Team Control Number

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61635

E-Z Merge

Problem Chosen

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2017

MCM/ICM

Summary Sheet

First of all, the shape, size and merging pattern are three essential parameters to determine the fan-in area. We first hypothesize the merge area is a quadrilateral. By adding important considerations of accident prevention capacity, we gradually specialize the area into a right trapezoid. In our design philosophy, we give throughput capacity the highest priority instead of cost and accident prevention capacity. Hence we calculate and determine the inner angle of the trapezoid.

In addition, we design the merging pattern of vehicles. That is to say there is a match of a running mode of vehicles and a certain merge area shape that can reach a high throughput capacity. Obviously, no matter how good a model is, it cannot guarantee a high throughput capacity in all cases. We can only design a good model that can guarantee a relatively high throughput capacity compared to other models in the normal running situation. The model can also guarantee a maximum theoretical value in some given premise. Particularly, the maximum theoretical value is equivalent to vehicles arranging closely in all L lanes. Once the merging pattern and shape are determined, the size of the model can be formulated. Thus the model is built. Then we quantify the throughput capacity, the cost and the accident prevention rate.

In part two, we make the premise less strict. Motorists drive vehicles according to their habitual behaviors. Thus our model is more practical. We control variables and test the model, mainly discuss performance of the model in light and heavy traffic, performance of the model when self-driving vehicles are added to traffic mix and performance when proportions of different kind of tollbooths. The entry point is trying to reach the theoretical maximum throughput capacity. By adding self-driving vehicles to traffic mix, controlling proportions of different kind of tollbooths and taking oversize vehicles into consideration, we can make the actual value largely approach the theoretical maximum value.

At last, we apply the data in reality into the model and validate that the model is of practical significance. Besides, we suggest innovations when funds are sufficient.

Dear New Jersey Turnpike Authority,

According to the fact that the Turnpike is the United States' sixth-busiest toll road, our team learns that New Jersey Turnpike is of great significance for land transportation in the United States. Hence the traffic condition of highways in New Jersey really needs to be paid high attention to. Considering the fact that the turnpike is one of the most heavily traveled highways in the United States, traffic condition in the Turnpike need to be improved.[1] Because traffic congestion in the barrier toll is a major cause of traffic jam in high ways, so we plan to design the fan-in area to reduce congestion in the barrier toll.

At first, we assume that the toll highway in New Jersey has L lanes of travel in each direction and a barrier toll containing B tollbooths $(B > L)$ in each direction. Considering of the accident prevention, throughput and cost, our team comes up with a solution which is presented in the following.

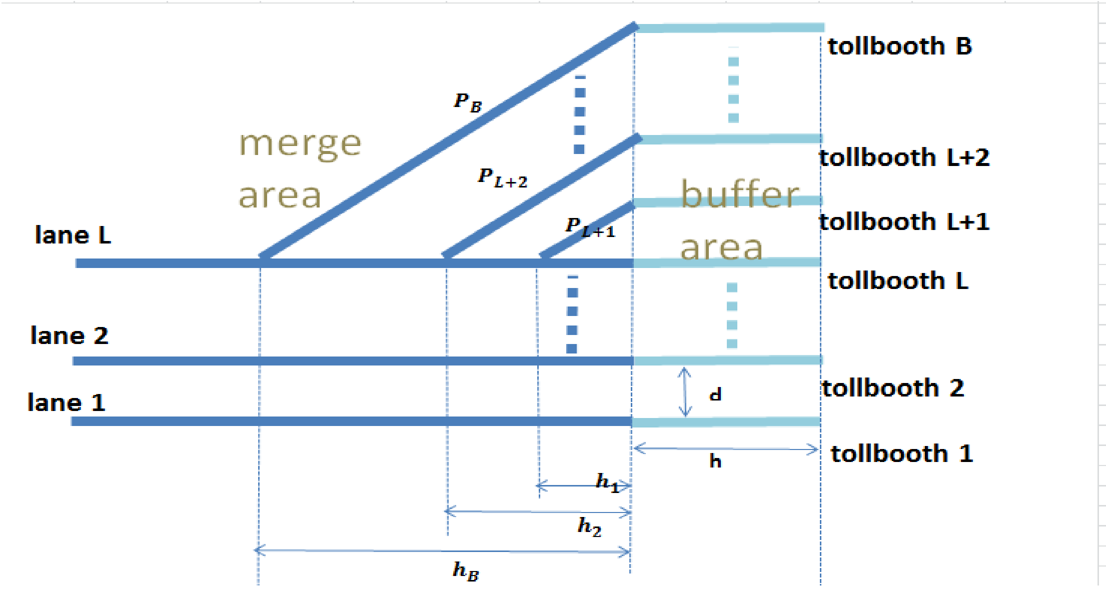


Figure 1

Firstly, a buffer area should be built following the roll barrier. The length of the buffer area should be $\frac{v_0 \times v_0}{2 \times a_0}$. In the formula, v_0 is the required speed in the merge area. a_0 is the average acceleration of vehicles.

Then, in the merge area, the tollbooth 1-L should be separately built in the same horizontal line of lane 1-L. Therefore, motorists in the lane 1-L don't need to change the direction in the fan-in area. The distance which vehicles keep to stay safe is s.

$$P_{L+k} = h_k + k \times (l + s)$$
$$h_k{}^2 + (k \times d)^2 = P_{L+k}{}^2$$

Therefore,

$$h_k = \frac{k(d^2 - (l + s)^2)}{2(l + s)}$$

There are several convincing strengths of our model as follows. First of all, our model's throughput capacity can reach a theoretical maximum. Although the theoretical value is not equal to the practical value in reality, this is the weakness that cannot avoid in all models. In addition, we can prove that we have methods to make the actual value approach the theoretical value. The methods include adding vehicles to traffic mix and controlling the proportions of different kinds of tollbooths in a barrier toll. In brief, the greatest strength of our model is the performance of throughput capacity. To prove it, we use value of numbers in reality to calculate in our model. Take the section whose number of lanes is 6 in the New Jersey Turnpike for example, the theoretical value of throughput is 2758 vehicles per hour. The actual value can be calculated is at around 1500 vehicles per hour.[2] Through the comparison, our model is worth applying.

Besides, if the funds for reconstructing toll plaza are sufficient, we can bring forth new ideas to our model. In reality, there are trucks and sedan cars in the traffic mix simultaneously. And the most common situation is that trucks use the outermost tollbooths. So the trucks have to change lanes to the main roads in the merge point. It is dangerous for trucks to change lanes to the lane which sedan cars are running. So if we can prevent trucks from such actions, it will surely increase the safety rate. To achieve this aim, we can build a viaduct for trucks in the toll plaza. Trucks will exactly enter the main roads when leave the viaduct. And other cars can change lanes through the underpass under the viaduct.

Hope our model will help you release the heavy traffic pressure on the New Jersey Turnpike.

Sincerely,

Team #61635

Contents

1. Problem statement

1.1. Background:

There is no doubt that keeping highways clear is of great importance for land transportation.[3] Studies have found that congestion in the barrier toll is, in fact, a major source of traffic jam. And congestion in the barrier toll is effected by several factors. For example, the ratio of tollbooths number to lanes number, the work efficiency of tollbooths, and especially what we will discuss in the paper, the design of the fan-in area. Therefore providing a solution which can apply to different kinds of highways has practical significance.

1.2. Problem restatement:

To design a fan-in area, three parameters of the area need to be determined as follows.

