

$$1. A = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$B = \{10, 12, 14, 16, 18, 20\}$$

$$C = \{1, 3, 5, 7, 9\}$$

$$(a) A \cap C \cup B = \{2, 3, 5, 7, 11, 13, 17, 19\} \cap \{1, 3, 5, 7, 9\} \cup \{10, 12, 14, 16, 18, 20\}$$

$$= \{3, 5, 7, 10, 12, 14, 16, 18, 20\}$$

$$(b) A \cap B \cup C = \{2, 3, 5, 7, 11, 13, 17, 19\} \cap \{10, 12, 14, 16, 18, 20\} \cup \{1, 3, 5, 7, 9\}$$

$$= \{1, 3, 5, 7, 9\}$$

$$P(A \cap B \cup C) = \{\emptyset, \{1\}, \{3\}, \{5\}, \{7\}, \{9\}, \{1, 3\}, \{1, 5\}, \{1, 7\}, \{1, 9\},$$

$$\{3, 5\}, \{3, 7\}, \{3, 9\}, \{5, 7\}, \{5, 9\}, \{7, 9\}, \{1, 3, 5\},$$

$$\{1, 3, 7\}, \{1, 3, 9\}, \{1, 5, 7\}, \{1, 5, 9\}, \{1, 7, 9\}, \{3, 5, 7\}, \{3, 5, 9\},$$

$$\{3, 7, 9\}, \{5, 7, 9\}, \{1, 3, 5, 7\}, \{1, 3, 5, 9\}, \{1, 5, 7, 9\}, \{3, 5, 7, 9\},$$

$$\{1, 3, 5, 7, 9\}\}$$

$$(c) A - C = \{2, 11, 13, 17, 19\}$$

$$(d) |A| = 8$$

$$|B| = 6$$

$$|C| = 5$$

$$(e) A \cap C = \{3, 5, 7\}$$

$$|P(A \cap C)| = 2^3 = 8$$

$$(f) B = \{10, 12, 14, 16, 18, 20\}$$

$$C' = \{2, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

$$(g) \therefore B \subset C' \text{ False}$$

$$(g) \text{ True}$$

$$2. (a) (A - C') \cup (B - C) = (A \cap C'') \cup (B \cap C') \text{ Set difference law}$$

$$= (A \cap C) \cup (B \cap C')$$

$$\therefore (A - C') \cup (B - C) \text{ is not equal to } A \cup B.$$

$$(b) (A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B') \text{ Set difference law}$$

$$= A \cup (B \cap B') \text{ Distributive law}$$

$$= A \cup \emptyset \text{ Complement law}$$

$$= A \text{ Identity law (shown)}$$

$$\therefore (A \cap B) \cup (A - B) \text{ is equal to } A$$



3. (a)  $S = \{a, b, c, d, e, f, g\}$

$T = \{h, i, j, k, l, m, n, p, q\}$

$E = \{r, s, t, u, v, w, x, y, z\}$

(b)  $T \cap E = \{p, q\}$

$S \times (T \cap E) = \{a, b, c, d, e, f, g\} \times \{p, q\}$

$= \{(a, p), (a, q), (b, p), (b, q), (c, p), (c, q), (d, p), (d, q), (e, p), (e, q), (f, p), (f, q), (g, p), (g, q)\}$

4. (a) TRUE

(b) TRUE

5. (a)

p	q	r	$\neg r$	$p \wedge r$	$q \vee \neg r$	$p \vee q$	$Q = (p \wedge r) \vee (q \vee \neg r)$	$R = (p \vee q) \vee \neg r$
T	T	T	F	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	T
T	F	F	T	F	T	T	T	T
F	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	F	F	F	F	F	F
F	F	F	T	F	T	F	T	T

$\therefore$  Since the truth values of Q and truth values of R are the same,  $Q \equiv R$ .

(b)

p	q	r	$\neg q$	$p \wedge r$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$q \vee r$	$Q = (p \wedge r) \vee \neg(p \wedge \neg q)$	$R = (p \wedge r) \rightarrow (q \vee r)$
T	T	T	F	T	F	T	T	T	T
T	T	F	F	F	F	T	T	T	T
T	F	T	T	T	T	F	T	T	F
T	F	F	T	F	T	F	F	F	T
F	T	T	F	F	F	T	T	T	T
F	T	F	F	F	F	T	T	T	T
F	F	T	T	F	F	T	T	T	T
F	F	F	T	F	F	T	F	T	T

$\therefore$  Since the truth values of Q and R are different,  $Q \not\equiv R$ .



6. (a) Not true. counterexample is 1.

1 is an odd number and less than 7.

(b) Not true. counterexample is 9.

9 is odd number and greater than 7.

7.  $\exists x (P(x) \wedge Q(x))$

8.  $a$  is odd.

let  $a = 2n+1$ , where  $n \in \mathbb{Z}$ .

$$a^2 - 3a = (2n+1)^2 - 3(2n+1)$$

$$= 4n^2 + 4n + 1 - (6n + 3)$$

$$= 4n^2 + 4n + 1 - 6n - 3$$

$$= 4n^2 - 2n - 2$$

$$= 2(2n^2 - n - 1)$$

$$= 2k \quad (\text{proven})$$

$\therefore$  There exists an integer  $k$ , such that  $a^2 - 3a = 2k$ . Therefore,  $a^2 - 3a$  is even.

9. Contradiction: Suppose  $n^2$  is an odd integer, then  $n$  is not odd. Hence,  $n$  is an even.

$$n = 2a, \text{ where } a \in \mathbb{Z}$$

$$n^2 = (2a)^2$$

$$= 4a^2$$

$$= 2(2a^2) \text{ where } 2a^2 \text{ is an integer and } k = 2a^2$$

$$= 2k$$