112-1 Calculus Chapter_6.1~6.4 Homework 2023/12/15

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P.330 #20, 23, 34
P.336 #37
P.341 #21, 39
P.346 #17, 20, 26

Find the integrals

1. $\int \frac{-1}{x(\ln x)^2}$ (*P*.330 #20)

Let $u = \ln x$

$du = \frac{1}{x}dx$

$$\int \frac{-1}{x(\ln x)^2}dx = -\int u^{-2}du$$
$$= \frac{1}{u} + C$$
$$= \frac{1}{\ln x} + C$$

2. $\int \frac{x^2}{x-1}dx$ (*P*.330 #23)

Long division

$$\frac{x^2}{x-1} = x + 1 + \frac{1}{x-1}$$

$$\int \frac{x^2}{x-1}dx = \int x \cdot dx + \int 1dx + \int \frac{1}{x-1}dx$$
$$= \frac{x^2}{2} + x + \ln|x-1| + C$$

Find $\frac{dy}{dx}$ *by logarithmic differentiation*

3. $y = \frac{(x^2+3)^{\frac{2}{3}}(3x+2)^2}{\sqrt{x+1}}$ (*P*.330 #34)

$$\ln y = \frac{2}{3}\ln(x^2+3) + 2\ln(3x+2) - \frac{1}{2}\ln(x+1)$$
$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{2}{3}\cdot\frac{2x}{x^2+3} + \frac{2\cdot 3}{3x+2} - \frac{1}{2(x+1)}$$
$$\frac{dy}{dx} = \frac{(x^2+3)^{\frac{2}{3}}(3x+2)^2}{\sqrt{x+1}}\left[\frac{4x}{3(x^2+3)} + \frac{6}{3x+2} - \frac{1}{2(x+1)}\right]$$

4. $f(x) = 3x^5 + x - 2$ (*P*.336 #37)

$f'(x) = 15x^4 + 1$

$y = 2$, $x = 1$

$$(f^{-1})'(2) = \frac{1}{f'(1)}$$
$$= \frac{1}{15+1}$$
$$= \frac{1}{16}$$

Find $D_x y$

5. $e^{xy} + xy = 2$ (*P*.341 #21) *Hint* : *Use implicit differentiation*

$$\frac{d}{dx}\left[e^{xy} + xy\right] = \frac{d}{dx}(2)$$

Find integral

6. $\int (x+3)e^{x^2+6x}dx$ (*P*.341 #39)

Find the indicated derivative or integral

7. $D_x\left(6^{2x}\right)$ (*P*.346 #17)

8. $D_x \log_{10}\left(x^3+9\right)$ (*P*.346 #20)

9. $\int_0^1 \left(10^{3x} + 10^{-3x}\right)dx$ (*P*.346 #26)