- the shape
- the size
- the merging pattern

In the process of determining these parameters, it is of great significance to meet some important considerations as follows.

- accident prevention
- throughput
- cost

After finishing the design of the fan-in area, that is to say, the model is built. Then the problem asks to do some tests on the model as follows.

- performance in light and heavy traffic
- performance of the model when self-driving vehicles are added
- performance of the model when proportions of conventional tollbooths, exact-change tollbooths, and electronic toll collection booths are different

Test results show the performance index of the model.

2. Premise

2.1. Assumptions

- Vehicles can arrange closely in the lanes.
- The tollbooth won't allow a vehicle come in unless the entire body of the front vehicle has left the tollbooth.
- All vehicles are divided into two types----sedan cars and oversize vehicles (e.g. trucks and buses). In our model, we roughly consider that oversize vehicles are twice as long as sedan cars.
- All the vehicles in the merge area are in the same speed. The speed is called "required speed" in the following.
- The time of starting a vehicle and reaching the required speed is a default value of number.
- All drivers should follow the same requirements of rules. Though the personal decisions of the driver are probabilistic, all the drivers have the same associated probabilities.
- The momentary process of drivers merging and changing lanes is negligible in the turning.
- All the motorists obey the traffic regulations.

2.2. Notations

variable	Definition
d	The distance between two close tollbooth
$v0$	The driving speed of vehicles in the merge area unit: m/h
l	The average length of sedan cars

$2l$	The average length of oversize vehicles
a_0	The average acceleration of vehicles in the buffer area
h	The length of road in the buffer area
h_1	The distance between the start of merge area and the first intersection
h_2	The distance between the start of merge area and the second intersection
h_3	The distance between the start of merge area and the third intersection
h_4	The distance between the start of merge area and the fourth intersection
h_5	The distance between the start of merge area and the fifth intersection
D	The distance between the close car in the same road
S	The distance that vehicles should keep to stay safe
Tp	number of vehicles per hour passing the point where the end of the plaza joins the L outgoing traffic lanes
B	The number of tollbooths
L	The number of lanes of travel
[ ]	Round down to the nearest whole unit. E.g. [4.4]=4
%	Take reminders after division. E.g. 5%3=2

3.Basic Model

3.1. Creation process of the first model:

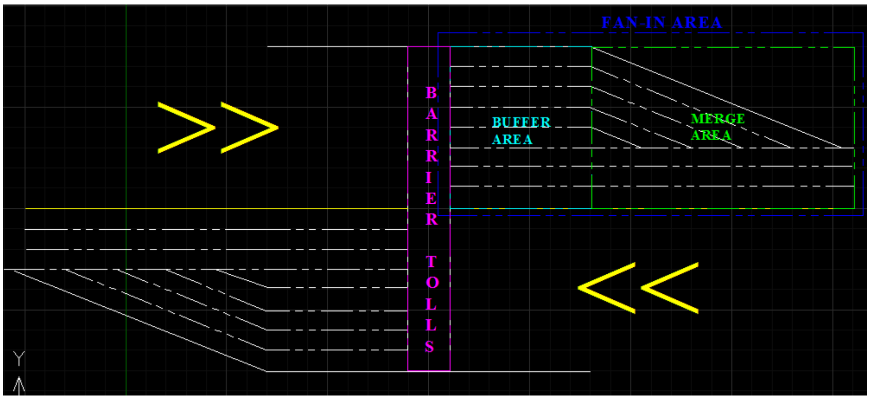


Figure 2

3.1.1. Brief Introduction: Figure 2 and Figure 3 are top views of the fan-in area of a toll plaza. The fan-in area consists of buffer area and merge area. After paying the toll, motorists start vehicles and reach the required speed before entering the merge area. And then motorists try to drive vehicles to three through lanes before leaving the merge area.

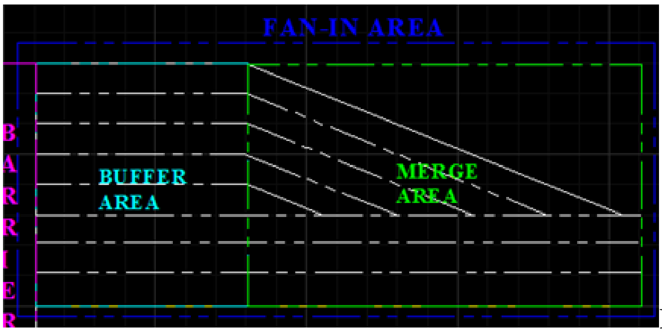


Figure3

3.1.2. The process of creating the model:

The creation of buffer area.

Buffer area is designed for motorists to start vehicles and reach the required speed. The purpose of the design is to increase safety factor, which means the occurrence rate of car accidents will be higher by lack of a buffer area. If there is not a buffer area, the motorists may have to speed up and deal with the merge action at the same time, obviously, which is dangerous and increase the possibility of car accidents.

The creation of merge area.

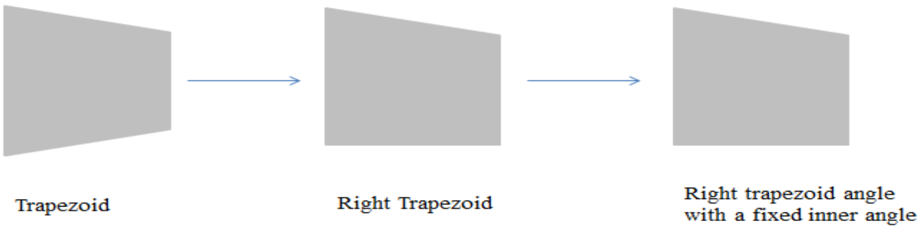


Figure 4

a. Trapezoid

The title of problem B is "Merge After Toll", so the premise is that the location of barrier tolls is already settled down. Upon most occasions, the barrier tolls are built to be perpendicular to the highway. Below this major premise, the merge area is obviously a trapezoid.

b. Right Trapezoid

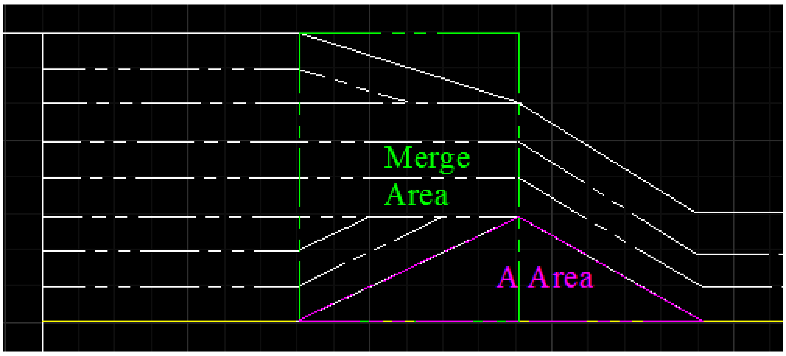


Figure 5

Figure 5 is a top-down view of the merge area below the premise that the merge area is a trapezoid. As shown in the figure, it is a most general condition, which means there is not any restriction of the trapezoid. There are mainly two disadvantages of this model. First of all, motorists finish the merge action before leaving the merge area. In the condition of figure XX, motorists still have to make two turns to reach the straight highways. The more turns motorists take, the higher car accident rate is in this section. Second, in this model, A Area is a unused area. Hence the land utilization of this part is relatively low. To sum up, it is better to keep lanes straight in the model. That means the shape of merge area is a right trapezoid.

c. Right trapezoid angle with a fixed inner angle.

