112-1 Discrete Mathematics Charpter 2-4~2-6

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chap2-4 ex2 , 10 ,12 , 16

2. a_8 ?

a) $2^7 = 128$
b) 7
c) $1+(-1)^8 = 2$
d) $-(-2)^8 = -256$

10. $a_0 \sim a_5$?

a) $a_n = -2a_{n-1},\ a_0 = -1$
$a_1 = 2$
$a_2 = -4$
$a_3 = 8$
$a_4 = -16$
$a_5 = 32$

b) $a_n = a_{n-1} - a_{n-2},\ a_0 = 2,\ a_1 = -1$
$a_2 = -1-(2) = -3$
$a_3 = -3-(-1) = -2$
$a_4 = -2-(-3) = 1$
$a_5 = 1-(-2) = 3$

c) $a_n = 3a_{n-1}^2,\ a_0 = 1$
$a_1 = 3$
$a_2 = 3\cdot 3^2 = 3^3$
$a_3 = 3\cdot 3^6 = 3^7$
$a_4 = 3\cdot 3^{14} = 3^{15}$
$a_5 = 3\cdot 3^{30} = 3^{31}$

d) $a_n = na_{n-1} + a_{n-2}^2,\ a_0 = -1,\ a_1 = 0$
$a_2 = 2\cdot 0 + (-1)^2 = 1$
$a_3 = 3\cdot 1 + 0^2 = 3$

$a_4 = 4 \cdot 3 + 1^2 = 13$
$a_5 \equiv 5 \cdot 13 \ \ + 3^2 = 74$

e)$a_n = a_{n-1} - a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = 1$, $a_2 = 2$
$a_3 = 2 - 1 + 1 = 2$
$a_4 = 2 - 2 + 1 = 1$
$a_5 = 1 - 2 + 2 = 1$

12. solution of the $a_n = -3a_{n-1} + 4a_{n-2}$

a) $a_n = 0$
$a_n = -3 \cdot 0 + 4 \cdot 0 = 0$

b) $a_n = 1$
$a_n = -3 \cdot 1 + 4 \cdot 1 = 1$

c) $a_n = (-4)^n$

$$\begin{aligned} a_n &= -3 \cdot (-4)^{n-1} + 4 \cdot (-4)^{n-2} \\ &= (-4)^{n-2}(12 + 4) \\ &= (-4)^{n-2} \cdot 4^2 \\ &= (-4)^n \end{aligned}$$

d) $a_n = 2(-4)^n + 3$

$$\begin{aligned} a_n &= -3 \cdot \left[2(-4)^{n-1} + 3\right] + 4 \cdot \left[2(-4)^{n-2} + 3\right] \\ &= -6(-4)^{n-1} - 9 + 8(-4)^{n-2} + 12 \\ &= (-4)^{n-2}(24 + 8) + 3 \\ &= 2 \cdot (-4)^n + 3 \end{aligned}$$

16. Find the solution of $\{a_n\}$

a) $a_n = -a_{n-1}$, $a_0 = 5$

$$
\begin{aligned}
a_n &= -a_{n-1} \\
&= -(-a_{n-2}) \\
&= (-1)^2 \cdot a_{n-2} \\
&\vdots \\
&= (-1)^n \cdot a_{n-n} \\
&= (-1)^n \cdot 5
\end{aligned}
$$

b) $a_n = a_{n-1} + 3,\ a_0 = 1$

$$
\begin{aligned}
a_n &= a_{n-1} + 3 \\
&= (a_{n-2} + 3) + 3 \\
&= a_{n-2} + 2 \cdot 3 \\
&\vdots \\
&= a_{n-n} + n \cdot 3 \\
&= 3n + 1
\end{aligned}
$$

c) $a_n = a_{n-1} - n,\ a_0 = 4$

$$
\begin{aligned}
a_n &= a_{n-1} - n \\
&= (a_{n-2} - n - 1) - n \\
&= [(a_{n-3} - n - 2) - n - 1] - n \\
&= a_{n-3} - 3n - 1 - 2 \\
&\vdots \\
&= a_{n-n} - n \cdot n - (1 + 2 + 3 \ldots n - 1) \\
&= a_0 - n^2 - \left(\frac{n(n-1)}{2}\right) \\
&= 4 - n^2 - \left(\frac{n(n-1)}{2}\right)
\end{aligned}
$$

d) $a_n = 2a_{n-1} - 3,\ a_0 = -1$

$$
\begin{aligned}
a_n &= 2a_{n-1} - 3 \\
&= 2[2a_{n-2} - 3] - 3 \\
&= 2^2 a_{n-2} - 2 \cdot 3 - 3 && = 2^2 a_{n-2} - 3(2+1) \\
&= 2^2[2a_{n-3} - 3] - 3(2+1) && = 2^3 a_{n-3} - 2^2 \cdot 3 - 3(2+1) \quad = 2^3 a_{n-3} - 3\left(2^2 + 2 + 1\right) \\
&\vdots \\
&= 2^n a_{n-n} - 3\left(2^0 + 2^1 + 2^2 + \dots + 2^{n-1}\right) && = 2^n a_0 - 3\frac{1 \cdot \left(2^n - 1\right)}{2-1} \quad = -2^n - 3\left(2^n - 1\right) \\
&= (-1)2^n(1+3) + 3 \\
&= (-1)2^{n+2} + 3
\end{aligned}
$$

