112-1 Calculus Chapter_5.3~5.4 Homework 2023/12/08

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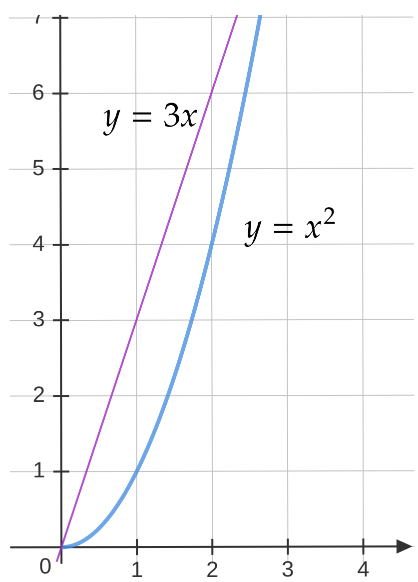
P.293 #8
P.300 #9 #22 #23

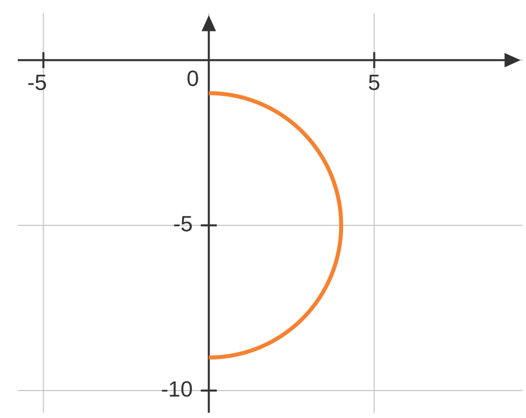
1. $(P.293\ \#8)$

$y = x^2,\ y = 3x;\ about\ the\ y-axis$



$$
\begin{aligned}
\Delta V &= 2\pi \cdot x \cdot \left(3x - x^2\right) \cdot \Delta x \\
V &= 2\pi \int_0^3 x \cdot \left(3x - x^2\right) dx \\
&= 2\pi \int_0^3 \left(3x^2 - x^3\right) dx \\
&= 2\pi \left[x^3 - \frac{x^4}{4}\right]_0^3 \\
&= 2\pi \left(27 - \frac{81}{4}\right) \\
&= \frac{27\pi}{2} \qquad \blacksquare
\end{aligned}
$$

2. $(P.300\ \#9)$



$x = 4\sin t, y = 4\cos t - 5$

$$
\begin{aligned}
\frac{dx}{dt} &= 4\cos t \\
\frac{dy}{dt} &= -4\sin t
\end{aligned}
$$

$$
\begin{aligned}
L &= \int_0^{\pi} \sqrt{(4\cos t)^2 + (-4\sin t)^2} \cdot dt \\
&= \int_0^{\pi} \sqrt{16\cos^2 t + 16\sin^2 t} \cdot dt \\
&= 4\int_0^{\pi} \sqrt{\cos^2 t + \sin^2 t} \cdot dt \\
&= 4\int_0^{\pi} 1 \cdot dt \\
&= 4\pi \qquad \blacksquare
\end{aligned}
$$

3. $(P.300\ \#\ 22)$

$Find\ \ the\ length\ of\ each\ curve.$

4. $(P.300\ \#\ 23)$

$Find\ the\ area\ of\ the\ surface\ generated\ by\ revolving\ the\ given\ curve\ about\ the\ x-axis$

(*a*)

$$
\begin{aligned}
y &= \int_{\pi/6}^{x}\sqrt{64\sin^2 u\cos^4 u-1}\cdot du,\ \frac{\pi}{6}\leqslant x\leqslant\frac{\pi}{3}\\
\frac{dy}{dx} &= \sqrt{64\sin^2 x\cos^4 x-1}\\
L &= \int_{\pi/6}^{\pi/3}\sqrt{1+64\sin^2 x\cdot\cos^4 x-1}\cdot dx\\
&= \int_{\pi/6}^{\pi/3}8\sin x\cos^2 x\cdot dx\\
&= \left[-\frac{8}{3}\cos^3 x\right]_{\pi/6}^{\pi/3}\\
&= -\frac{1}{3}+\sqrt{3} \qquad \blacksquare
\end{aligned}
$$

$$
\begin{aligned}
y &= 6x, \qquad 0\leqslant x\leqslant 1\\
\frac{dy}{dx} &= 6\\
A &= 2\pi\int_0^1 6x\sqrt{1+36}\cdot dx\\
&= 12\sqrt{37}\pi\int_0^1 x\cdot dx\\
&= 12\sqrt{37}\pi\left[\frac{1}{2}x^2\right]_0^1\\
&= 6\sqrt{37}\pi \qquad \blacksquare
\end{aligned}
$$

(*b*)

$$
\begin{aligned}
x &= a\cos t+at\sin t\\
y &= a\sin t-at\cos t,\ -1\leqslant t\leqslant 1
\end{aligned}
$$

$$
\begin{aligned}
\frac{dx}{dt} &= -a\sin t+a\sin t+at\cos t\\
&= at\cos t\\
\frac{dy}{dt} &= a\cos t-a\cos t+at\sin t\\
&= at\sin t
\end{aligned}
$$

$$
\begin{aligned}
L &= \int_{-1}^{1}\sqrt{a^2t^2\cos^2 t+a^2t^2\sin^2 t}\cdot dt\\
&= \int_{-1}^{1}|at|\cdot dt\\
&= \int_0^1 at\cdot dt-\int_{-1}^{0}at\cdot dt\\
&= \left[\frac{a}{2}t^2\right]_0^1-\left[\frac{a}{2}t^2\right]_{-1}^{0}\\
&= \frac{a}{2}+\frac{a}{2}\\
&= a \qquad \blacksquare
\end{aligned}
$$