To determine the inner angle of the right trapezoid, we have to simulate the vehicle stream in the merge area. There is a match of a running mode and a certain merge area shape that can reach a high throughput capacity. Obviously, no matter how good a model is, it cannot guarantee a high throughput capacity in all cases. (e.g. Traffic flow is extremely large. e.g. Motorists break traffic regulations by changing route.) We can only design a good model that can guarantee a relatively high throughput capacity compared to other models in the normal running situation. That is our design criterion and the aim we want to achieve.

The ideal running mode can be expressed as follows, by switching lines and merging to reach a result before leaving the merge area: vehicles come from the first two tollbooths run on the first lane, vehicles come from the next two tollbooths run on the second lane, and so on. To this end, the inner angle of the right trapezoid has to be fixed. The calculation and result of the inner angle are shown in the latter part in the paper.

3.2. Model Description:

The situations of the highway in the different areas, of course, are different. So the numbers of tollbooths and lanes are different. For better understanding and clearer expression, we use 8 tollbooths and 3 lanes in the model. Constructing a toll plaza like the figure 6 can help get a relatively lower accident prevention, higher throughput and lower cost. The details of the model are displayed in the following.

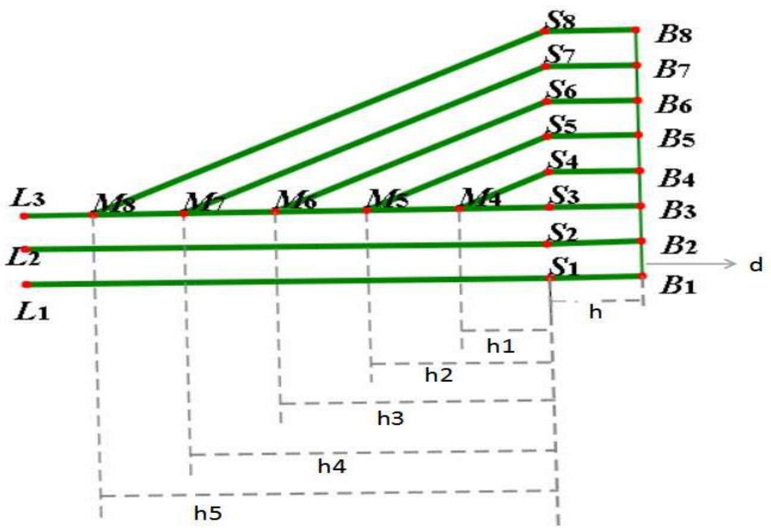


Figure 6

In the figure 6, B_1-B_8 represent 8 different tollbooths, L_1-L_3 represent 3 different lanes, M_4-M_8 represent 5 different intersections and S_1-S_8 represent 8 different exits of buffer area. The distance between two close tollbooths uses d said. The average length of sedan car uses l said. The driving speed of vehicles in the merge area uses v_0 said. The average acceleration of vehicles in the buffer area uses a_0 said. And h, h_1, h_2, h_3, h_4, h_5 have been expressed in the figure.

$$S_4M_4 = S_3M_4 + (l + \mathrm{s})$$

$$S_5M_5 = S_3M_5 + 2(l + \mathrm{s})$$
$$S_6M_6 = S_3M_6 + 3(l + \mathrm{s})$$
$$S_7M_7 = S_3M_7 + 4(l + \mathrm{s})$$
$$S_8M_8 = S_3M_8 + 5(l + \mathrm{s})$$

In conclusion,

$$S_{k+3}M_{k+3} = h_k + k(l + \mathrm{s})$$

According to the Pythagorean Theorem,

when $\mathrm{k} = 1,2,3 \ldots \mathrm{B} - \mathrm{L}$:

$$(h_k + (l + s) \times k)^2 = {h_k}^2 + (k \times d)^2$$
$$(l + s)^2 \times k^2 + 2 \times h_k \times (l + s) \times k = k^2 \times d^2$$
$$h_k = \frac{k(d^2 - (l + s)^2)}{2(l + s)}$$

So,

$$h_k = kh_1$$

Because,

$$S_3\mathrm{S}_{k+3} = k \times d = kS_3\mathrm{S}_4$$

So,
$$\Delta S_3S_4M_4 \sim \Delta S_3S_5M_5 \sim \Delta S_3S_6M_6$$

In general,

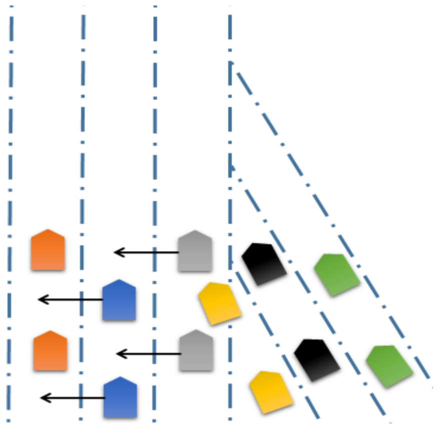
$$\Delta S_3S_4M_4 \sim \Delta S_kM_k \ \ , \ k = 4,5,6,7,8$$

So,
$$S_4M_4 // S_5M_5 // S_6S_6 // S_7S_7 // S_8S_8$$
$$h = \frac{v_0 \times v_0}{2 \times a_0}$$

Also, there are some rules that motorists should follow. Firstly, oversize vehicles can only pass the fan-in area through the road $B_8S_8M_8L_8$. Secondly, sedan cars can pass the fan-in area through all the roads except $B_8S_8M_8L_3$. Thirdly, in the intersection M_8, sedan cars should let oversize vehicles go first and sedan cars can choose to wait or move to road L_2.

3.3. Throughput capacity:

Firstly, we ignore the oversize vehicles and then add them in our model later. Suppose there are three lanes only and that the distance between the adjacent sedan cars is one length of a sedan car’s body plus two safe distance, we should design a plan to form the circumstance that the three lanes are full of sedan cars one by one with the only gap of safe distance and that no car need to wait. This is the plan which simple make those which comes from the first and the second tollbooth all merge into the first lane. The third and fourth ones are required to merge in the second lane, with the fifth and sixth ones just doing as the above management. Three lanes, that is to say, can extremely endure six tollbooths full of sedan cars coming continuously in the situation that no car was limited to go. In this case, this plan is optimal. The sketch map shows the regulation how we make these vehicles keep pace with each other.



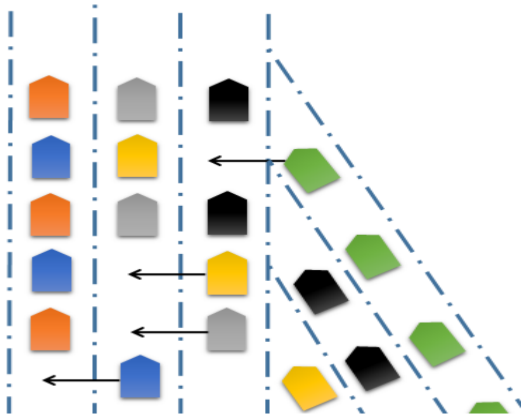
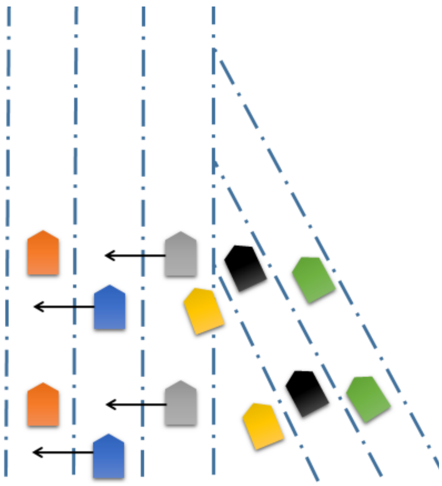


Figure 7 Figure 7

When vehicle distance extend to be n times of a sedan car's body length and n+1 times of safe distance, the lanes then can endure n times numbers of the capacity of lanes above in the consistent ways. What should be done is to merge n+1 sedan cars in one lanes from the high-speed side to the low-speed side. The whole lanes can extremely endure 3*(n+1) tollbooths full of sedan cars coming continuously according to the gap described in the beginning of the paragraph in the situation that no car was limited to go.

