112-1 Calculus Charpter 2.4~2.5 Homework

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Use differential formulas to prove the following expressions.

1. *Proof*: $D_x \tan x = \sec^2 x$

$$\begin{aligned} D_x \tan x &= D_x \frac{\sin x}{\cos x} \\ &= \frac{D_x(\sin x)\cos x - \sin x\, D_x(\cos x)}{\cos^2 x} \\ &= \frac{\cos x \cos x - \sin x\,(-\sin x)}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \qquad \blacksquare \end{aligned}$$

2. *Proof*: $D_x \sec x = \sec x \tan x$

$$\begin{aligned} D_x \sec x &= D_x \frac{1}{\cos x} \\ &= \frac{D_x(1)\cos x - 1 \cdot D_x(\cos x)}{\cos^2 x} \\ &= \frac{0 \cdot \cos x - (-\sin x)}{\cos^2 x} \\ &= \frac{\sin x}{\cos^2 x} \\ &= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \\ &= \sec x \tan x \qquad \blacksquare \end{aligned}$$

3. *Proof*: $D_x \cot x = -\csc^2 x$

$$
\begin{aligned}
D_x \cot x &= D_x \frac{\cos x}{\sin x} \\
&= \frac{D_x(\cos x)\,\sin x - \cos x\, D_x(\sin x)}{\sin^2 x} \\
&= \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x} \\
&= -\left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x}\right) \\
&= -\frac{1}{\sin^2 x} \\
&= -\csc^2 x \qquad\qquad \blacksquare
\end{aligned}
$$

4. *Proof*: $D_x \csc x = -\csc x \cot x$

$$
\begin{aligned}
D_x \csc x &= D_x \frac{1}{\sin x} \\
&= \frac{D_x(1)\,\sin x - 1 \cdot D_x(\sin x)}{\sin^2 x} \\
&= \frac{0 \cdot \sin x - (\cos x)}{\sin^2 x} \\
&= -\left(\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}\right) \\
&= -\csc x \cot x \qquad\qquad \blacksquare
\end{aligned}
$$

(Page 120, example 4)

5. *If* $y = \sin 2x$, *find* $\frac{dy}{dx}$ (*don't use the Chain Rule*) ($\frac{dy}{dx}$ *will be* $2\cos 2x$)

Let $y = 2 \sin x \cos x$. *Then*

$$
\begin{aligned}
\frac{dy}{dx} &= 2 \cdot [\cos x \cos x + \sin x(-\sin x)] \\
&= 2 \cdot (\cos x \cos x - \sin x \sin x) \\
&= 2 \cos 2x \qquad\qquad \blacksquare
\end{aligned}
$$

(Page 121, example 6)

6. *Find* $D_x\left(\dfrac{x^2(1-x)^3}{1+x}\right)$ (*use the Product Rule with* 3 *functions*).

($D_x\left(\dfrac{x^2(1-x)^3}{1+x}\right)$ *will be* $\dfrac{(1+x)(1-x)^2x(2-5x)-x^2(1-x)^3}{(1+x)^2}$)

The Product Rule with 3 *functions*: $(fgh)' = f'gh + fg'h + fgh'$

$$
\begin{aligned}
D_x\left(\frac{x^2(1-x)^3}{1+x}\right) &= D_x\left[x^2(1-x)^3(1+x)^{-1}\right] \\
&= \left[2x\cdot(1-x)^3(1+x)^{-1}\right] + \left\{x^2\cdot\left[3(1-x)^2\cdot -1\right]\cdot(1+x)^{-1}\right\} + \left\{x^2(1-x)^3\left[-1(1+x)^{-2}\cdot 1\right]\right\} \\
&= \left[2x\cdot(1-x)^3(1+x)^{-1}\right] + \left\{-3x^2(1-x)^2\cdot(1+x)^{-1}\right\} + \left[-x^2(1-x)^3(1+x)^{-2}\right] \\
&= \frac{2x\cdot(1-x)^3}{(1+x)} + \frac{-3x^2(1-x)^2}{(1+x)} + \frac{-x^2(1-x)^3}{(1+x)^2} \\
&= \frac{2x\cdot(1-x)^3\cdot(1+x)}{(1+x)^2} + \frac{-3x^2(1-x)^2\cdot(1+x)}{(1+x)^2} + \frac{-x^2(1-x)^3}{(1+x)^2} \\
&= \frac{2x\cdot(1-x)^3\cdot(1+x) + \left[-3x^2(1-x)^2\cdot(1+x)\right] + \left[-x^2(1-x)^3\right]}{(1+x)^2} \\
&= \frac{(1+x)(1-x)^2\cdot\left[2x\cdot(1-x) - 3x^2\right] - x^2(1-x)^3}{(1+x)^2} \\
&= \frac{(1+x)(1-x)^2\cdot\left(2x-5x^2\right) - x^2(1-x)^3}{(1+x)^2} \\
&= \frac{(1+x)(1-x)^2\cdot x(2-5x) - x^2(1-x)^3}{(1+x)^2} \qquad \blacksquare
\end{aligned}
$$