112-2 Linear algebra Chapter_2 assignment

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Operations with Metrices

Perform the matrix operation

(1) $\begin{bmatrix} 1 & 5 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} 6 & -2 & 8 \\ 4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 26 & -2 & 8 \\ -4 & -4 & 16 \end{bmatrix}$

Solving a System of Linear Equations

(1) $\begin{bmatrix} 2 & 3 & 1 \\ 2 & -3 & -3 \\ 4 & -2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 22 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 & 1 & 10 \\ 2 & -3 & -3 & 22 \\ 4 & -2 & 3 & -2 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -6 \end{bmatrix}$ (*Gauss – Jordan elimination*)

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -6 \end{bmatrix}$

Finding and Multiplying with a Transpose

Find A^T, A^TA, *and* AA^T

(1)

$A = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$

$A^TA = \begin{bmatrix} 13 & -3 \\ -3 & 1 \end{bmatrix}$

$$AA^T = \begin{bmatrix} 10 & 6 \\ 6 & 4 \end{bmatrix}$$

Find the inverse of matrix

(1) $A = \begin{bmatrix} 4 & -1 \\ -8 & 2 \end{bmatrix}$

$det(A) = 4*2 + (-1)(-8) = 0$

the inverse of A doesn't exist.

(2) $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & -3 & -3 \\ 4 & 0 & 3 \end{bmatrix}$

$$\begin{bmatrix} 2 & 3 & 1 & 1 & 0 & 0 \\ 2 & -3 & -3 & 0 & 1 & 0 \\ 4 & 0 & 3 & 0 & 0 & 1 \end{bmatrix} r_{12}^{(-1)}, r_{13}^{(-2)}, r_{1}^{(1/2)} \Longrightarrow \begin{bmatrix} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & -6 & -4 & -1 & 1 & 0 \\ 0 & -6 & 1 & -2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & -6 & -4 & -1 & 1 & 0 \\ 0 & -6 & 1 & -2 & 0 & 1 \end{bmatrix} r_{23}^{(1)}, r_{21}^{(3/12)}, r_{2}^{(-1/6)}, r_{31}^{(1/2)}, r_{32}^{(-2/3)} \Longrightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{3}{20} & \frac{3}{20} & \frac{1}{10} \\ 0 & 1 & 0 & \frac{3}{10} & -\frac{1}{30} & -\frac{2}{15} \\ 0 & 0 & 1 & -\frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{3}{20} & \frac{3}{20} & \frac{1}{10} \\ \frac{3}{10} & -\frac{1}{30} & -\frac{2}{15} \\ -\frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

Using the Inverse of Matrix

(1) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$, *then* $A^{-1} = \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{3}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A^{-1}b = \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{3}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(2) $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 2 & 1 \\ 4 & -3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -7 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{5}{18} & \frac{1}{9} & \frac{1}{6} \\ -\frac{8}{9} & \frac{4}{9} & -\frac{1}{3} \\ \frac{17}{18} & -\frac{2}{9} & \frac{1}{6} \end{bmatrix}$

$x = A^{-1}b = \begin{bmatrix} \frac{5}{18} & \frac{1}{9} & \frac{1}{6} \\ -\frac{8}{9} & \frac{4}{9} & -\frac{1}{3} \\ \frac{17}{18} & -\frac{2}{9} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -7 \end{bmatrix} = \begin{bmatrix} -\frac{23}{18} \\ \frac{17}{9} \\ -\frac{17}{18} \end{bmatrix}$

Solving the Matrix Equation

Find A.

(1) $(2A)^{-1} = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$A = \left(A^{-1}\right)^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & \frac{1}{2} \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix}$$

Nonsingular Matrix

Find x such that the matrix A is nonsingular.

(1) $A = \begin{bmatrix} 2 & x \\ 1 & 4 \end{bmatrix}$

$2 \times 4 - x \neq 0 \implies 8 - x \neq 0 \implies x \neq 8$

Finding the Inverse of an Elementary Matrix

(1) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Finding a Sequence of Elementary Matries

(1) $A = \begin{bmatrix} -3 & 13 \\ 1 & -4 \end{bmatrix}$

$A \cdot R_{12} \cdot R_{12}^{(3)} = I$

$A = (R_{12})^{-1} \cdot \left(R_{12}^{(3)}\right)^{-1}$

(2) $A = \begin{bmatrix} 3 & 0 & 6 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$

Find the LU – Factorization of a matrix

(1) $A = \begin{bmatrix} -3 & 1 \\ 12 & 0 \end{bmatrix}$

(2) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

Solving a Linear System Using LU – Factorization

(1) $A = \begin{bmatrix} 2 & 1 & 1 & -1 & 7 \\ 0 & 3 & 1 & -1 & -3 \\ 0 & 0 & -2 & 0 & 2 \\ 2 & 1 & 1 & -2 & 8 \end{bmatrix}$