Another significant scope is when the vehicle distance is not an integer multiples of the vehicle length. According to the assumptions we make, sedan cars can only keep their speed or slow down to wait instead of accelerate to overtake, so its maximum throughput is certainly its round down times of the capacity of one lane in the consistent ways. The second sedan car in the first lane, for example, can't speed up to shorten the distance with the first sedan car in the first lane, which means these length is the minimum length between these two sedan cars. What we can make throughput larger is only able to insert one sedan car into these two sedan cars to obtain the maximum throughput.



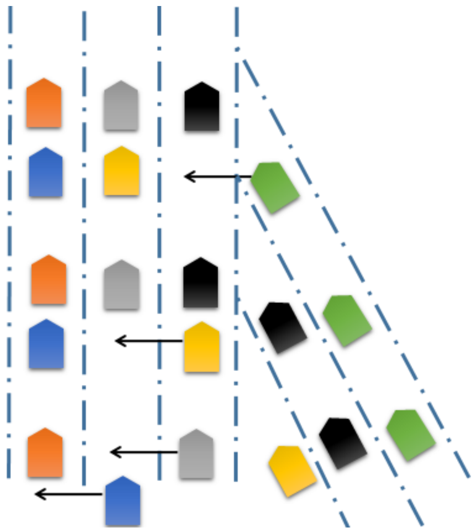


Figure 9 Figure 8

Now we make out a formula about the maximum throughput in our model.

When there are 2L sedan cars in the barrier toll, they start at the same time, so that the max throughput of the fin-in area can be calculated.

$$\begin{cases} k = \left\lfloor \dfrac{D-s}{l+s} \right\rfloor \\ Tp = \dfrac{1}{\dfrac{(k+1)(l+s)}{v} + \dfrac{(D-kl-(k+1)s)}{v}} \times (k+1) \times L \end{cases}$$

$\frac{(k+1)(l+s)}{v}$ represents the time which 2L sedan cars spend in passing the point where the end of the plaza joins the L outgoing traffic lanes.

$\frac{(D-kl-(k+1)s)}{v}$represents the time at which no cars pass the point.

Suppose oversize vehicles are double length of sedan cars' length. Because based on the statistics and the special lanes for oversize vehicles, we can easily calculate the time of oversize vehicles passing through the observation point during a period. The rule established in our model is that these vehicles possess the higher priority than sedan cars. We can take advantage of the results above to acquire the maximum throughput concerning the only sedan cars. If there are oversize vehicles, just use the special lanes, so we can also gain the maximum throughput in this condition. As for the actual total maximum throughput, it should consist of maximum throughput of the only sedan cars and that of the included oversize vehicles according to the percentage of the observation time whether oversize vehicles appeared.

Next we introduce the situation that there are oversize vehicles in the tollbooths. Suppose the part A where oversize vehicles included are invariable wouldn't influence another part B where only sedan cars are included. The sketch map will present the rules of the part B in our model. The figure 11 and 12 is when the oversize vehicle distance is the even times length of one sedan car's body and the figure 13 and 14 is when the oversize vehicle distance is the odd times length of one sedan car's body.

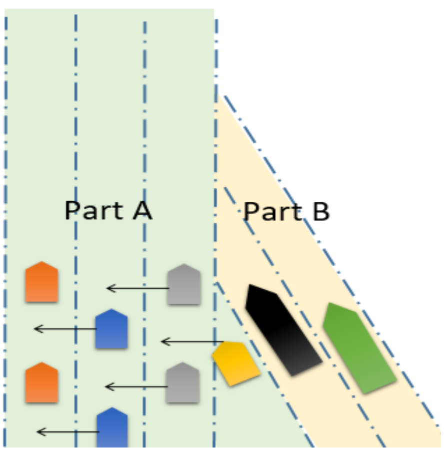


Figure 11

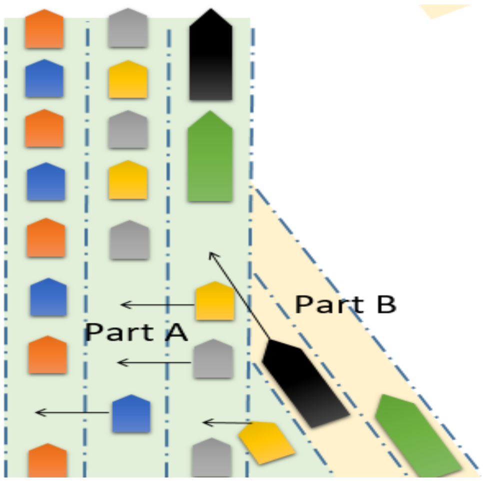


Figure 9

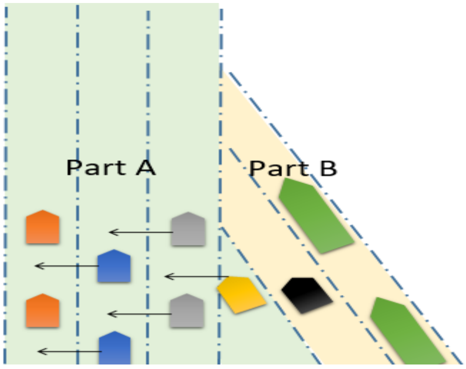


Figure 10

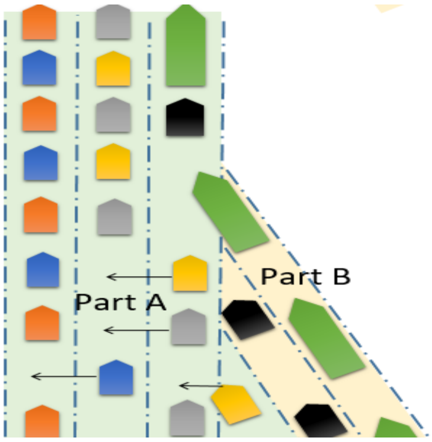


Figure 11

3.4. Cost:

Analysis about the cost:

The shape of the fan-in area consists of a right trapezoid (merge area) and a rectangle (buffer area), which is determined in the creation and description of the model. According to the area formula of a trapezoid:

$$S = \frac{1}{2}(a + b)h$$

In the model, $a = L \times d$, $b = B \times d$, $h = h_k$.

And the area formula of a rectangle:

$$S = a \times b$$

In the model, $a = L \times d$, $b = h$.

In the premise of the problem B, the number of lanes and the number of tollbooths are given. Which means the value of area only depends on h_k and h.

The cost of the buffer area.

