112-2 Linear algebra Chapter_1 assignment

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Linear Equations

Determine whether the equation is linear in the variables x and y.

(1) $2xy - 6y = 0$

Nonlinear

(2) $e^{-2}x + 5y = 8$

Linear

(3) $\frac{x}{2} - \frac{y}{4} = 0$

Linear

Parametric Representation

Find a parametric representation of the solution set of the linear equation.

(1) $3x_1 + 2x_2 - 4x_3 = 0$

$$\begin{array}{lccccccc}
Let & x_1 & = & t & , & x_2 & = & t \\
 & 3t & + & 2t & - & 4x_3 & = & 0 \\
 & & & & - & 4x_3 & = & -5t \\
 & & & & & x_3 & = & \frac{5}{4}t
\end{array}$$

$$Solution\ set: \left\{\left(t, t, \frac{5}{4}t\right) \mid t \in \mathrm{R}\right\}$$

Graphical Analysis

Graph the system of linear equations. Solve the system and interpret your answer.

(1)

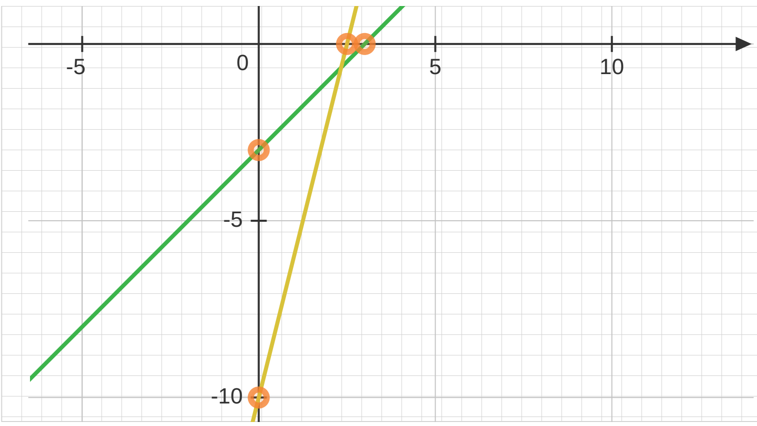
$$
\begin{array}{rcrcr}
x & = & y & + & 3 \\
4x & = & y & + & 10
\end{array}
\qquad
\begin{array}{l}
(0,-3),\ (3,0) \\
(0,-10),\ \left(\frac{5}{2},0\right)
\end{array}
$$

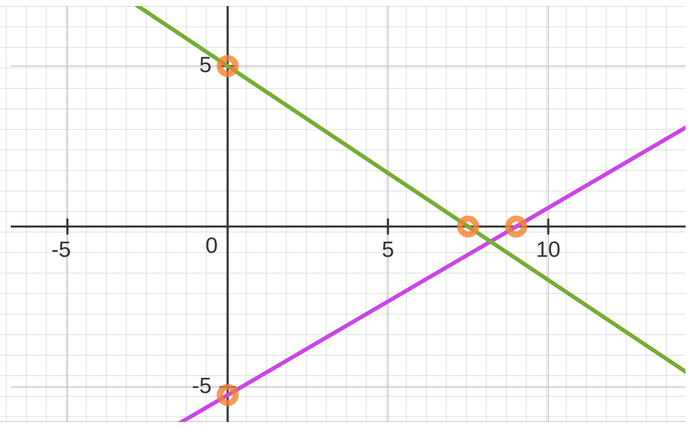
Exactly one solution

(2)

$$
\begin{array}{rcrcr}
\frac{1}{3}x & - & \frac{4}{7}y & = & 3 \\
2x & + & 3y & = & 15
\end{array}
\qquad
\begin{array}{l}
\left(0,-\frac{21}{4}\right),\ (9,0) \\
(0,5),\ \left(\frac{15}{2},0\right)
\end{array}
$$

Exactly one solution





Matrix Size

`Determine the size of the matrix.`

(1) $\begin{bmatrix} 2 & 1 \\ -4 & -1 \\ 0 & 5 \end{bmatrix}$

The size of the matrix is 3×2

Augmented Matrix

`Find the solution set of the system of linear equations represented by the augmented matrix.`

(1) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x + 2y + 3z = 0$

$$\begin{array}{rcccccl} Let \ \ x & = & t & , & y & = & t \\ t & + & 2t & + & 3z & = & 0 \\ & & & & 3z & = & -3t \\ & & & & z & = & -t \end{array}$$

$Solution\ set: \{(t, t, -t) |\ t \in \mathrm{R}\}$

Row – Echelon Form

Determine whether the matrix is in row-echelon form. If it is, determine whether it is also in reduced row-echelon form.

$$(1) \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$It\ is\ in\ row-echelon\ form,\ but\ it\ isn't\ in\ reduced\ row-echelon\ form.$

$$(2) \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$It\ is\ in\ row-echelon\ form,\ it\ is\ also\ in\ reduced\ row-echelon\ form.$

System of Linear Equations

Solve the system using either Gaussian elimination with back-substitution or Gauss-Jordan elimination.

