112-1 Discrete Mathematics Charpter 2-3

姓名：許嘉隆　　學號：412770116

2. Determine whether f is a function from $\mathbb{Z}$ to $\mathbb{R}$.

a) $f(n) = \pm n$ is not a function, because it have two y-values.

b) $f(n) = \sqrt{n^2+1}$ is a function from $\mathbb{Z}$ to $\mathbb{R}$

c) $f(n) = \dfrac{1}{n^2-4}$ is not a function, because if n = 2 or -2, $f(n)$ doesn't exist.

8. Find these values.

a) $\lfloor 1.1 \rfloor = 1$

b) $\lceil 1.1 \rceil = 2$

c) $\lfloor -0.1 \rfloor = -1$

d) $\lceil -0.1 \rceil = 0$

e) $\lceil 2.99 \rceil = 3$

f) $\lceil -2.99 \rceil = -2$

g) $\left\lfloor \dfrac{1}{2} + \left\lceil \dfrac{1}{2} \right\rceil \right\rfloor = \left\lfloor \dfrac{1}{2} + 1 \right\rfloor = 1$

h) $\left\lceil \left\lfloor \dfrac{1}{2} \right\rfloor + \left\lceil \dfrac{1}{2} \right\rceil + \dfrac{1}{2} \right\rceil = \left\lceil 0 + 1 + \dfrac{1}{2} \right\rceil = 2$

12. Determine whether each of these fucntions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one.

a) $f(n) = n-1$ is one-to-one, because $f(a) = a-1 = f(b) = b-1 \Longrightarrow a = b$.

b) $f(n) = n^2+1$ is not one-to-one, because $f(1) = f(-1) = 2\ and\ 1 \neq -1$.

c) $f(n) = n^3$ is one-to-one, because $f(a) = a^3 = f(b) = b^3 \Longrightarrow a = b$.

d) $f(n) = \left\lceil \dfrac{n}{2} \right\rceil$ is not one-to-one, because $f(1) = f(2) = 2\ and\ 1 \neq 2$.

14. Determine whether $f:\mathbb{Z}\times\mathbb{Z}\to\mathbb{Z}$ is onto.

a) $f(m,n)=2m-n$

$when\ m=0,\ n=-n$

$f(0,-n)=n$

$\therefore\ it\ is\ onto$

b) $f(m,n)=m^2-n^2$

$Let\ f(m,n)=2 \quad = \quad m^2-n^2$

$2 \quad = \quad (m+n)(m-n)$

$\Longrightarrow \quad Can't\ find\ m\ and\ n\ in\ the\ domain\ of\ definition\ is\ \mathbb{Z}$

$\therefore\ it\ is\ not\ onto$

c) $f(m,n)=m+n+1$

$when\ m=-1 \quad f(-1,n)=n$

$\therefore\ it\ is\ onto$

d) $f(m,n)=|m|-|n|$

$when\ m=0 \quad f(0,n)=-|n|$

$when\ n=0 \quad f(m,0)=|m|$

$\therefore\ it\ is\ onto$

e) $f(m,n)=m^2-4$

$Let\ f(m,n)=2 \quad = \quad m^2-4$

$2 \quad = \quad (m+2)(m-2)$

$\Longrightarrow \quad Can't\ find\ m\ in\ the\ domain\ of\ definition\ is\ \mathbb{Z}$

$\therefore\ it\ is\ not\ onto$