The purpose to build a buffer area is to provide a distance for motorists to speed up and reach the required speed before entering the merge area. Because motorist have to fan in from B tollbooth egress lanes down to L lanes of traffic in the merge area, it is safer if motorists have reached a stable and appropriate speed before taking such action.

To calculate the h, it depends on vehicles' accelerated velocity and the required speed.

$$h = \frac{{v_0}^2}{2a} \ ; S = \frac{{v_0}^2 Ld}{2a}$$

The result of the cost of the buffer area:

$$C_B = \frac{{v_0}^2 Ldc}{2a}$$

The cost of the merge area.

In the premise that models' shape are right trapezoid. Any model of which height bigger than h_k will have a larger area than our model. So only models of which height smaller than h_k need to be taken into consideration. Obviously, models of which height smaller than h_k have a smaller area than our model. But the throughput capacity must be weaker. It has been proved that our model has the highest throughput capacity compared to other models. In this case, there is not a model which has both low cost and high throughput capacity. The cost shows negative relation with throughput capacity. According to our design philosophy,

throughput capacity has a higher priority, so we meet this condition first.
To calculate the cost of the merge area:

$$h_k = \frac{k(d^2 - (l+s)^2)}{2(l+s)}$$

The result of the cost of the merge area:

$$C_M = \frac{(Ld + Bd)k(d^2 - (l+s)^2)}{4(l+s)}$$

The total cost:

$$C = \frac{(Ld + Bd)k(d^2 - (l+s)^2)}{4(l+s)} + \frac{{v_0}^2 Ldc}{2a}$$

3.5. Accident prevention

There are two factors affecting the accident prevention. They are taking turns and changing lanes.

Take turn

Firstly, to avoid the bend, the way between B_k and L_k is straight ($k = 1,2,3..L$), which can decrease the possibility of traffic accidents. Secondly, there is no angle between the fan-in area and L lanes of travel, which means motorists don't need to change direction to enter a lane when they drive out of the fin-in area. So the probability of accident can be greatly decreased.

Change lanes

The probability of traffic accident will increase when the times of changing lanes become greater.

In a general model, to reach the maximum throughput, the least times of changing lanes can be calculated. The process will be presented in the following.

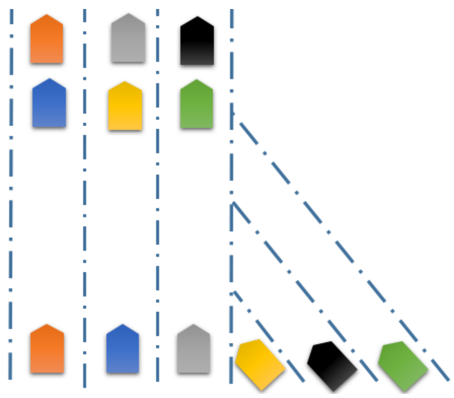


Figure 12

At first, all the vehicles are in the start of the merge area. Finally, they should all be in the one of lanes of highway. In order to reach the maximum throughput, they should arrange into a shape which has been presented in the figure xx.

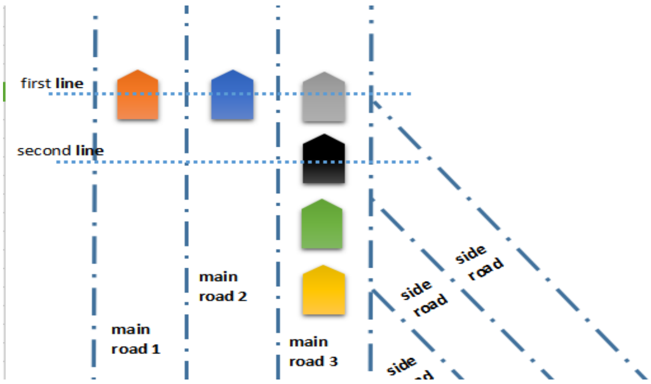


Figure 13

To arrange in that way, all the vehicles in the side roads should enter the main roads at first, which need $B-L$ times of changing lane.

Then to enter the second lane, $L-1$ cars should separately change lanes$L-1, L-2, L-3.....1$ times.

The total times of changing lanes to enter the second line is:

$$L-1+L-2+L-3+L-4+......+2+1=\frac{L\times(L-1)}{2}$$

The number of lines we need to form is:

$$\mathrm{N}=[\frac{B}{L}]$$

In the last line, the number of vehicles is :

$$la=B\%L$$

The total times of changing lanes to enter the last line is:

$$1+2+3+...+la-1=\frac{la\times(la-1)}{2}$$

So the least total times of changing lane (TOTAL TIMES) is :

When $B\%L=0$

$$TOTAL\ TIMES\ =\ B-L+\frac{L\times(L-1)\times[\frac{B}{L}]}{2}$$

Else,

$$TOTAL\ TIMES=B-L+\frac{L\times(L-1)\times\left(\left[\frac{B}{L}\right]-1\right)}{2}+\frac{la\times(la-1)}{2}$$

In the model presented before, if there are 8 cars in the barrier toll, by changing lanes 9 times, the throughput can be max.

Then, in the model: $B=8, L=3$

Put the value of B and L into the formula,

$$TOTAL\ TIMES=8-5+\frac{3\times2\times1}{2}+\frac{2\times1}{2}=9$$

Hence we can prove that the number of changing lanes is exactly the minimum value. And the car accident rate will be the lowest.

3.6. Conclusion:

All in all, in our design philosophy, throughput capacity has the highest priority. Hence we design a model that guarantee the highest throughput capacity, and it has a rather low cost and good capacity for car accident prevention at the same time.

4. Performance Index of the Model

4.1. Premise of 2nd model:

The following parts are performance of the model in light and heavy traffic, performance of the model when self-driving vehicles are added and performance of the model when proportion of different kind of tollbooths is different. Actually, these performances are results of testing our model in different situation. To make these tests have practical significance, we make some adjustments. Since motorists cannot always drive cars as our merge pattern describes in the first model in reality, we give another pattern which most motorist will do in reality.

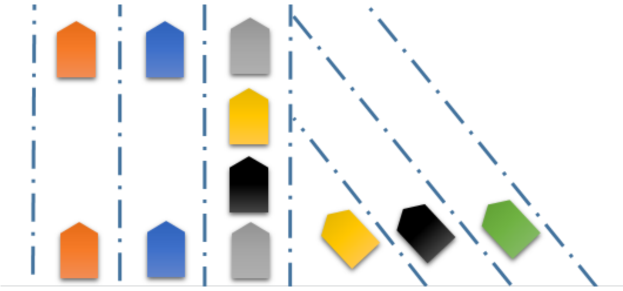


Figure 17

Figure 17 is a top view of the actual pattern (e.g.$B = 6,\ L = 3$). Motorists on the first and second lane will not changing lanes without external factors. In other words, motorists will not make room for other vehicles on their own initiative. Hence vehicles from the fourth to sixth tollbooths can only merge in the third lane.

Spread the pattern to B and L, motorists on the first and No.$(L-1)$ will always drive on their own lane without changing lanes. The rest vehicles will try to merge in the No. L lane. In addition, to make test results comparable, vehicles are uniformly regulated to leave tollbooths at the same time for analyzing.

