outline

- What is Volatility Spillover
- What is Wavelet Transform
- An Example of Wavelet Transform on Artificial Signal
- Empirical Analysis on FTSE100 and USD-GBP

Volatility Spillover

- What is Volatility Spillover

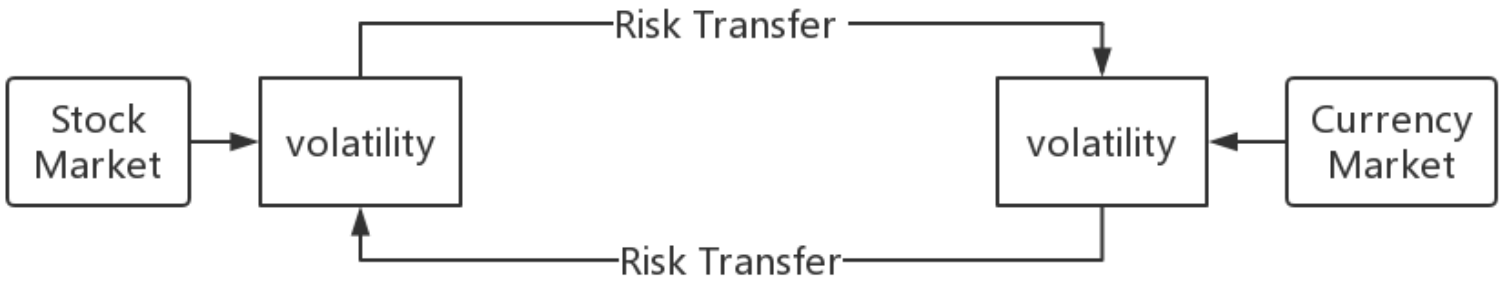


Figure 1: Volatility Spillover is actually the risk transfer from different markets

- Equity flows increase
- Portfolio risk management

Wavelet Transform Basic Idea

- Using a series of waveform curves to capture the features of original signal

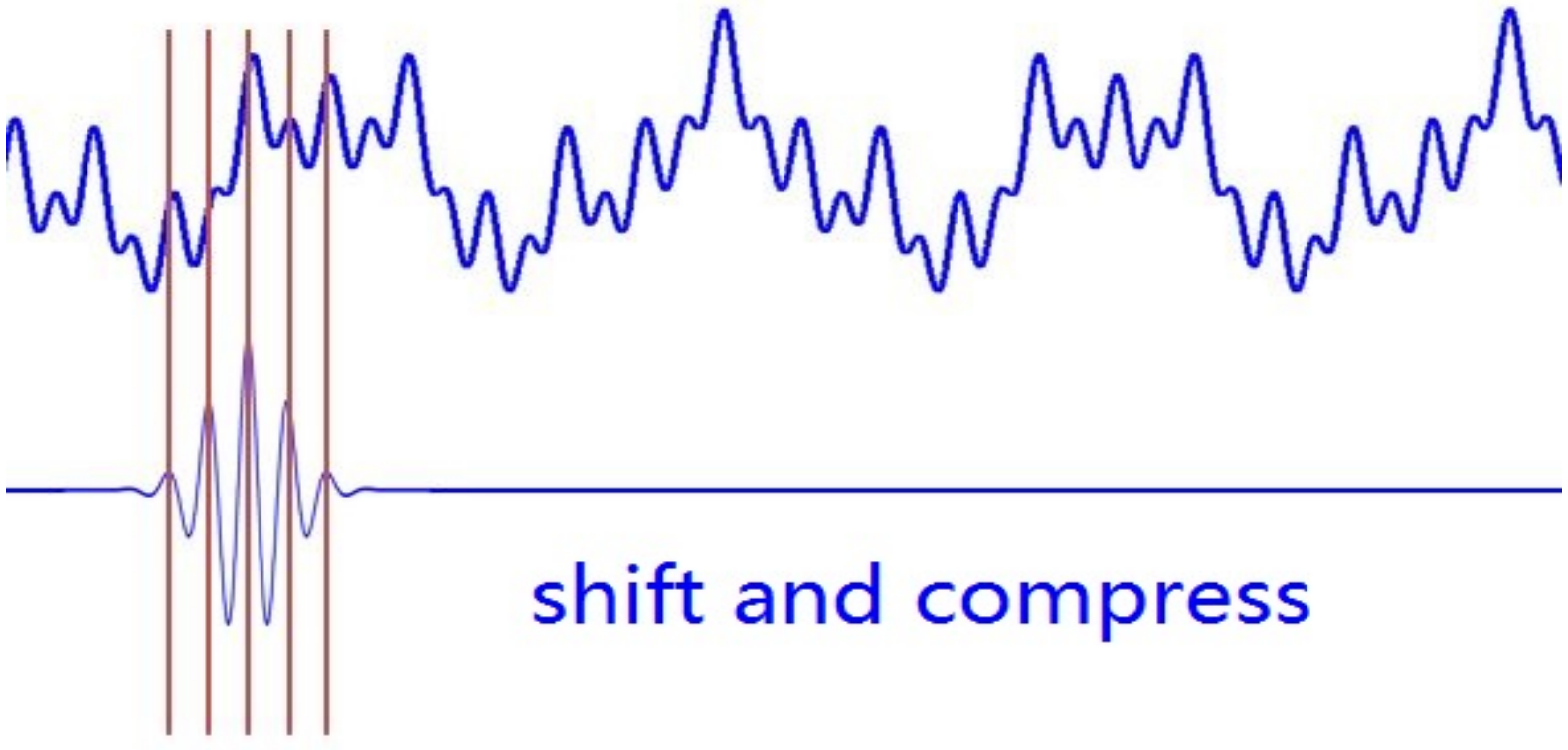


Figure 2: the wavelet capture the features

How Wavelet Generates

- Definition: A wavelet is a waveform of effectively limited duration that has an average value of zero.

