112-1 Calculus Chapter-4 Homework 2023/11/24

學號：412770116　　　　　　姓名：許嘉隆

P.241 #22 #24 #26　P.251 #35 # 45 #55
P.258 #18 # 20　　P.259 #35 #37

1. (p.241 #22) $G(x) = \int_1^x xt\,dt$

$$
\begin{aligned}
G'(x) &= \frac{d}{dx}\left[x \cdot \int_1^x t\,dt\right] \\
&= 1 \cdot \int_1^x t + x \cdot x \quad (\textit{product rule}) \\
&= \left[\frac{1}{2}t^2\right]_1^x + x^2 \\
&= \frac{x^2-1}{2} + x^2 \\
&= \frac{3}{2}x^2 - \frac{1}{2}
\end{aligned}
$$

2. (p.241 #24) $G(x) = \int_1^{x^2+x} \sqrt{2z + \sin z} \cdot dz$

$$
\begin{aligned}
Let\ u &= x^2 + x \\
G'(x) &= \frac{d}{du}\int_1^u \sqrt{2z+\sin z} \cdot dz \cdot \frac{du}{dx} \\
&= \sqrt{2u + \sin u} \cdot (2x+1) \\
&= (2x+1) \cdot \sqrt{2\left(x^2+x\right) + \sin\left(x^2+x\right)}
\end{aligned}
$$

3. (p.241 #26) $G(x) = \int_{\cos x}^{\sin x} t^5 dt$

$$
\begin{aligned}
G'(x) &= \frac{d}{dx}\left[\int_{\cos x}^{\sin x} t^5 dt\right] \\
&= \frac{d}{dx}\left[\int_0^{\sin x} t^5 dt + \int_{\cos x}^0 t^5 dt\right] \\
&= \frac{d}{dx}\left[\int_0^{\sin x} t^5 dt - \int_0^{\cos x} t^5 dt\right] \\
&= \frac{d}{dx}\left[\int_0^{\sin x} t^5 dt\right] - \frac{d}{dx}\left[\int_0^{\cos x} t^5 dt\right] \\
&= \sin^5 x \cdot \cos x + \cos^5 x \cdot \sin x
\end{aligned}
$$

4. (p.251 #35) $\int_0^1 \left(x^2+1\right)^{10}(2x)\,dx$

$$
\begin{aligned}
Let\ u &= x^2 + 1 \\
du &= 2x \cdot dx
\end{aligned}
$$

$$
\begin{aligned}
\int_0^1 \left(x^2+1\right)^{10}(2x)\,dx &= \int_1^2 (u)^{10} du \\
&= \left[\frac{u^{11}}{11}\right]_1^2 \\
&= \frac{2048-1}{11} \\
&= \frac{2047}{11}
\end{aligned}
$$

5. (p.251 #45) $\int_0^1 (x+1)\left(x^2+2x\right)^2 dx$

6. (p.251 #55) $\int_0^{\frac{\pi}{2}} \sin x \sin(\cos x)\,dx$

$$\begin{aligned} \text{Let } u &= x^2 + 2x \\ du &= (2x+2)\,dx \\ &= 2(x+1)\,dx \end{aligned}$$

$$\begin{aligned} \int_0^1 (x+1)\left(x^2+2x\right)^2 dx &= \frac{1}{2}\int_0^3 u^2\,du \\ &= \left[\frac{u^3}{6}\right]_0^3 \\ &= \frac{27}{6} \\ &= \frac{9}{2} \end{aligned}$$

$$\begin{aligned} \text{Let } u &= \cos x \\ du &= -\sin x\,dx \end{aligned}$$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \sin x \sin(\cos x)\,dx &= -\int_1^0 \sin(u)\,du \\ &= [\cos u]_1^0 \\ &= 1 - \cos 1 \end{aligned}$$

7. (p.258 #18) $f(x) = x(1-x);\ [0,1]$

$$\begin{aligned} \frac{1}{(1-0)}\int_0^1 x(1-x)\,dx &= c(1-c) \\ \int_0^1 \left(-x^2+x\right) dx &= -c^2 + c \\ \left[-\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1 &= -c^2 + c \\ \frac{1}{6} &= -c^2 + c \\ c^2 - c + \frac{1}{6} &= 0 \\ c &= \frac{3 \pm \sqrt{3}}{6} \end{aligned}$$

8. (p.258 #20) $f(x) = |x|\ ;\ [-2,2]$

$$\begin{aligned} \frac{1}{2+2}\int_{-2}^{2} |x|\,dx &= |c| \\ \int_0^2 x + \int_{-2}^{0} -x &= 4 \cdot |c| \\ \left[\frac{x^2}{2}\right]_0^2 + \left[\frac{-x^2}{2}\right]_{-2}^0 &= 4 \cdot |c| \\ 2 + 2 &= 4 \cdot |c| \\ |c| &= 1 \\ c &= \pm 1 \end{aligned}$$

9. (p.259 #35) $\int_{-\pi}^{\pi} (\sin x + \cos x)\,dx$

$$\begin{aligned} \int_{-\pi}^{\pi} (\sin x + \cos x)\,dx &= \int_{-\pi}^{\pi} \sin x\,dx + 2\int_0^{\pi} \cos x\,dx \\ &= 0 + 2[\sin x]_0^{\pi} \\ &= 0 \end{aligned}$$

10. (p.259 #37) $\int_{-\pi/2}^{\pi/2} \frac{\sin x}{1+\cos x}\,dx$

$$\int_{-\pi/2}^{\pi/2} \frac{\sin x}{1+\cos x}\,dx = 0,$$

since $\frac{\sin x}{1+\cos x}$ *is odd.*