112-1 Discrete Mathematics Charpter 3-2

姓名：許嘉隆　　　　學號：412770116

chapter 3-2 ex2,6,8,14,34(a)

2. Determine whether each of these functions is $O\left(x^2\right)$?

a) $f(x) = 17x + 11$

$x > 1,\ 17x + 11 \leqslant 17x^2 + 11x^2 = 28x^2$

$k = 1,\ C = 28$

$\therefore f(x) = O\left(x^2\right)$

b) $f(x) = x^2 + 1000$

$x^2 + 1000 \leqslant x^2 + x^2 = 2x^2$

$x > \sqrt{1000}$

$k = \sqrt{1000},\ C = 2$

$\therefore\ f(x) = O\left(x^2\right)$

c) $f(x) = x \log x$

$x > 1,\ \log x \leqslant x \implies x \log x \leqslant x^2$

$k = 1,\ C = 1$

$\therefore f(x) = O\left(x^2\right)$

d) $f(x) = \dfrac{x^4}{2}$

$x > k,\ \dfrac{x^4}{2} \leqslant Cx^2$

$can't\ find\ k,\ C = \dfrac{x^2}{2}\ (not\ a\ constant)$

$\therefore f(x)\ is\ not\ O\left(x^2\right)$

e) $f(x) = 2^x$

$2^x \leqslant x^2, x = 1,\ C = 1,\ can't\ find\ x > k$

$\therefore f(x)\ is\ not\ O\left(x^2\right)$

f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

$\lfloor x \rfloor \cdot \lceil x \rceil \leqslant x(x+1) \leqslant x \cdot 2x = 2x^2$
$x > 1,\ C = 2,\ k = 1$
$\therefore f(x) = O\left(x^2\right)$

6. *Show that* $\dfrac{\left(x^3 + 2x\right)}{(2x+1)}$ *is* $O\left(x^2\right)$

$$x > 1,\ \frac{x^3 + 2x}{2x+1} \leqslant \frac{x^3 + 2x}{2x} \leqslant \frac{x^3 + 2x^3}{2x} = \frac{3x^3}{2x} = \frac{3}{2}x^2$$

$$k = 1,\ C = \frac{3}{2}$$

$$\therefore \frac{x^3 + 2x}{2x+1} \text{ is } O\left(x^2\right)$$

8. *Find the least integer n such that* $f(x)$ *is* $O\left(x^n\right)$ *for each of these functions?*

a) $f(x) = 2x^2 + x^3 \log x$
$x > 1,\ \log x \leqslant x \implies x^3 \log x \leqslant x^4$
$\implies 2x^2 + x^3 \log x \leqslant 2x^4 + x^4 = 3x^4$
$k = 1,\ C = 3$
$\therefore f(x) = O\left(x^4\right)$

b) $f(x) = 3x^5 + (\log x)^4$
$x > 1,\ 3x^5 + (\log x)^4 \leqslant 3x^5 + x^5 = 4x^5$
$k = 1,\ C = 4$
$\therefore f(x) = O\left(x^5\right)$

c) $f(x) = \dfrac{x^4 + x^2 + 1}{x^4 + 1}$

$$x > 1,\ \frac{x^4 + x^2 + 1}{x^4 + 1} \leqslant \frac{x^4 + x^4 + x^4}{x^4} \leqslant 3 \cdot x^0$$

$k = 1,\ C = 3$
$\therefore f(x) = O\left(x^0\right)$

d) $f(x) = \dfrac{x^3 + 5\log x}{x^4 + 1}$

$$x > 1, \frac{x^3 + 5\log x}{x^4 + 1} \leqslant \frac{x^3 + 5x^3}{x^4} \leqslant \frac{6}{x}$$

$k = 1,\ C = 6$

$\therefore f(x) = O\left(x^{-1}\right)$

14. *Determine whether x^3 is $O(g(x))$ for each of these functions $g(x)$.*

a) $g(x) = x^2$
$x > k,\ x^3 \leqslant Cx^2$
$can't\ find\ k,\ C = x\ (not\ a\ constant)$
$\therefore x^3\ is\ not\ O\left(x^2\right)$

b) $g(x) = x^3$
$x > 1,\ x^3 \leqslant x^3$
$k = 1,\ C = 1$
$\therefore x^3\ is\ O\left(x^3\right)$

c) $g(x) = x^2 + x^3$
$x > 1, x^3 \leqslant x^3 + x^2$
$k = 1,\ C = 1$
$\therefore x^3\ is\ O\left(x^2 + x^3\right)$

d) $g(x) = x^2 + x^4$
$x > 1,\ x^3 \leqslant x^4 + x^2$
$k = 1,\ C = 1$
$\therefore x^3\ is\ O\left(x^2 + x^4\right)$

e) $g(x) = 3^x$
$x^3 \leqslant 3^x,\ x > 1,$
$k = 1,\ C = 1$
$x^3\ is\ O\left(3^x\right)$

f) $g(x) = \dfrac{x^3}{2}$
$x > 1,\ k = 1,\ C = 2$
$x^3 \leqslant 2\dfrac{x^3}{2} = x^3$
$\therefore x^3\ is\ O\left(\dfrac{x^3}{2}\right)$

34. *Show that* $3x^2 + x + 1$ *is* $\Theta\left(3x^2\right)$ *by directly finding the constants* $k,\ C_1,$ *and* C_2.

a)

1. $x > 1,\ 3x^2 + x + 1 \geqslant 3x^2,\ k = 1,\ C_1 = 1$
2. $x > 1,\ 3x^2 + x + 1 \leqslant 3x^2 + x^2 + x^2 = 5x^2 = \frac{5}{3}\left(3x^2\right)$

   $k = 1,\ C_2 = \frac{5}{3}$

$\Longrightarrow x > 1,\ 3x^2 \leqslant 3x^2 + x + 1 \leqslant \frac{5}{3} \cdot 3x^2$

$\therefore 3x^2 + x + 1$ *is* $\Theta\left(3x^2\right)$