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0$$

$$\int_{-\infty}^{+\infty} \psi^2(t)dt = 1$$

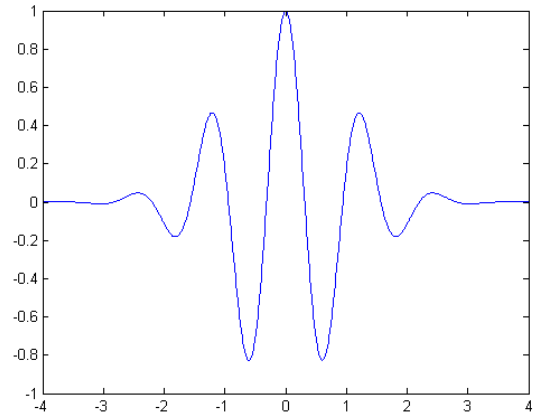


Figure 3: the Morlet wavelet.

How Wavelet Compress and Shift

$$\psi_{s,\tau}(t) = |s|^{-\frac{1}{2}}\psi(\frac{t-\tau}{s})$$

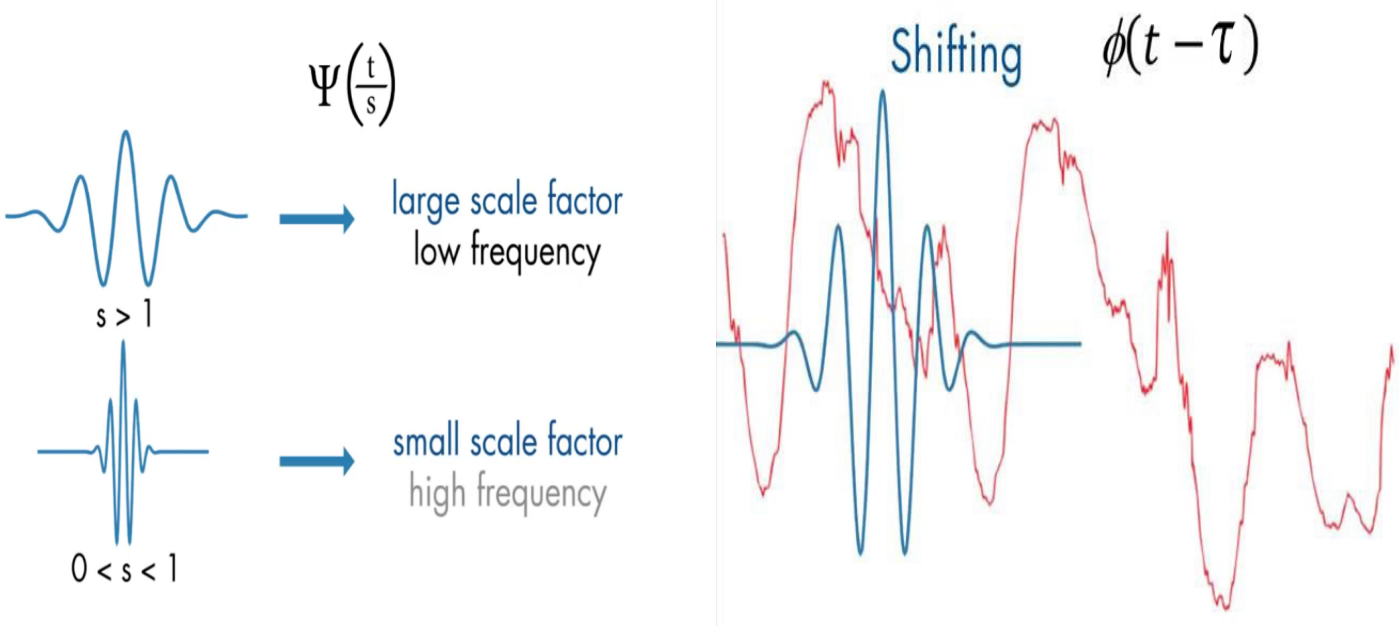
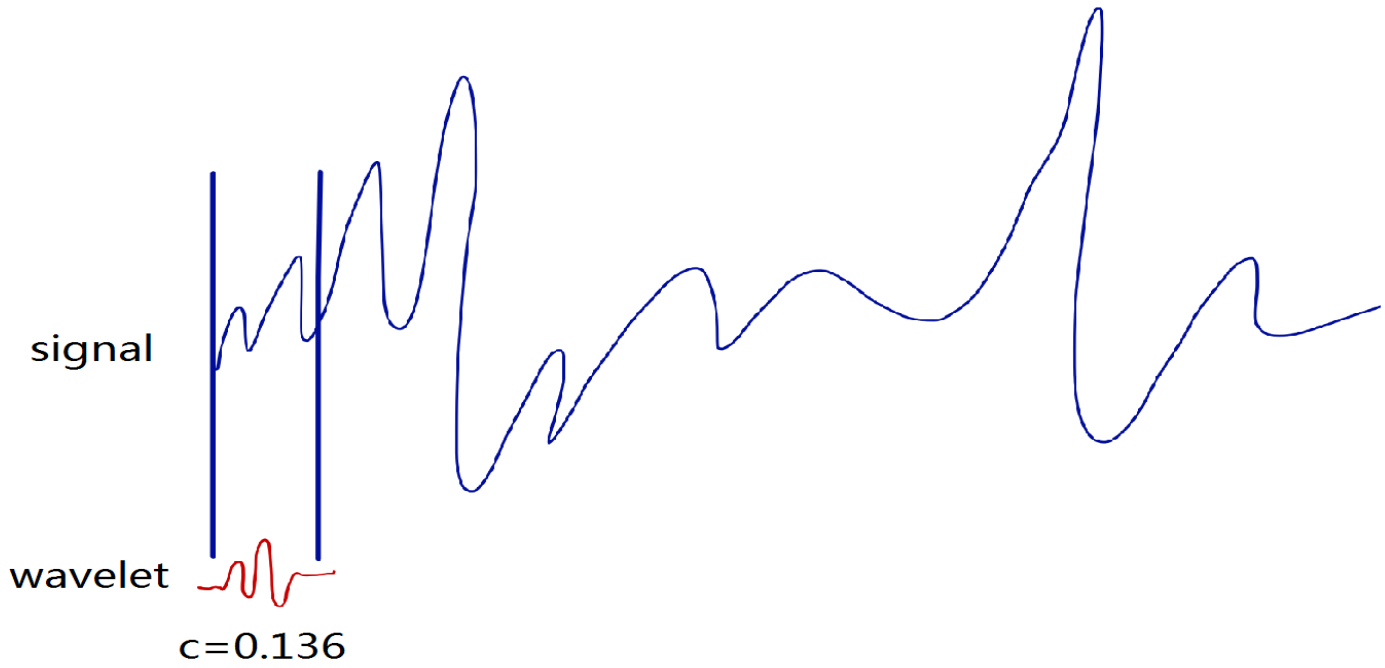