4.2. Performance of the model in light and heavy traffic:

The throughput capacity of the model with the merge pattern in reality can be calculated:

Situation 1 (heavy traffic): the number of vehicles coming out of tollbooth n is bigger than $L + k$

a. First kind: $\boldsymbol{B > L + k}$

$$K = \left[\frac{D - s}{l + s}\right]$$

$$Tp = \frac{1}{\frac{(k+1)(l+s)}{v} + \frac{(D - kl - (k+1)s)}{v}} \times (k + L)$$

Explanation of the throughput capacity formula:

$$\frac{(k+1)(l+s)}{v}$$

represents the time which 2L sedan cars spend in passing the point where the end of the plaza joins the L outgoing traffic lanes.

$$\frac{(D-kl-(k+1)s)}{v}$$

represents the time at which no cars pass the point.

During the time

$$\frac{(k+1)(l+s)}{v}+\frac{(D-kl-(k+1)s)}{v}$$

There are $(k+L)$ vehicles passing the point.

b. First kind: $\boldsymbol{B \le L+k}$

$$K=\left\lceil\frac{D-s}{l+s}\right\rceil$$

$$Tp=\frac{1}{\frac{(k+1)(l+s)}{v}+\frac{(D-kl-(k+1)s)}{v}}\times B$$

The calculation method is just similar to the former one in 1. The only difference is the number of vehicles passing the point during the time:

$$\frac{(k+1)(l+s)}{v}+\frac{(D-kl-(k+1)s)}{v}$$

Situation 2 (heavy traffic): the number of vehicles coming out of tollbooth n is not bigger than $L+k$

$$K=\left\lceil\frac{D-s}{l+s}\right\rceil$$

$$Tp=\frac{1}{\frac{(k+1)(l+s)}{v}+\frac{(D-kl-(k+1)s)}{v}}\times n$$

4.3. Performance of the model when self-driving vehicles are added

Our solution change as more autonomous vehicles are added to the traffic mix in the following four situations. Firstly when all the vehicles are all self-driving cars which can exactly act up to our thinking to maximum the throughput, our model is completely applicative. Secondly we choose the condition in which all the vehicles are made up of common cars rather than self-driving cars, which is easy to discuss and whose driver ordinarily lack the awareness to give somebody the right of the way to achieve our throughput goal, and we have describe it in the part X in detail. Now we focus on the most sophisticated circumstances in which common cars and self-driving cars are mixed up. If the rate of the numbers of the self-driving cars are higher than the calculated value

$$\mathrm{r}=\frac{L+k-2}{B}$$

where: r is the rate of the numbers of the self-driving cars;

L is the numbers of lanes;

k is the numbers of sedans common cars able to insert between the continuously cars given out in one lane;

B is the numbers of tollbooths.

we can use the self-driving cars to force the common cars do as our model's ways. Suppose as a driver if the car's front area exit another car and its left lane is empty, he/she will make the car turn left. An example of explanations considering only one batch of cars is presented in the sketch map.

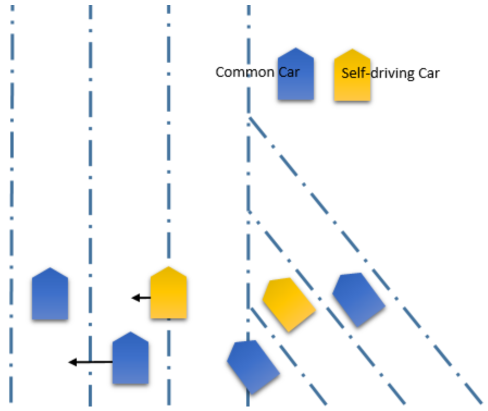


Figure18

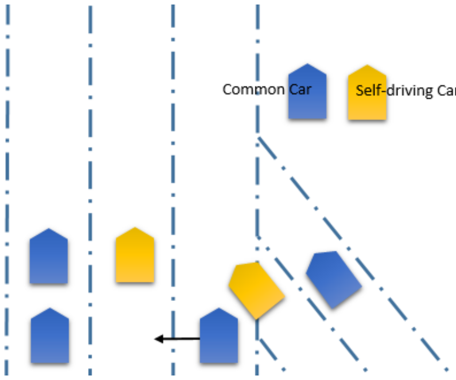


Figure 19

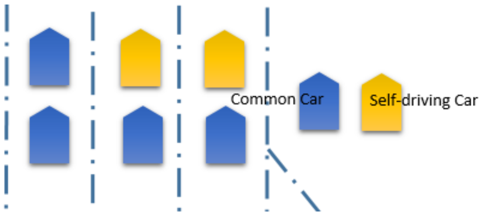


Figure 20

In this situation, the lanes' maximum throughput is the completely consistent with our model:

$$K = \left[\frac{D-s}{l+s}\right]$$

$$Tp = \frac{1}{\frac{(k+1)(l+s)}{v} + \frac{(D-kl-(k+1)s)}{v}} \times (k+L)$$

When the rate of the numbers of the self-driving cars are lower than the calculated value, just manage autonomous vehicles to try them best to cause compact lanes in the left straight lanes but last (the first and second lanes count from left side to right side in the sketch map) according to the first situation explained above and make common vehicles do as the second situation described.

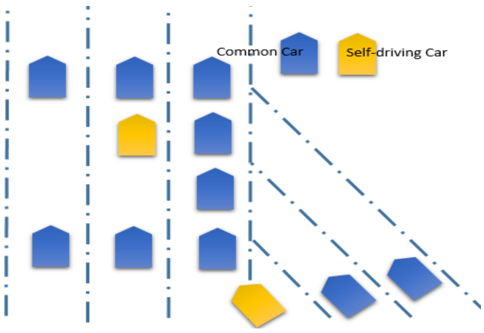


Figure 21

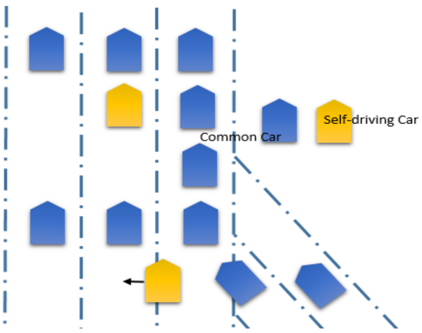


Figure 22

$$K = \left\lceil \frac{D - s}{l + s} \right\rceil$$

$$Tp = \frac{1}{\frac{(k+1)(l+s)}{v} + \frac{(D - kl - (k+1)s)}{v}} \times (k + L + a)$$

Where: a is the numbers of self-driving cars.

4.4. Performance of the model when proportions of conventional tollbooths, exact-change tollbooths, and electronic toll collection booths are different

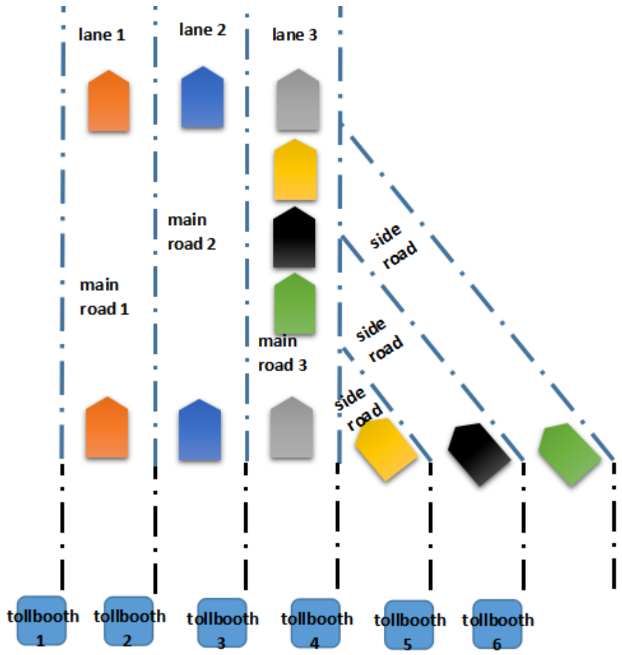


Figure 23

The time motorists spend in the tollbooth 1 uses t_1 said. The time motorists spend in the tollbooth 2 uses t_2 said...... The time motorists spend in the tollbooth k uses t_k said. The throughput of the lane 1 uses T_{p1} said. The throughput of the lane 2 uses T_{p2} said..... So the throughput of the lane k uses T_{pk} said.