$$(1) \begin{bmatrix} 4 & 2 & 1 & 18 \\ 4 & -2 & -2 & 28 \\ 2 & -3 & 2 & -8 \end{bmatrix} \quad r_{12}^{(-1)}, r_{13}^{\left(-\frac{1}{2}\right)}$$

$$\Longrightarrow \begin{bmatrix} 4 & 2 & 1 & 18 \\ 0 & -4 & -3 & 10 \\ 0 & -4 & \frac{3}{2} & -17 \end{bmatrix} \quad r_{23}^{(-1)}, r_2^{(-4)}, \ \ r_3^{\left(\frac{2}{9}\right)}$$

$$\Longrightarrow \begin{bmatrix} 4 & 2 & 1 & 18 \\ 0 & 1 & 1 & -\frac{5}{2} \end{bmatrix} \quad r_{32}^{(-1)}, r_{31}^{(-1)}, \ r_{21}^{(-2)}, r_1^{\left(\frac{1}{4}\right)}$$

$$(2) \begin{bmatrix} 2 & 1 & 2 & 4 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{bmatrix} \quad r_1^{\left(\frac{1}{2}\right)}, r_{12}^{(-2)}, \ r_{13}^{(-2)}$$

$$\Longrightarrow \begin{bmatrix} 1 & \frac{1}{2} & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & 4 & -2 \end{bmatrix} r_{21}^{\left(-\frac{1}{2}\right)}, r_{23}^{(2)}$$

$$\Longrightarrow \begin{bmatrix} 1 & 0 & 2 & \frac{3}{2} \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & -6 \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -6 \end{bmatrix}$$

$x = 5$
$y = 2$
$z = -6$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$let\ z = t$
$x = -t + \frac{3}{2}$
$y = 2t + 1$

$$(3) \begin{bmatrix} 2 & 0 & 6 & -9 \\ 3 & -2 & 11 & -16 \\ 3 & -1 & 7 & -11 \end{bmatrix} \quad r_1^{\left(\frac{1}{2}\right)}, r_{12}^{(-3)},\ r_{13}^{(-3)}$$

$$\Longrightarrow \begin{bmatrix} 1 & 0 & 3 & -\frac{9}{2} \\ 0 & -2 & 2 & -\frac{5}{2} \\ 0 & -1 & -2 & \frac{5}{2} \end{bmatrix} \quad r_{23},\ r_2^{(-1)}, r_{23}^{(2)}$$

$$\Longrightarrow \begin{bmatrix} 1 & 0 & 3 & -\frac{9}{2} \\ 0 & 1 & 2 & -\frac{5}{2} \\ 0 & 0 & 6 & -\frac{15}{2} \end{bmatrix} \quad r_3^{\left(\frac{1}{6}\right)}, r_{31}^{(-3)}, r_{32}^{(-2)}$$

$$\Longrightarrow \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} \end{bmatrix}$$

$x = -\frac{3}{4}$
$y = 0$

$$(4) \begin{bmatrix} 2 & 5 & -19 & 34 \\ 3 & 8 & -31 & 54 \end{bmatrix} \quad r_1^{\left(\frac{1}{2}\right)}, r_{12}^{(-3)}$$

$$\Longrightarrow \begin{bmatrix} 1 & \frac{5}{2} & -\frac{19}{2} & 17 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 3 \end{bmatrix} r_2^{(2)}, r_{21}^{\left(-\frac{5}{2}\right)}$$

$$\Longrightarrow \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & -5 & 6 \end{bmatrix}$$

$let\ x_3 = t$
$x_1 = -3t + 2$
$x_2 = 5t + 6$

$z = -\frac{5}{4}$

Homogeneous System

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Solve the homogeneous system of linear equations.
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(1) $\begin{bmatrix} 2 & 4 & -7 & 0 \\ 1 & -3 & 9 & 0 \end{bmatrix} r_{12},\ r_{12}^{(-2)}$

$\Longrightarrow \begin{bmatrix} 1 & -3 & 9 & 0 \\ 0 & 10 & -25 & 0 \end{bmatrix} r_2^{\left(\frac{1}{10}\right)}, r_{21}^{(3)}$

$\Longrightarrow \begin{bmatrix} 1 & 0 & \frac{3}{2} & 0 \\ 0 & 1 & -\frac{5}{2} & 0 \end{bmatrix}$

Let $x_3 = t$

$x_1 = -\left(\frac{3}{2}\right)t$

$x_2 = \frac{5}{2}t$

(2) $\begin{bmatrix} 1 & 3 & 5 & 0 \\ 1 & 4 & \frac{1}{2} & 0 \end{bmatrix} r_{12}\ ^{(-1)}$

$\Longrightarrow \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & -\frac{9}{2} & 0 \end{bmatrix} r_{21}\ ^{(-3)}$

$\Longrightarrow \begin{bmatrix} 1 & 0 & \frac{37}{2} & 0 \\ 0 & 1 & -\frac{9}{2} & 0 \end{bmatrix}$

Let $x_3 = t$

$x_1 = -\left(\frac{37}{2}\right)t$

$x_2 = \frac{9}{2}t$