Figure 4: How scale coefficient s and shift coefficient τ Works.

How Wavelet Transform Works Step by Step

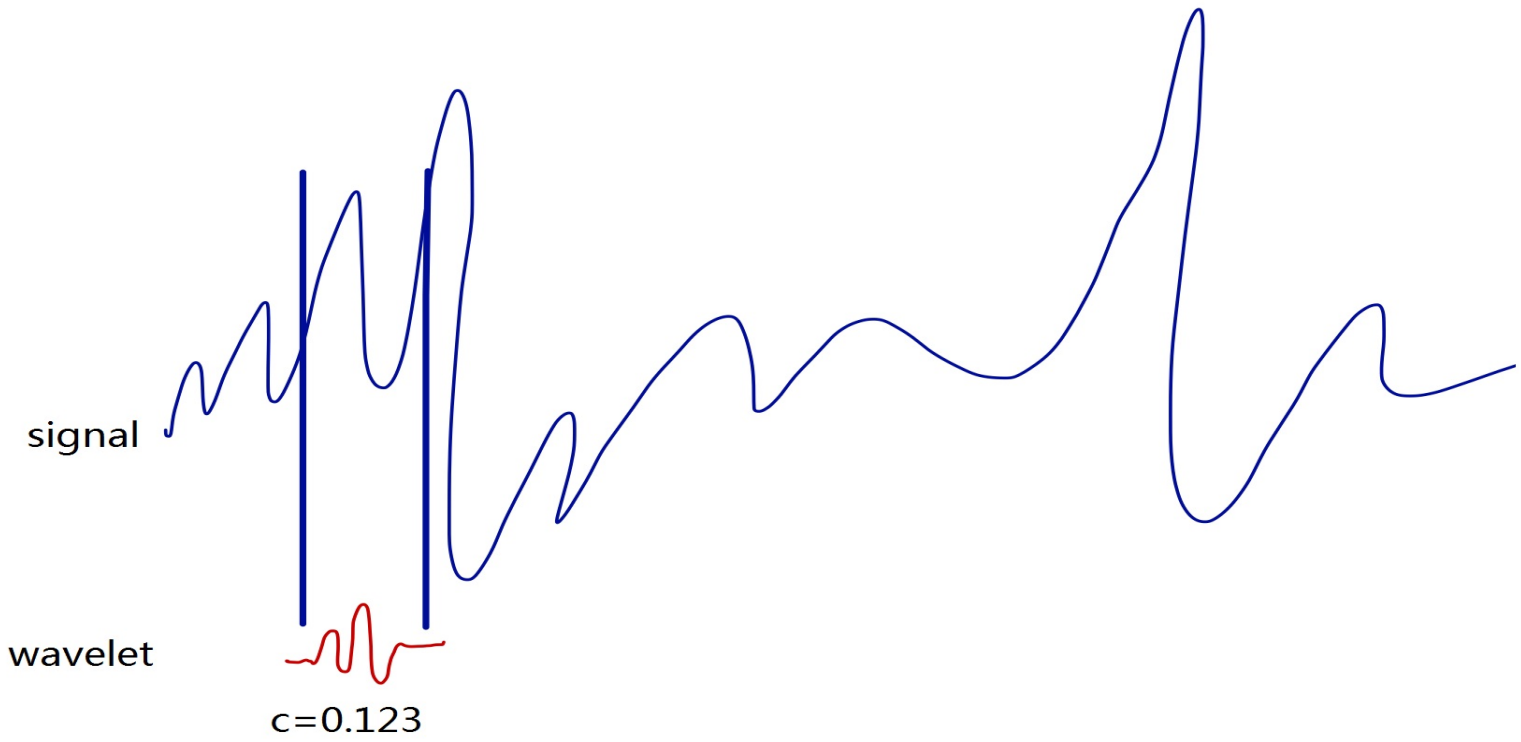
- Take a wavelet from the start of the signal,given the first scale factor.



- Calculate the detail coefficient
  $C(scale, position) = \int_{-\infty}^{\infty} f(t)\psi(scale, position, t)dt$
- The coefficient measures how the wavelet related closely to the original signal