Assumption: There are B vehicles in the start of the merge area at first. Then they enter the merge area at the same time.

In most of the time, motorists in the main road won't change their lane. So, the throughput of the lane k is: When k=1, 2, 3, 4,…,L-1,

$$T_{pk} = \frac{1}{\frac{t_k \times v + s}{v}} \times 1$$

The time that conventional (human-staffed) tollbooth will take to collect toll uses t_a said. Then the throughput of the lane which uses this kind of tollbooth is:

$$T_{pa} = \frac{1}{\frac{t_a \times v + s}{v}} \times 1$$

The time that exact-change (automated) tollbooths will take to collect toll uses t_b said. Then the throughput of the lane which uses this kind of tollbooth is:

$$T_{pb} = \frac{1}{\frac{t_b \times v + s}{v}} \times 1$$

The time that electronic toll collection booths will take to collect toll uses t_c said. Then the throughput of the lane which uses this kind of tollbooth is:

$$T_{pc} = \frac{1}{\frac{t_c \times v + s}{v}} \times 1$$

The number of vehicles entering in lane L from the side road uses A said. So, the throughput of the lane L is:

$$T_{pL} = \frac{1}{\frac{t_L \times v + s}{v}} \times (1 + A)$$

The total throughput is:

$$\text{Total throughput} = T_{pL} + \sum_{k=1}^{L-1} T_{pk}$$

Through the formula of the total throughput, it is easy to know that the proportion of conventional (human-staffed) tollbooths, exact-change (automated) tollbooths, and electronic toll collection booths (such as electronic toll collection via a transponder in the vehicle) will affect the total throughput.

The number of conventional (human-staffed) tollbooths uses a said. The number of exact-change (automated) tollbooths uses b said. The number of electronic toll collection booths (such as electronic toll collection via a transponder in the vehicle) uses c said.

If the distance between the vehicles in the lane L is the lowest, when all vehicles leave out of merge area, then the TPL will be the largest, then we place other faster tollbooths in the lane 1, 2, 3, …, L-1. As a result, we can get a relatively greater total throughput. May be it is not the largest throughput, but the difference value between it and maximum throughput must be very small.

According to the research, we've found that:

$t_a = 20\ s$; $t_b = 15\ s$; $t_c = 3\ s$;[4]

The length vehicle: $\mathrm{l} = 5\ \mathrm{m}$

Average speed: $\mathrm{v} = 20\ \mathrm{km/h} = 5.56\mathrm{m/s}$[5]

Safe distance: $\mathrm{s} = 5\mathrm{m}$

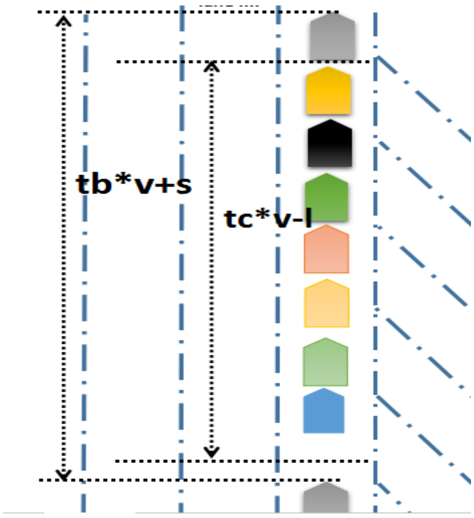


Figure 25

In the distance between vehicles in the lane L which use exact-change tollbooths, 7 vehicles can be placed there.

$$\left[\frac{t_b \times v - l}{l + s}\right] = 7$$

The throughput of lane L is:

$$8 \times T_{pb} = 1811$$

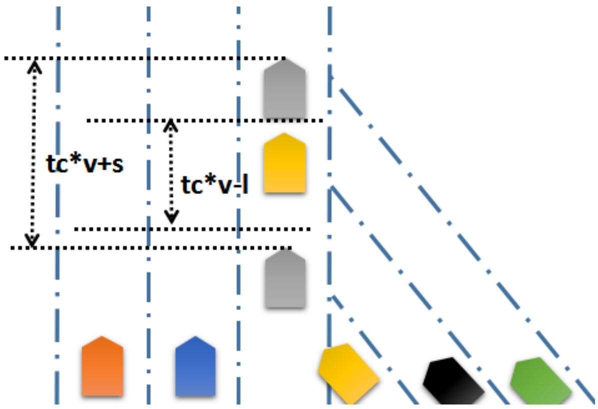


Figure 24

In the distance between vehicles in the lane L which use electronic toll collection booths, only 1 vehicle can be placed there.

$$\left[\frac{t_c \times v - l}{l + s}\right] = 1$$

The throughput of lane L is:

$$2 \times T_{pc} = 1847$$

In the distance between vehicles in the lane L which use conventional (human-staffed) tollbooths, only 10 vehicles can be placed there.

$$\left[\frac{t_a \times v - l}{l + s}\right] = 10$$

The throughput of the lane L ist: $11 \times T_{pb} = 1893$

To analyze the effect, we divide it into four parts.

1. $a >= 11$

We all use conventional (human-staffed) tollbooths in the tollbooth $L, L+1, L+2, L+$

$3, L+4, \ldots, L+10$. Then in the lane L, there will be ten vehicles entering in lane L from the side road, which means: $A = 10$;

If $c >= L-1$, We all use electronic toll collection booths in the tollbooth $1,2,3,\ldots,L-1$. Then the total throughput is:

$$TT_P = (L-1) \times T_{pc} + (1+10) \times T_{pa}$$

If $c < L-1, c+b >= L-1$, We all use electronic toll collection booths in the tollbooth $1,2,3,\ldots,c$. Then We all use exact-change tollbooths in the tollbooth $c+1, c+2,\ldots,L-1$. So the total throughput is:

$$TT_P = c \times T_{pc} + (L-1-c) \times T_{pb} + (1+10) \times T_{pa}$$

If $c < L-1, c+b < L-1$, We all use electronic toll collection booths in the tollbooth $1,2,3,\ldots,c$. Then We all use exact-change tollbooths in the tollbooth $c+1, c+2,\ldots,c+b$. Then We all use conventional (human-staffed) tollbooths in the tollbooth $c+b+1, c+b+2,\ldots.L-1$. So the total throughput is:

$$TT_P = c \times T_{pc} + b \times T_{pb} + (L-1-b-c) \times T_{pa} + (1+10) \times T_{pa}$$

2. $\boldsymbol{a < 11}$ & $\boldsymbol{c >= 2}$

We use electronic toll collection booths in the tollbooth $L, L+1$. Then in the lane L, there will be one vehicles entering in lane L from the side road, which means: $A = 1$;

