112-1 Discrete Mathematics Charpter 1-3

姓名：許嘉隆　　　　　　學號：412770116

4.

p	q	r	$p \vee q$	$p \wedge q$	$q \vee r$	$q \wedge r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T	F	F
T	F	T	T	F	T	F	T	T	F	F
T	F	F	T	F	F	F	T	T	F	F
F	T	T	T	F	T	T	T	T	F	F
F	T	F	T	F	T	F	T	T	F	F
F	F	T	F	F	T	F	T	T	F	F
F	F	F	F	F	F	F	F	F	F	F

6.

p	q	$-p$	$-q$	$p \wedge q$	$-(p \wedge q)$	$-p \vee -q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

10.

a) $-p \to -q$

$\equiv -(-p) \vee -q$ *by the conditional – disjunction equivalence*

$\equiv p \vee -q$ *by the double negation law*

b) $(p \vee q) \to -p$

$\equiv -(p \vee q) \vee -p$ *by the conditional – disjunction equivalence*

$\equiv (-p \wedge -q) \vee -p$ *by the De Morgan's laws*

$\equiv -p$ *by the absorption laws*

c) $(p \to -q) \to (-p \to q)$

$\equiv -(p \to -q) \vee (-p \to q)$ *by the conditional – disjunction equivalence*

$\equiv -(-p \vee -q) \vee (-(-p) \vee q)$ *by the conditional – disjunction equivalence*

$\equiv (p \wedge q) \vee p \vee q$ *by the De Morgan's laws*

$\equiv p \vee q$ *by the absorption laws*

12.

a,c)

p	q	$-p$	$p \vee q$	$[-p \wedge (p \vee q)] \to q$	$p \to q$	$[p \wedge (p \to q)] \to q$
T	T	F	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	F	T	F	T	T	T

b,d)

p	q	r	$p \vee q$	$p \to q$	$q \to r$	$p \to r$	$\left[(p \to q) \wedge (q \to r)\right] \to (p \to r)$	$\left[(p \vee q) \wedge (p \to r) \wedge (q \to r)\right] \to r$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	T	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

16.

a) $[-p \wedge (p \vee q)] \to q$

$\equiv -[-p \wedge (p \vee q)] \vee q$ *by the conditional – disjunction equivalence*

$\equiv p \vee -(p \vee q) \vee q$ *by the De Morgan's laws*

$\equiv (p \vee q) \vee -(p \vee q)$ *by the commutative and associative laws*

$\equiv T$ *because* $p \vee \neg p \equiv T$

b) $\left[(p \to q) \wedge (q \to r)\right] \to (p \to r)$

$\equiv -\left[(p \to q) \wedge (q \to r)\right] \vee (p \to r)$ *by the conditional – disjunction equivalence*

$\equiv -(p \to q) \vee -(q \to r) \vee (p \to r)$ *by the De Morgan's laws*

$\equiv -(-p \vee q) \vee -(-q \vee r) \vee (-p \vee r)$ *by the conditional – disjunction equivalence*

$\equiv (p \wedge -q) \vee (q \wedge -r) \vee -p \vee r$ *by the De Morgan's laws*

$\equiv [(p \wedge -q) \vee -p] \vee [(q \wedge -r) \vee r]$ *by the commutative and associative laws*

$\equiv [(p \vee -p) \wedge (-q \vee -p)] \vee [(q \vee r) \wedge (-r \vee r)]$ *by the distributive laws*

$\equiv [T \wedge (-q \vee -p)] \vee [(q \vee r) \wedge T]$ *because* $p \vee \neg p \equiv T$

$\equiv (-q \vee -p) \vee (q \vee r)$ *by the identity laws*

$\equiv (-q \vee q) \vee (-p \vee r)$ *by the commutative and associative laws*

$\equiv T \vee (-p \vee r)$ *because* $p \vee \neg p \equiv T$

$\equiv T$ *by the domination laws*

c) $\left[p \wedge (p \to q)\right] \to q$

$\equiv -\left[p \wedge (p \to q)\right] \vee q$ *by the conditional – disjunction equivalence*

$\equiv -p \vee -(p \to q) \vee q$ *by the De Morgan's laws*

$\equiv -p \vee -(-p \vee q) \vee q$ *by the conditional – disjunction equivalence*

$\equiv -p \vee (p \wedge -q) \vee q$ *by the De Morgan's laws*

$\equiv (-p \vee p) \wedge (-p \vee -q \vee q)$ *by the distributive and associative laws*

$\equiv T \wedge (-p \vee T)$ *because* $p \vee \neg p \equiv T$

$\equiv -p \vee T$ *by the identity laws*

$\equiv T$ *by the domination laws*

d) $\left[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)\right] \rightarrow r$

$\equiv$	$-\left[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)\right] \vee r$	*by the conditional – disjunction equivalence*
$\equiv$	$-(p \vee q) \vee -(p \rightarrow r) \vee -(q \rightarrow r) \vee r$	*by the De Morgan's laws*
$\equiv$	$-(p \vee q) \vee -(-p \vee r) \vee -(-q \vee r) \vee r$	*by the conditional – disjunction equivalence*
$\equiv$	$(-p \wedge -q) \vee (p \wedge -r) \vee (q \wedge -r) \vee r$	*by the De Morgan's laws*
$\equiv$	$(-p \wedge -q) \vee (p \wedge -r) \vee (q \vee r) \wedge (-r \vee r)$	*by the distributive laws*
$\equiv$	$(-p \wedge -q) \vee (p \wedge -r) \vee (q \vee r) \wedge T$	*because* $p \vee \neg p \equiv T$
$\equiv$	$(-p \wedge -q) \vee [(p \wedge -r) \vee r] \vee q$	*by the identity, commutative and associative laws*
$\equiv$	$(-p \wedge -q) \vee [(p \vee r) \wedge (-r \vee r)] \vee q$	*by the distributive laws*
$\equiv$	$(-p \wedge -q) \vee [(p \vee r) \wedge T] \vee q$	*because* $p \vee \neg p \equiv T$
$\equiv$	$[(-p \wedge -q) \vee p] \vee r \vee q$	*by the identity and associative laws*
$\equiv$	$[(-p \vee p) \wedge (-q \vee p)] \vee r \vee q$	*by the distributive laws*
$\equiv$	$[T \wedge (-q \vee p)] \vee r \vee q$	*because* $p \vee \neg p \equiv T$
$\equiv$	$(-q \vee q) \vee p \vee r$	*by the identity, commutative and associative laws*
$\equiv$	$T \vee p \vee r$	*because* $p \vee \neg p \equiv T$
$\equiv$	T	*by the domination laws*