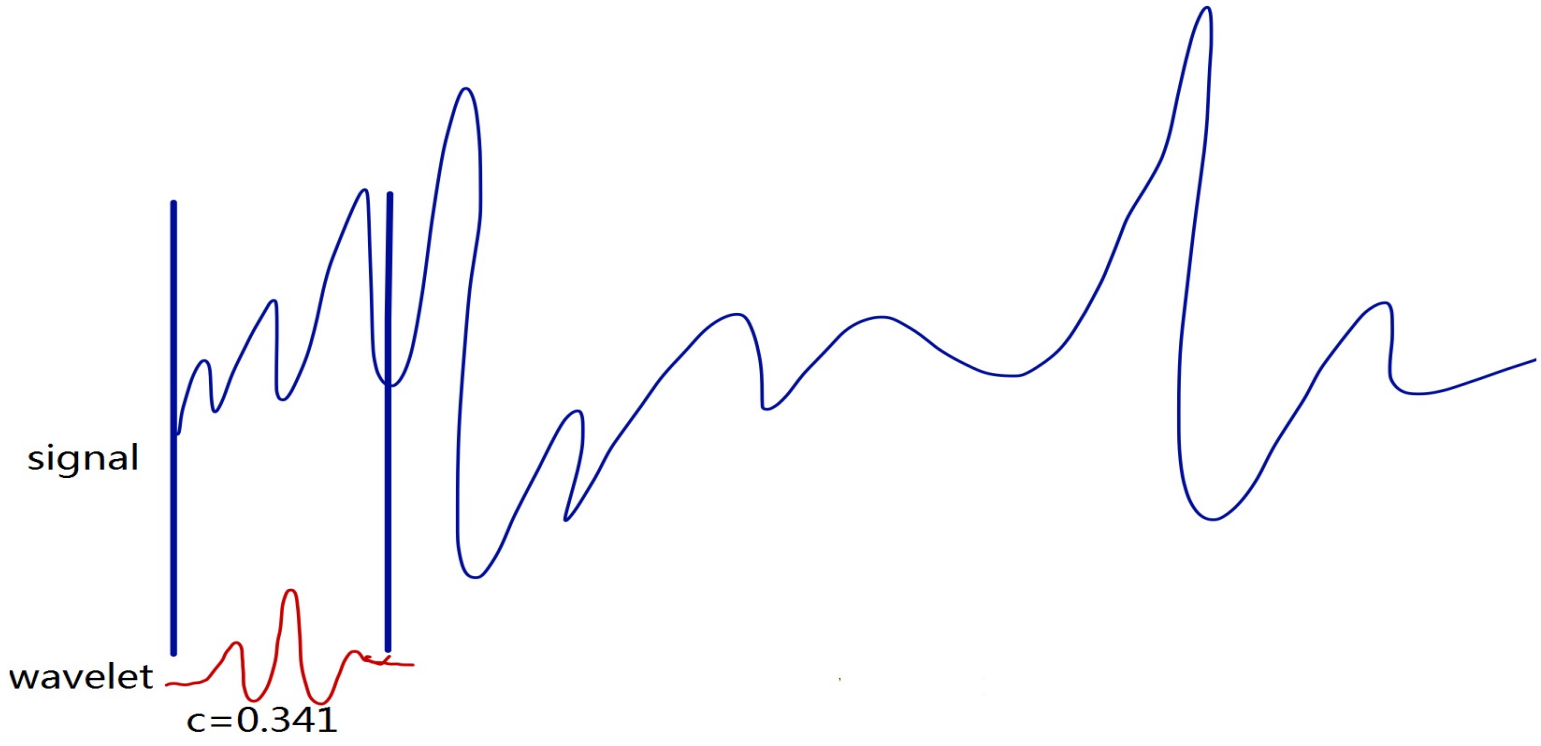
How Wavelet Transform Works

- Shift the wavelet to the next data point and repeat the previous steps until cover the whole signal.



How Wavelet Transform Works

- change the scale(stretch) the wavelet for low frequency details



- Repeat the previous steps for all scales
- why we use wavelet transform

Why Wavelet Transform

- 1.transform result contains the time information.
- 2.Given low scale factor= Compressed wavelet = Rapidly changing details = High frequency Component.
- 3.Given high scale factor= Stretched wavelet = Slowly changing, coarse features = Low frequency Component.
- 4.Another advantage of wavelet is that its different scale factors can make people investigate the signal at various time scales,which is also called multiresolution.
- next, I give a example of wavelet transform result.

Example of Wavelet Transform

- $$dS = 0.001Sdt + 0.01Sdx, t \in (0, 1000)$$
$$dS = 0.001Sdt + 0.05Sdx, t \in (1000, 2000)$$
$$dS = 0.001Sdt + 0.02Sdx, t \in (2000, 3000)$$

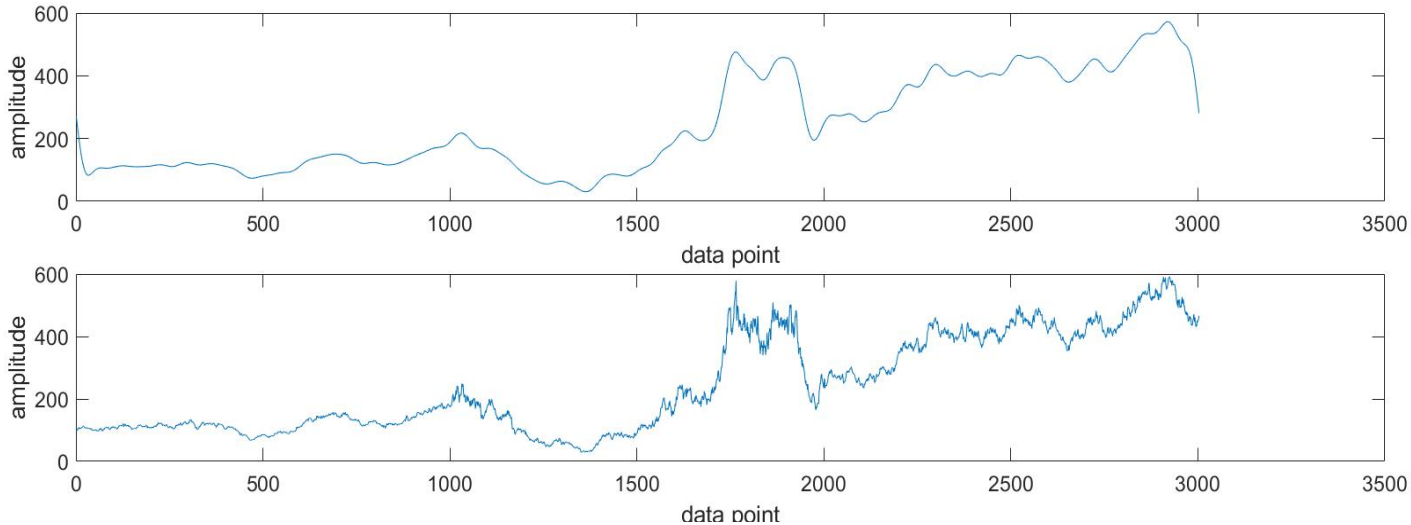


Figure 5: How Wavelet Transform Works.