e) $a_n = (n+1)a_{n-1},\ a_0 = 2$

$$
\begin{aligned}
a_n &= (n+1)(n \cdot a_{n-2}) && = (n+1)n \cdot a_{n-2} \\
&= (n+1)n[(n-1)a_{n-3}] && = (n+1)n(n-1)a_{n-3} \\
&\vdots \\
&= (n+1)(n)(n-1)\dots \cdot 2 \\
&= (n+1)! \cdot a_0 \\
&= 2(n+1)!
\end{aligned}
$$

f) $a_n = 2na_{n-1},\ a_0 = 3$

$$
\begin{aligned}
a_n &= 2 \cdot n[2 \cdot (n-1) \cdot a_{n-2}] && = 2 \cdot n[2 \cdot (n-1) \cdot a_{n-2}] \\
&= 2^2 \cdot n \cdot (n-1) \cdot a_{n-2} \\
&\vdots \\
&= 2^n \cdot a_{n-n} \cdot (n \cdot (n-1) \cdot \dots \cdot 1) \\
&= 2^n \cdot 3 \cdot n!
\end{aligned}
$$

g) $a_n = -a_{n-1} + n - 1,\ a_0 = 7$

$$
\begin{aligned}
a_n &= -[-a_{n-2} + n - 2] + n - 1 && = (-1)^2 a_{n-2} + [(n-1) - (n-2)] \\
&= (-1)^2[-a_{n-3} + n - 3] + [(n-1) - (n-2)] && = (-1)^3 \cdot a_{n-3} + [(n-1) - (n-2) + (n-3)] \\
&\vdots \\
&= (-1)^n \cdot a_{n-n} + \left[(n-1) - (n-2) + \dots + (-1)^{n-1}(n-n)\right] \\
&= \frac{2n - 1 + (-1)^n}{4}(-1)^n \cdot 7
\end{aligned}
$$

chap2-5 ex2,4

2. Determine whether each of these sets is finite, countably infinite, or uncountable.

a) *the integers greater than* 10
$11, 12, 13, \ldots \quad countably\ \ infinite$

b) *the odd negative integers*
$-1, -3, -5, \ldots \quad countably\ \ infinite$

c) *the integers with absolute value less than* $1,000,000$
$-999.999, \ \ldots, 0, 1, \ \ldots 999.999 \quad finite$

d) *the real numbers between* 0 *and* 2
$0.???, \ 1.??? \quad uncountable$

e) *the set* $A \times \mathbb{Z}^{+}$ *where* $A = \{2, 3\}$
$(2, 1), \ (3, 1), \ (2, 2), \ (3, 2), \ldots \quad countably\ infinite$

f) *the integers that are multiples of* 10
$10, \ 20, \ 30, \ \ldots \quad countably\ infinite$

4. Determine whether each of these sets is countable or uncountable.

a) *integers not divisible by* 3
$1, 2, 4, 5, 7, 8 \ldots \quad countable$

b) *integers divisible by* 5 *but not by* 7
$5, \ 10, \ \ldots \ 30, \ 40, \ldots \quad countable$

c) *the real numbers with decimal representations consisting of all* $1s$

$$\begin{matrix} .1 & .11 & .111 & .1111 & \ldots \\ 1 & 1.1 & 1.11 & 1.111 & \ldots \\ 11 & 11.1 & 11.11 & 11.111 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \end{matrix} \quad countable$$

d) *the real numbers with decimal representations of all* $1s$ *or* $9s$

$$\begin{matrix} .1 & .9 & .11 & .19 & .91 & .111 & .??? \\ 1.9 & 1.1 & 1.19 & 1.91 & 1.111 & 1.119 & 1.??? \end{matrix} \quad uncountable$$

chap2-6 4,28

4. Find the product AB, where

a) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 2 & 3 \\ -1 & 0 & 3 & -1 \\ -3 & -2 & 0 & 2 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 4 & -1 & -7 & 6 \\ -7 & -5 & 8 & 5 \\ 4 & 0 & 7 & 3 \end{bmatrix}$$

c) $A = \begin{bmatrix} 0 & -1 \\ 7 & 2 \\ -4 & -3 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 & 2 & 3 & 0 \\ -2 & 0 & 3 & 4 & 1 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 2 & 0 & -3 & -4 & -1 \\ 24 & -7 & 20 & 29 & 2 \\ -10 & 4 & -17 & -24 & -3 \end{bmatrix}$$

28. Find the Boolean product of A and B, where

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \textit{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A \odot B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$