# 112-1 Discrete Mathematics Charpter 1-5~7

姓名：許嘉隆　　　　學號：412770116

## Chapter 1-5

4.

a) There is a student in class has taken a computer science course at school.
b) There is a student in class has taken all computer science courses at school.
c) All students in class have taken a computer science course at school.
d) There is a computer science course at school has taken by all students in class.
e) All computer science courses at school have taken by a student in class.
f) All students in class have taken all computer science courses at school.

6.

a) Randy Goldberg is enrolled in class CS 252.
c) Carol Sitea is enrolled all classes at school.
e) There are two different people, and the second person has attended all the courses that the first person has attended.

10.

a) $\forall x F(x, Fred)$
b) $\forall x F(Evelyn,\ x)$
c) $\forall x \exists y\ F(x, y)$
d) $\neg \exists x \forall y\ F(x, y)$
e) $\forall y \exists x\ F(x, y)$
f) $\neg \exists x (F(x, Fred) \land F(x, Jerry))$
g) $\exists y_1 \exists y_2 \Big[ F(Nancy,\ y_1) \land F(Nancy,\ y_2) \land (y_1 \neq y_2) \land \forall y \big( F(Nancy, y) \to (y = y_1) \lor (y = y_2) \big) \Big]$
h) $\exists y \Big[ \forall x F(x, y) \land \forall z \big( \forall x F(x, z) \to z = y \big) \Big]$
i) $\neg \exists x F(x, x)$
j) $\exists x \exists y \Big[ F(x, y) \land (x \neq y) \land \forall z \big( F(x, z) \to z = y \big) \Big]$

12.

b) $\neg C(Rachel, Chelsea)$

16. $Let\ P(s, c, m)\ =\ "Student\ s\ has\ class\ standing\ c\ and\ is\ majoring\ in\ m"$

a) $\exists s \exists m P(s,\ junior,\ m) \implies True$

b) $\forall s \exists c P(s, c, computer\ science) \implies False$

c) $\exists s \exists c \exists m[P(s, c, m) \wedge (m \neq mathemastics) \wedge (c \neq junior)] \implies True$

d) $\forall s P(s, sophomore, computer\ science) \implies False$

e) $\exists m \forall c \exists s\ P(s, c, m) \implies False$

# Chapter 1-6

6. *Let* $r =$ "*It rains*", $f =$ "*It is foggy*", $s =$ "*The sailing race will be held*", $l =$ "*The life saving demonstration*" *and* $t =$ "*The trophy will be awarded*".

$$\implies Premise: \begin{cases} 1. & (\neg r \vee \neg f) \to (s \wedge l) \\ 2. & s \to t \\ 3. & \neg t \end{cases} \qquad Conclusion:\ r$$

| Step | | Reason |
|---|---|---|
| 1. | $\neg t$ | *Premise* 3 |
| 2. | $s \to t$ | *Premise* 2 |
| 3. | $\neg s$ | *Modus tollens from* (1) *and* (2) |
| 4. | $(\neg r \vee \neg f) \to (s \wedge l)$ | *Premise* 1 |
| 5. | $\neg(s \wedge l) \to \neg(\neg r \vee \neg f)$ | *Contrapositive of* (4) |
| 6. | $\neg s \vee \neg l \to r \wedge f$ | *De Morgan's law from* (5) |
| 7. | $r \wedge f$ | *Modus ponens from* (2) *and* (6) |
| 8. | $r$ | *Simplification from* (7) |

14.

a)

$$\begin{aligned} Let\ s(x) &= \text{"x is a student in this class"} \\ r(x) &= \text{"x is one owns a red convertible"} \\ t(x) &= \text{"x is one got a speeding ticket"} \end{aligned}$$

$$\implies Premise: \begin{cases} 1. & s(Linda) \wedge r(Linda) \\ 2. & \forall x \big(r(x) \to t(x)\big) \end{cases} \qquad Conclusion:\ \exists x(s(x) \wedge t(x))$$

| Step | | | Reason |
|---|---|---|---|
| 1. | $s(Linda) \wedge r(Linda)$ | | *Premise* 1 |
| 2. | $s(Linda)$ | | *Simplification from* (1) |
| 3. | $r(Linda)$ | | *Simplification from* (2) |
| 4. | $\forall x\big(r(x) \to t(x)\big)$ | | *Premise* 2 |
| 5. | $r(y) \to t(y)$ | *for some y* | *Universal instantiation from* (4) |
| 6. | $t(y)$ | *for some y* | *Modus ponens from* (3) *and* (5) |
| 7. | $s(y) \wedge t(y)$ | *for some y* | *Conjunction from* (2) *and* (6) |
| 8. | $\exists x(s(x) \wedge t(x))$ | | *Existential generalization from* (7) |

b)

$$\begin{aligned} \text{Let } r(x) &= \text{"x is one of the five roommates listed"} \\ d(x) &= \text{"x has taken a course in discrete mathematics"} \\ a(x) &= \text{"x can take a course in algorithms"} \end{aligned}$$

$$\Longrightarrow Premise: \begin{cases} 1. & \forall x\big(r(x) \to d(x)\big) \\ 2. & \forall x\big(d(x) \to a(x)\big) \end{cases} \qquad Conclusion: \ \forall x\big(r(x) \to a(x)\big)$$