Example of Wavelet Transform

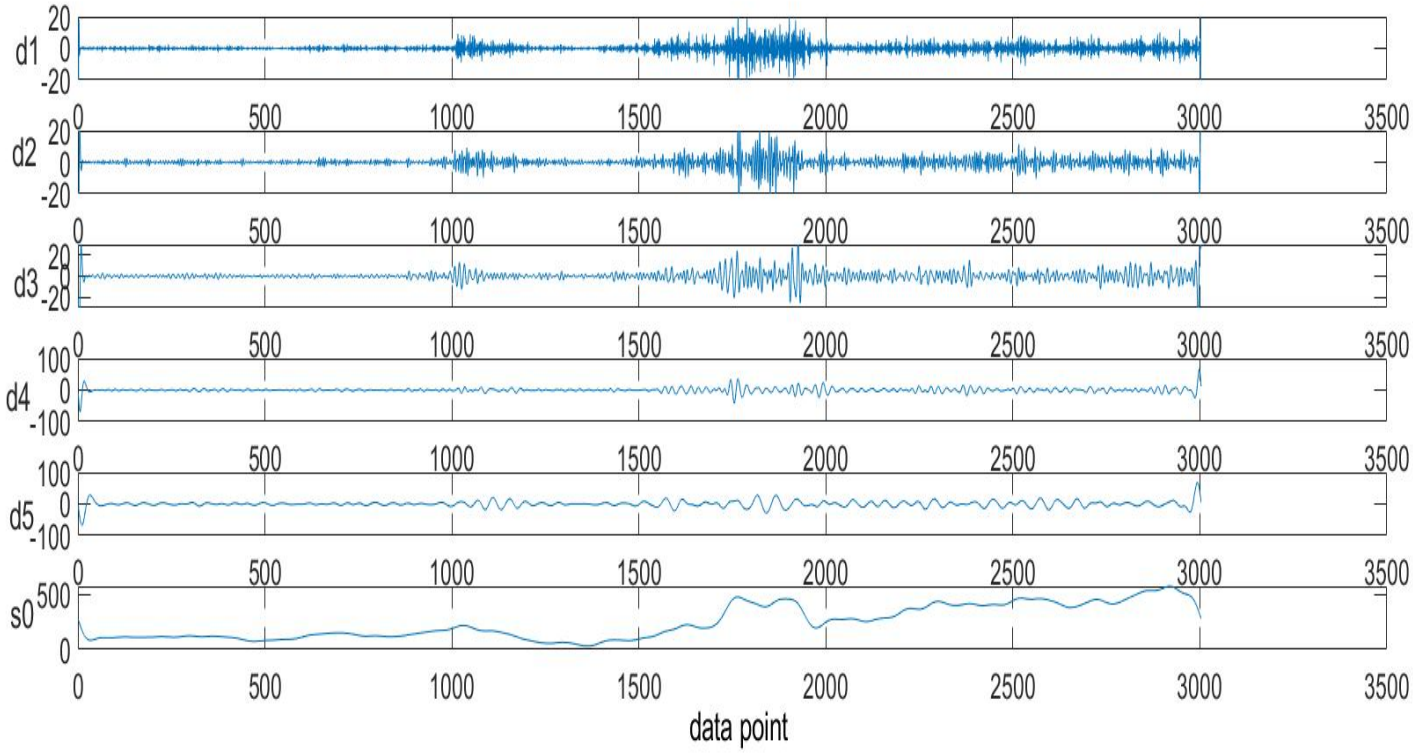


Figure 6: The Result of Wavelet Transform

Empirical Data Analysis

- The half-year 30-minute data of GBP-USD Exchange Rate and FTSE100 Index

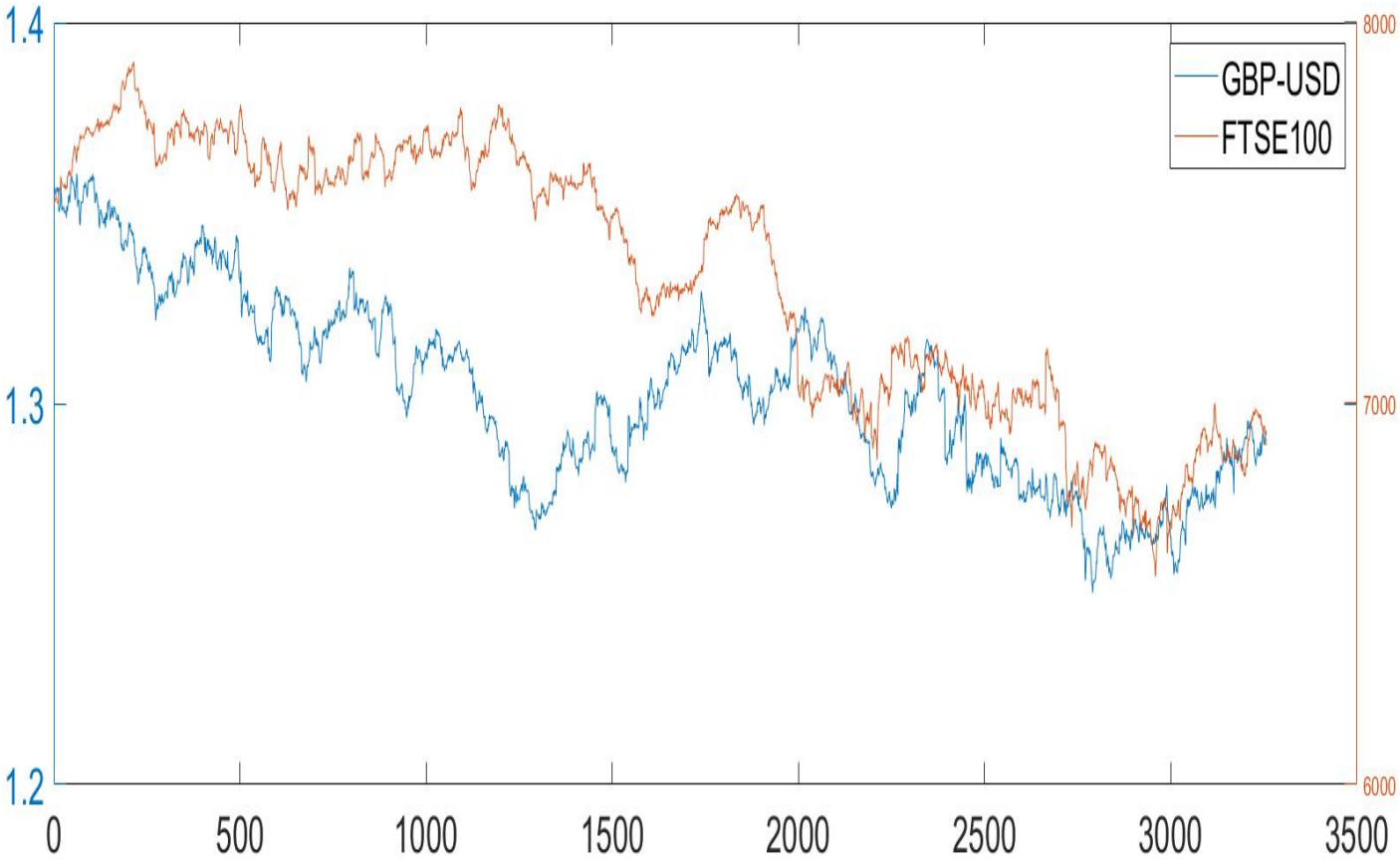


Figure 7: The data

Correlation by Scale-Wavelet Coefficients

- Use cross-correlation equation for scale τ_j at lag-term,which is defined as:

- $\rho_{x,y}(\tau_j) = \frac{\mathrm{cov}(\bar{w}_{j,t}^{(x)}, \bar{w}_{j,t+lag-term}^{(y)})}{(\mathrm{var}(\bar{w}_{j,t}^{(x)}) * \mathrm{var}(\bar{w}_{j,t+lag-term}^{(y)}))^{\frac{1}{2}}}$

Correlation by Scale-Wavelet Coefficients

- scale1 means the details existing in 0.5-1hour scale,scale2 means 1-2hours,scale 3 means 2-4 hours scale 4 means 4-8 hours(one day),scale 5 means 8-16 hours(two days),scale 6 means weeks scale, scale 7 means half-months scale.