If c-2>=L-1, We all use electronic toll collection booths in the tollbooth $1,2,3,\ldots,L-1$. Then total throughput is:

$$TT_P = (L-1) \times T_{pc} + (1+1) \times T_{pc}$$

If $c-2 < L-1, c-2+b >= L-1$, We all use electronic toll collection booths in the tollbooth $1,2,3,\ldots,c-2$. Then We all use exact-change tollbooths in the tollbooth $c-1, c+2,\ldots,L-1$. So the total throughput is:

$$TT_P = (c-2) \times T_{pc} + (\mathrm{L}-1-(\mathrm{c}-2)) \times T_{pb} + (1+1) \times T_{pc}$$

If $c-2 < L-1, c-2+b < L-1$, We all use electronic toll collection booths in the tollbooth $1,2,3,\ldots,c-2$. Then We all use exact-change tollbooths in the tollbooth $c-1, c+2,\ldots,c-2+b$. Then We all use conventional (human-staffed) tollbooths in the tollbooth $c+b-1, c+b+2,\ldots.L-1$. So the total throughput is:

$$TT_P = (c-2) \times T_{pc} + \mathrm{b} \times T_{pb} + (\mathrm{L}-1-\mathrm{b}-(\mathrm{c}-2)) \times T_{pa} + (1+1) \times T_{pc}$$

3. $\boldsymbol{a < 11}$ & $\boldsymbol{c < 2}$ & $\boldsymbol{b >= 8}$;

We use exact-change tollbooths in the tollbooth$L, L+1, L+2, L+3, L+4, L+5, L+6, L+7$. Then in the lane L, there will be 7 vehicles entering in lane L from the side road, which means: $A = 7$;

If $c >= L-1$, We all use electronic toll collection booths in the tollbooth $1,2,3,\ldots,L-1$. Then total throughput is:

$$TT_P = (L-1) \times T_{pc} + (1+7) \times T_{pb}$$

If $c < L-1, c+b-8 >= L-1$, We all use electronic toll collection booths in the tollbooth $1,2,3,\ldots,c$. Then We all use exact-change tollbooths in the tollbooth $c+1, c+2,\ldots,L-1$. So the total throughput is:

$$TT_P = c \times T_{pc} + (\mathrm{L}-1-\mathrm{c}) \times T_{pb} + (1+7) \times T_{pb}$$

If $c < L-1, c+b-8 < L-1$, We all use electronic toll collection booths in the tollbooth $1,2,3,\ldots,c$. Then We all use exact-change tollbooths in the tollbooth $c+1, c+2,\ldots,c+b-8$. Then We all use conventional (human-staffed) tollbooths in the tollbooth $c+b-7, c+b+2,\ldots.L-1$. So the total throughput is:

$$TT_P = c \times T_{pc} + (\mathrm{b} - 8) \times T_{pb} + (\mathrm{L} - 1 - \mathrm{c} - (\mathrm{b} - 8)) \times T_{pa} + (1 + 7) \times T_{pb}$$

4. $\boldsymbol{a < 11}$ & $\boldsymbol{c < 2}$ & $\boldsymbol{b < 8}$

We all use conventional (human-staffed) tollbooths in the tollbooth $L, L+1, L+2, L+3, L+4, \ldots, L+a-1$. Then in the lane L, there will be $a-1$ vehicles entering in lane L from the side road, which means: $A = a - 1$; The condition is similar to the first situation, so we won't explain it.

5. Validation

Taking the actual numerical values in New Jersey Turnpike into consideration, some results of our model can be calculated to be compared with the practical results. Firstly, we should calculate the throughput of the tollbooths.

$$K = \left\lfloor \frac{D - s}{l + s} \right\rfloor$$

$$Tp = \frac{1}{\frac{(k+1)(l+s)}{v} + \frac{(D - kl - (k+1)s)}{v}} \times (k + L)$$

Where: The server time of conventional tollbooth is approximately 20 seconds, and the average speed is 20 kilometers per hour, so the $D = \frac{20 \times 20}{3.6} = 111m$ because the speeding up process is extremely short;

$$\mathrm{s = 5ml{=}5m}; v = 20km/h; L = 6.$$

So,

$$k = 10\ ;\ Tp = 2758\ /h$$

That is to say, when all the vehicles are common vehicles and the tollbooths are all conventional tollbooths, we calculate the throughput of 6 lanes in New Jersey Turnpike is 2758 per hour in our model.

Then we calculate the surface of the "fan in" area in New Jersey Turnpike according to our model. The length of the buffer area is $h = \frac{v_0 \times v_0}{2 \times a_0}$

Where: $v0 = \frac{20km}{h}$; [6] $a = 2.5m/s^2$

so, $h = 6.17m.$

And the width of one lane is d, 12 feet, approximately 3.66 meters. So the surface of buffer area is $sf = h \times d = 22.6m2.$

Considering the merging part, the length of it is

$$h_k = \frac{k((kd)^2 - (l+s)^2)}{2l}$$

Where:

$$k = B - L = 16 - 6 = 10; d = 3.66; l = 5m;$$

so,

$$h_{10} = 382m$$

$$sm = \frac{(L+B) \times d \times h10}{2} = 15380m^2$$

so,

$$s = sf + sm = 15402m2$$

At last we talk about the times cars change the lanes.

$$TOTAL\ TIMES = B - L + \frac{L \times (L-1) \times \left(\left[\frac{B}{L}\right] - 1\right)}{2} + \frac{la \times (la-1)}{2}$$

Where:

$$B = 16, L = 6, la = 4,$$

So,

$$TOTAL\ TIMES = 31 \text{ per batch}$$

the times cars change the lanes per hour is

$$T = \frac{1}{\frac{(k+1)(l+s)}{v} + \frac{(D - kl - (k+1)s)}{v}} \times (TOTAL\ TIMES)$$

$$T = 5344 \text{ per hour}$$

6. Strengths and Weakness

Strengths:

In the premise of the given fan-in area shape and merge pattern, the throughput capacity can reach a theoretical maximum compared to all other models (It has been proved in the X.X):

$$\begin{cases} k = \left[\frac{D-s}{l+s}\right] \\ Tp = \frac{1}{\frac{(k+1)(l+s)}{v} + \frac{(D - kl - (k+1)s)}{v}} \times (k+1) \times L \end{cases}$$

Along with self-driving vehicles added in the future, the theoretical maximum of the throughput will become reality. In another word, with the development of science and technology, theoretical model will be the same as actual situation. The model we design has a long lifespan and it will be even better as time goes by.

In the existing level of science and technology, we can still make the theoretical level approach actual level by controlling proportions of different kinds of tollbooths in a barrier toll.

Our model is easy to be understood and put into practice, so it has a good portability. The model can be ported to barrier tolls in different highways.

Weakness:

Because we take the throughput capacity as the highest priority in our model, the cost is rather low but not the lowest. And the accident prevention capacity is good but not the best.

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[1]https://tieba.baidu.com/f?kz=4948420452&mo_device=1&ssid=0&from=1086k&uid=0&pu=usm@0,sz@320_1002,ta@iphone_2_6.0_2_6.0&bd_page_type=1&baiduid=0E80B9465F1E655F9201232E73E4A7FE&tj=www_normal_4_0_10_title?pn=0&

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[4]http://wenku.baidu.com/view/10c53d35b90d6c85ec3ac6dc.html

[5]

[6] Dilwyn Edwards, Mike Hamson. 1989. Guide to Mathematical Modeling. London: Macmillan Education LTD.