112-1 Calculus Chapter_5.1~5.2 Homework 2023/12/01

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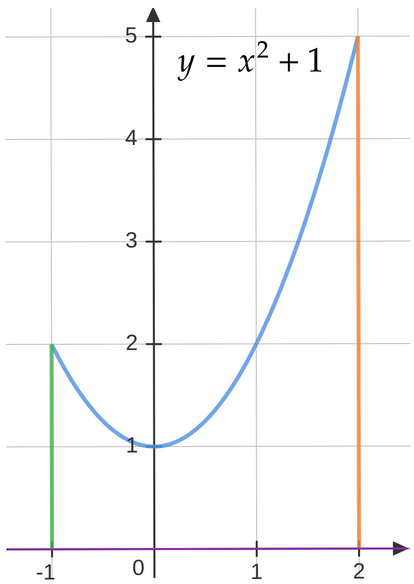
P.280#1 #5 #30
P.287#7 #11 #18

Evaluate an intergral (or integrals) for the area of the indicated region.

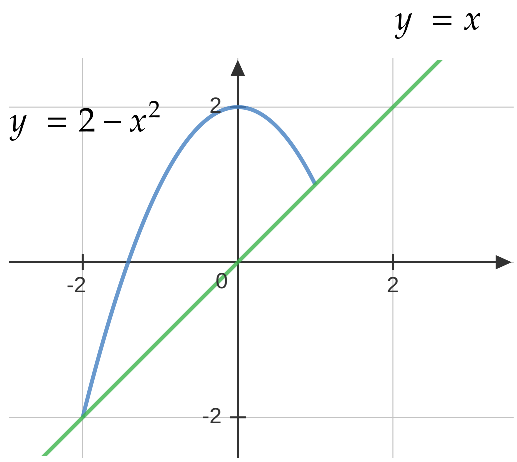
1. (*P*.280 #1)



Area :

$$\begin{aligned} A &= \int_{-1}^{2} \left(x^2+1\right)\, dx \\ &= \left[\frac{1}{3}x^3 + x\right]_{-1}^{2} \\ &= 6 \quad \blacksquare \end{aligned}$$

2. (*P*.280 #5)



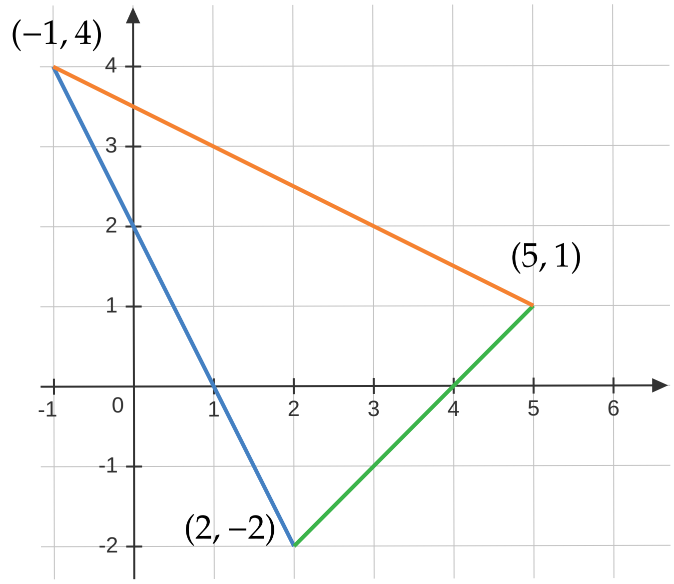
Intersection points :

$$\begin{aligned} 2 - x^2 &= x \\ x^2 + x - 2 &= 0 \\ x &= -2,\ 1 \end{aligned}$$

Area :

$$\begin{aligned} A &= \int_{-2}^{1} \left(-x^2 - x + 2\right) dx \\ &= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{-2}^{1} \\ &= \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right) \\ &= \frac{9}{2} \quad \blacksquare \end{aligned}$$

3. (P.280 #30)



Equation of the line through $(-1,4)$ *and* $(5,1)$:

$$3x+6y=21$$

$$y=-\frac{1}{2}x+\frac{7}{2}$$

Equation of the line through $(-1,4)$ *and* $(2,-2)$:

$$6x+3y=6$$

$$y=-2x+2$$

Equation of the line through $(2,-2)$ *and* $(5,1)$:

$$3x-3y=12$$

$$y=x-4$$

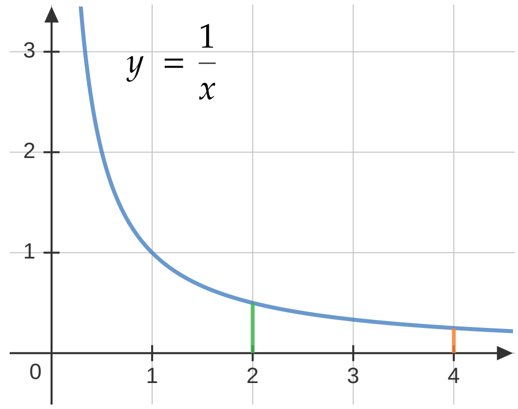
Area :