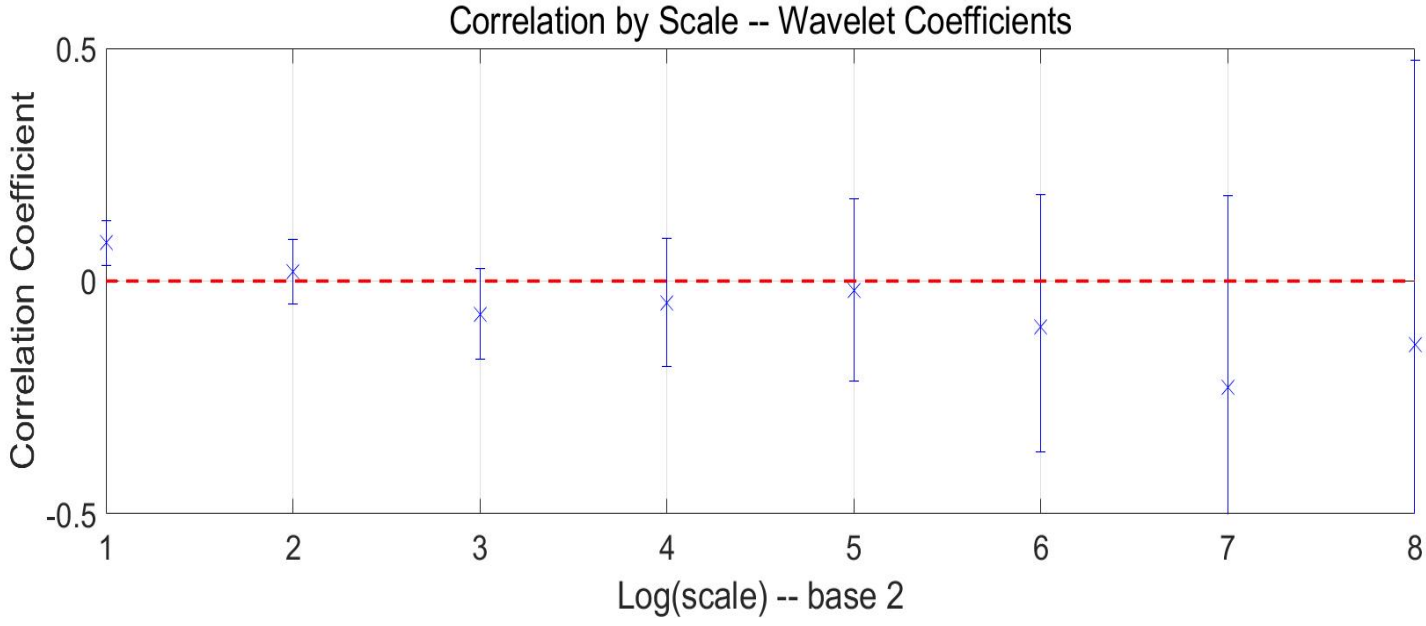


Figure 8: Correlation by Scale-Wavelet Coefficients

Correlation by Scale-Wavelet Coefficients for Lead-Lag Analysis

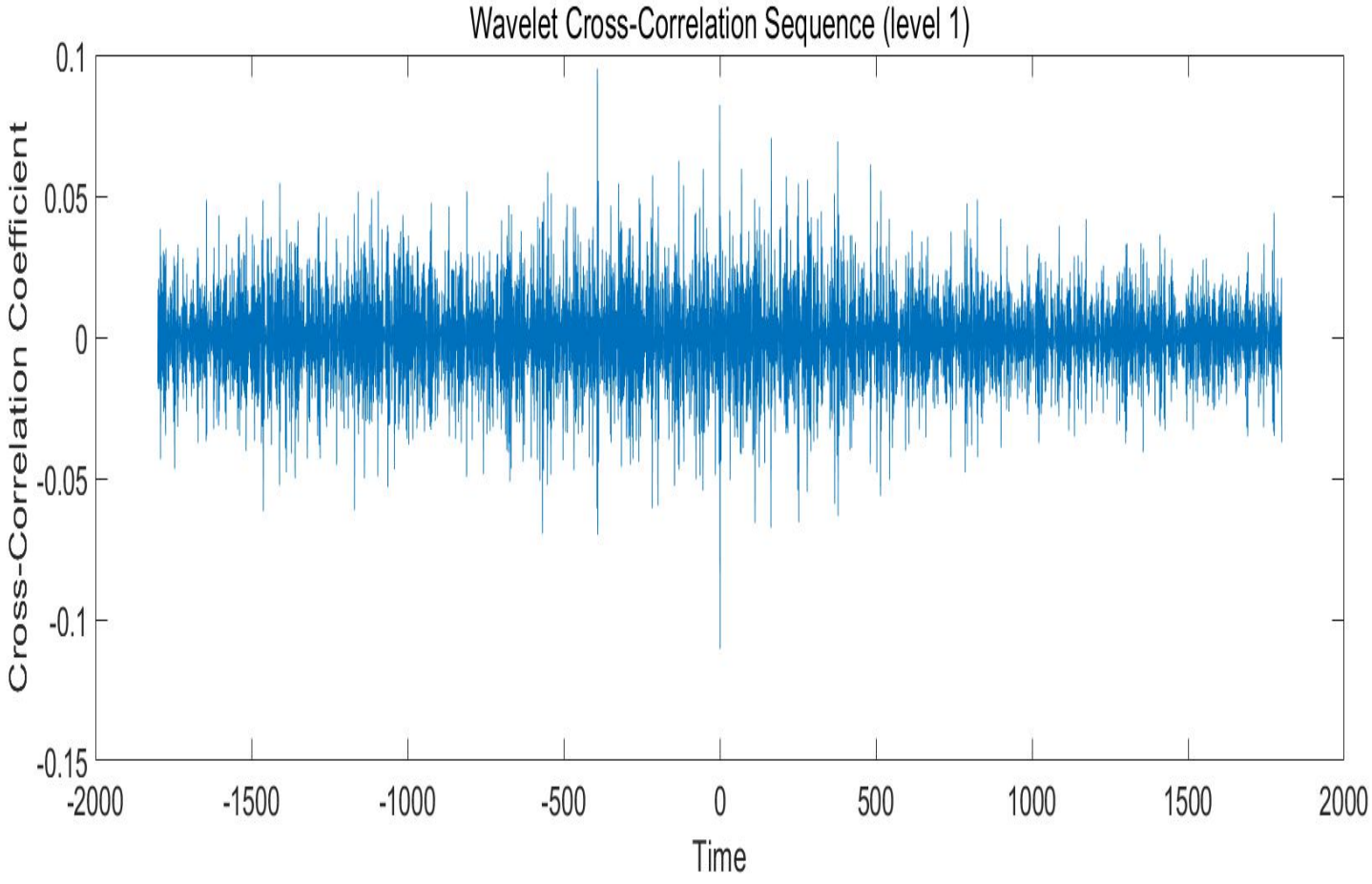


Figure 9: Correlation by Scale-Wavelet Coefficients level 1

Correlation by Scale-Wavelet Coefficients for Lead-Lag Analysis