| Step | | | Reason |
|---|---|---|---|
| 1. | $\forall x\big(r(x) \to d(x)\big)$ | | *Premise* 1 |
| 2. | $r(y) \to d(y)$ | *for every y* | *Universal instantiation from* (1) |
| 3. | $\forall x\big(d(x) \to a(x)\big)$ | | *Premise* 2 |
| 4. | $d(y) \to a(y)$ | *for every y* | *Universal instantiation from* (3) |
| 5. | $r(y) \to a(y)$ | *for every y* | *Hypothetical syllogism from* (2) *and* (4) |
| 6. | $\forall x\big(r(x) \to a(x)\big)$ | | *Universal generalization from* (5) |

c)

$$\begin{aligned} \text{Let } m(x) &= \text{"movie produced by x"} \\ w(x) &= \text{"x is a wonderful movie"} \\ t(x) &= \text{"x is the theme of the movie"} \end{aligned}$$

$$\Longrightarrow Premise: \begin{cases} 1. & \forall x\big(m(John\ Sayles) \rightarrow w(x)\big) \\ 2. & m(John\ Sayles) \wedge t(coal\ miners) \end{cases} \qquad Conclusion: \ \exists x(w(x) \wedge t(coal\ miners))$$

| Step | | | Reason |
|---|---|---|---|
| 1. | $m(John\ Sayles) \wedge t(coal\ miners)$ | | *Premise* 2 |
| 2. | $m(John\ Sayles)$ | | *Simplification from* (1) |
| 3. | $t(coal\ miners)$ | | *Simplification from* (2) |
| 4. | $\forall x\big(m(John\ Sayles) \rightarrow w(x)\big)$ | | *Premise* 1 |
| 5. | $m(John\ Sayles) \rightarrow w(y)$ | *for every y* | *Universal instantiation from* (4) |
| 6. | $w(y)$ | *for every y* | *Modus ponens from* (2) *and* (5) |
| 7. | $w(y) \wedge t(coal\ miners)$ | *for every y* | *Conjunction from* (3) *and* (6) |
| 8. | $\exists x(w(x) \wedge t(coal\ miners))$ | | *Existential generalization from* (7) |

d)

$$\begin{aligned} Let\ c(x) &= \text{"}x \text{ is in this class"} \\ f(x) &= \text{"}x \text{ has been to France"} \\ l(x) &= \text{"}x \text{ has visited the Louvre"} \end{aligned}$$

$$\Longrightarrow Premise: \begin{cases} 1. & \exists x(c(x) \wedge f(x)) \\ 2. & \forall x\big(f(x) \rightarrow l(x)\big) \end{cases} \qquad Conclusion: \ \exists x(c(x) \wedge l(x))$$

| Step | | | Reason |
|---|---|---|---|
| 1. | $\exists x(c(x) \wedge f(x))$ | | *Premise* 1 |
| 2. | $c(y) \wedge f(y)$ | *for some y* | *Existential instantiation from* (1) |
| 3. | $c(y)$ | *for some y* | *Simplification from* (2) |
| 4. | $f(y)$ | *for some y* | *Simplification from* (2) |
| 5. | $\forall x\big(f(x) \rightarrow l(x)\big)$ | | *Premise* 2 |
| 6. | $f(y) \rightarrow l(y)$ | *for some y* | *Universal instantiation from* (5) |
| 7. | $l(y)$ | *for some y* | *Modus ponens from* (4) *and* (6) |
| 8. | $c(y) \wedge l(y)$ | *for some y* | *Conjunction from* (3) *and* (7) |
| 9. | $\exists x(c(x) \wedge l(x))$ | | *Existential generalization from* (8) |

28. $Prove: \ If \ n \ is \ a \ positive \ integer, \ then \ n \ is \ even \ if \ and \ only \ if \ 7n+4 \ if \ even.$

$$We \ need \ to \ prove: \begin{cases} 1. \quad n \ is \ even \rightarrow 7n+4 \ is \ even \\ 2. \quad 7n+4 \ is \ even \rightarrow n \ is \ even \end{cases}$$

1. $If \ n \ is \ even \Longrightarrow \ let \ n = 2k, \ k = 1,2,3, \ \ldots$
$then \ 7n+4 = 7(2k)+4 = 14k+4 = 2(7k+2)$
$\therefore \ 7n+4 \ is \ even$

2. $Suppose \ n \ is \ odd, \ then \ 7n+4 \ is \ odd \Longrightarrow \ let \ n = 2k+1, \ k = 0,1,2, \ \ldots$

$$\begin{aligned} then \ 7n+4 &= 7(2k+1)+4 \\ &= 14k+11 \\ &= 2(7k+5)+1 \\ \\ &\therefore \ 7n+4 \ is \ odd \end{aligned}$$

$$\because \ n \ is \ odd \rightarrow 7n+4 \ is \ odd \Longrightarrow True$$
$$\therefore \ 7n+4 \ is \ even \rightarrow n \ is \ even \Longrightarrow True \ (Contrapositive)$$

$By \ (1) \ and \ (2), \ n \ is \ even \leftrightarrow 7n+4 \ is \ even \quad \blacksquare$