$$\int_{-1}^{2}\left[\left(-\frac{1}{2}x+\frac{7}{2}\right)-(-2x+2)\right]dx+\int_{2}^{5}\left[\left(-\frac{1}{2}x+\frac{7}{2}\right)-(x-4)\right]dx$$

$$=\left[\frac{3}{4}x^2+\frac{3}{2}x\right]_{-1}^{2}+\left[-\frac{3}{4}x^2+\frac{15}{2}x\right]_{2}^{5}$$

$$=\frac{27}{2}=13.5 \qquad \blacksquare$$

4. (P.287 #7)



$$\Delta V \approx \pi\left(\frac{1}{x}\right)^2\Delta x$$

$$=\pi\left(\frac{1}{x^2}\right)\Delta x$$

$$V=\pi\int_2^4\left(\frac{1}{x^2}\right)\cdot dx$$

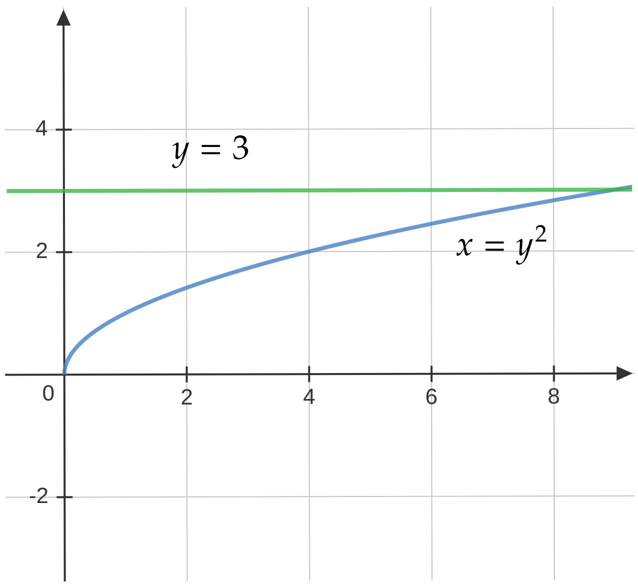
$$=\pi\left[-\frac{1}{x}\right]_2^4$$

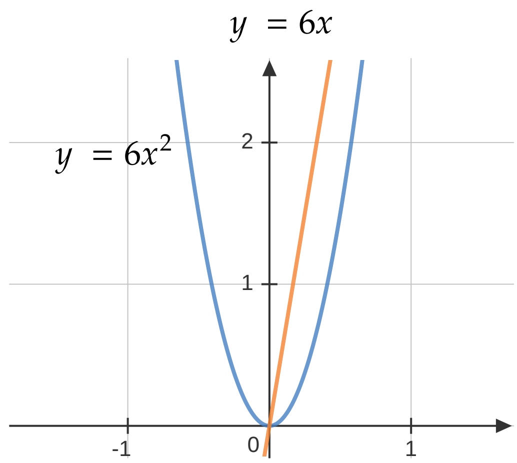
$$=\pi\left(-\frac{1}{4}+\frac{1}{2}\right)$$

$$=\frac{\pi}{4} \qquad \blacksquare$$

5. (P.28 #11)

6. (P.287 #18)





$$\begin{aligned}
\Delta V &\approx \pi\left(y^2\right)^2 \Delta y \\
&= \pi y^4 \Delta y \\
V &= \pi \int_0^3 \left(y^4\right) dy \\
&= \pi\left[\frac{1}{5} y^5\right]_0^3 \\
&= \frac{243\pi}{5} \quad \blacksquare
\end{aligned}$$

Intersection points :

$$\begin{aligned}
\frac{x}{2} &= 2\sqrt{x} \\
x^2 - 16x &= 0 \\
x(x-16) &= 0 \\
x &= 0, 16
\end{aligned}$$

$$\begin{aligned}
\Delta V &\approx \pi\left[\left(2\sqrt{x}\right)^2 - \left(\frac{x}{2}\right)^2\right] \Delta x \\
&= \pi\left(4x - \frac{x^2}{4}\right) \Delta x \\
V &= \pi \int_0^{16} \left(4x - \frac{x^2}{4}\right) dx \\
&= \pi\left[2x^2 - \frac{x^3}{12}\right]_0^{16} \\
&= \pi\left(512 - \frac{1024}{3}\right) \\
&= \frac{512\pi}{3} \quad \blacksquare
\end{aligned}$$