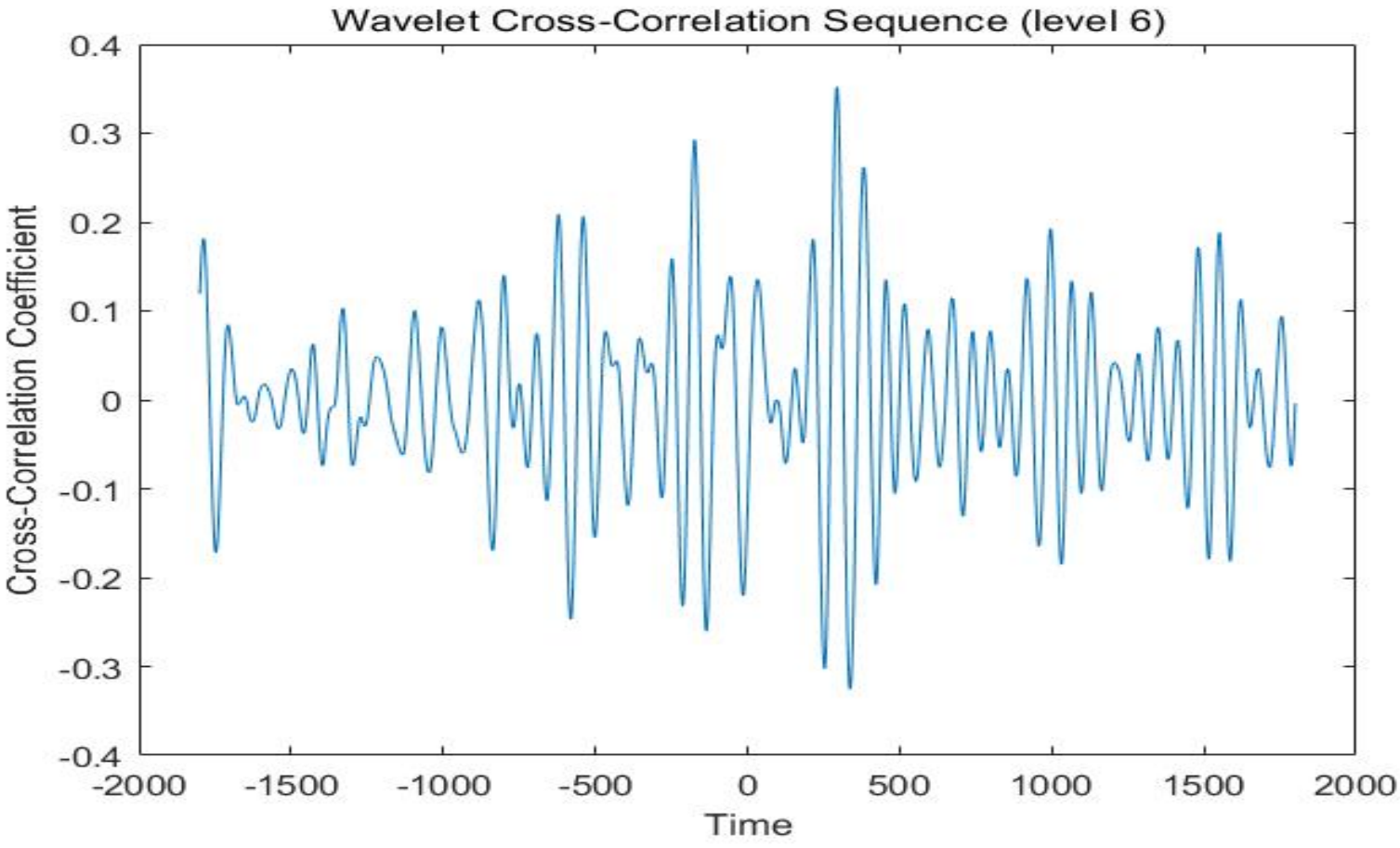


Figure 10: Correlation by Scale-Wavelet Coefficients level 6

Correlation by Scale-Wavelet Coefficients for Lead-Lag Analysis

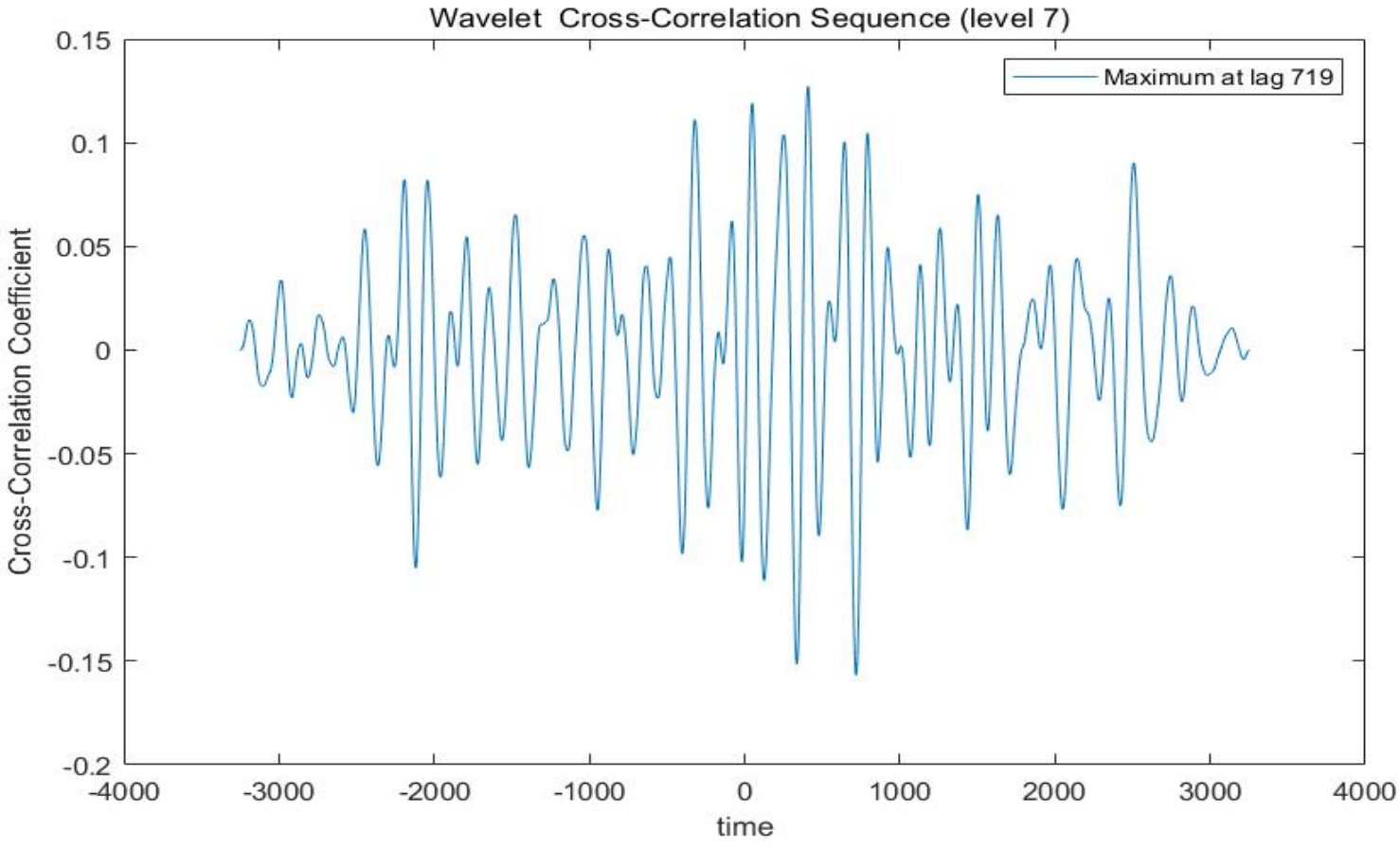


Figure 11: Correlation by Scale-Wavelet Coefficients level 7

Conclusion

- For short time scale such 30min-60min and two day data the lead-lag relation is not significant, the volatility spillover is not significant

- For a longer time scale such weekly scale and half-month scale the lead-lag relation is significant, there exists the stock market volatility spillover to currency exchange market.