

# 112-1 Discrete Mathematics Chapter 1-5~7

姓名：許嘉隆

學號：412770116

## Chapter 1-5

4.

- a) There is a student in class has taken a computer science course at school.
- b) There is a student in class has taken all computer science courses at school.
- c) All students in class have taken a computer science course at school.
- d) There is a computer science course at school has taken by all students in class.
- e) All computer science courses at school have taken by a student in class.
- f) All students in class have taken all computer science courses at school.

6.

- a) Randy Goldberg is enrolled in class CS 252.
- c) Carol Sitea is enrolled all classes at school.
- e) There are two different people, and the second person has attended all the courses that the first person has attended.

10.

- a)  $\forall x F(x, Fred)$
- b)  $\forall x F(Evelyn, x)$
- c)  $\forall x \exists y F(x, y)$
- d)  $\neg \exists x \forall y F(x, y)$
- e)  $\forall y \exists x F(x, y)$
- f)  $\neg \exists x (F(x, Fred) \wedge F(x, Jerry))$
- g)  $\exists y_1 \exists y_2 [F(Nancy, y_1) \wedge F(Nancy, y_2) \wedge (y_1 \neq y_2) \wedge \forall y (F(Nancy, y) \rightarrow (y = y_1) \vee (y = y_2))]$
- h)  $\exists y [\forall x F(x, y) \wedge \forall z (\forall x F(x, z) \rightarrow z = y)]$
- i)  $\neg \exists x F(x, x)$
- j)  $\exists x \exists y [F(x, y) \wedge (x \neq y) \wedge \forall z (F(x, z) \rightarrow z = y)]$

12.

- b)  $\neg C(Rachel, Chelsea)$

16. Let  $P(s, c, m) = \text{"Student } s \text{ has class standing } c \text{ and is majoring in } m\text{"}$

- a)  $\exists s \exists m P(s, \text{junior}, m) \implies \text{True}$

- b)  $\forall s \exists c P(s, c, \text{computer science}) \implies \text{False}$   
c)  $\exists s \exists c \exists m [P(s, c, m) \wedge (m \neq \text{mathematics}) \wedge (c \neq \text{junior})] \implies \text{True}$   
d)  $\forall s P(s, \text{sophomore, computer science}) \implies \text{False}$   
e)  $\exists m \forall c \exists s P(s, c, m) \implies \text{False}$

## Chapter 1-6

6. Let  $r = \text{"It rains"}$ ,  $f = \text{"It is foggy"}$ ,  $s = \text{"The sailing race will be held"}$ ,  
 $l = \text{"The life saving demonstration"}$  and  $t = \text{"The trophy will be awarded"}$ .

$$\implies \text{Premise: } \begin{cases} 1. (\neg r \vee \neg f) \rightarrow (s \wedge l) \\ 2. s \rightarrow t \\ 3. \neg t \end{cases} \quad \text{Conclusion: } r$$

Step	Reason
1. $\neg t$	Premise 3
2. $s \rightarrow t$	Premise 2
3. $\neg s$	Modus tollens from (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Premise 1
5. $\neg(s \wedge l) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6. $\neg s \vee \neg l \rightarrow r \wedge f$	De Morgan's law from (5)
7. $r \wedge f$	Modus ponens from (2) and (6)
8. $r$	Simplification from (7)

14.

a)

Let  $s(x) = \text{"x is a student in this class"}$

$r(x) = \text{"x is one owns a red convertible"}$

$t(x) = \text{"x is one got a speeding ticket"}$

$$\implies \text{Premise: } \begin{cases} 1. s(\text{Linda}) \wedge r(\text{Linda}) \\ 2. \forall x (r(x) \rightarrow t(x)) \end{cases} \quad \text{Conclusion: } \exists x (s(x) \wedge t(x))$$

Step		Reason
1.	$s(Linda) \wedge r(Linda)$	Premise 1
2.	$s(Linda)$	Simplification from (1)
3.	$r(Linda)$	Simplification from (2)
4.	$\forall x(r(x) \rightarrow t(x))$	Premise 2
5.	$r(y) \rightarrow t(y)$ <i>for every y</i>	Universal instantiation from (4)
6.	$t(y)$	Modus ponens from (3) and (5)
7.	$s(y) \wedge t(y)$	Conjunction from (2) and (6)
8.	$\exists x(s(x) \wedge t(x))$	Existential generalization from (7)

b)

Let  $r(x)$  = "x is one of the five roommates listed"

$d(x)$  = "x has taken a course in discrete mathematics"

$a(x)$  = "x can take a course in algorithms"

$\Rightarrow$  Premise:  $\begin{cases} 1. \forall x(r(x) \rightarrow d(x)) \\ 2. \forall x(d(x) \rightarrow a(x)) \end{cases}$       Conclusion:  $\forall x(r(x) \rightarrow a(x))$

Step		Reason
1.	$\forall x(r(x) \rightarrow d(x))$	Premise 1
2.	$r(y) \rightarrow d(y)$ <i>for every y</i>	Universal instantiation from (1)
3.	$\forall x(d(x) \rightarrow a(x))$	Premise 2
4.	$d(y) \rightarrow a(y)$ <i>for every y</i>	Universal instantiation from (3)
5.	$r(y) \rightarrow a(y)$	Hypothetical syllogism from (2) and (4)
6.	$\forall x(r(x) \rightarrow a(x))$	Universal generalization from (5)

c)

Let  $m(x)$  = "movie produced by x"

$w(x)$  = "x is a wonderful movie"

$t(x)$  = "x is the theme of the movie"

$$\Rightarrow \text{Premise: } \begin{cases} 1. \forall x(m(\text{John Sayles}) \rightarrow w(x)) \\ 2. m(\text{John Sayles}) \wedge t(\text{coal miners}) \end{cases} \quad \text{Conclusion: } \exists x(w(x) \wedge t(\text{coal miners}))$$

Step		Reason
1.	$m(\text{John Sayles}) \wedge t(\text{coal miners})$	Premise 2
2.	$m(\text{John Sayles})$	Simplification from (1)
3.	$t(\text{coal miners})$	Simplification from (2)
4.	$\forall x(m(\text{John Sayles}) \rightarrow w(x))$	Premise 1
5.	$m(\text{John Sayles}) \rightarrow w(y)$	for every $y$ Universal instantiation from (4)
6.	$w(y)$	Modus ponens from (2) and (5)
7.	$w(y) \wedge t(\text{coal miners})$	Conjunction from (3) and (6)
8.	$\exists x(w(x) \wedge t(\text{coal miners}))$	Existential generalization from (7)

d)

Let  $c(x)$  = "x is in this class"

$f(x)$  = "x has been to France"

$l(x)$  = "x has visited the Louvre"

$$\Rightarrow \text{Premise: } \begin{cases} 1. \exists x(c(x) \wedge f(x)) \\ 2. \forall x(f(x) \rightarrow l(x)) \end{cases} \quad \text{Conclusion: } \exists x(c(x) \wedge l(x))$$

Step		Reason
1.	$\exists x(c(x) \wedge f(x))$	Premise 1
2.	$c(y) \wedge f(y)$	for some $y$ Existential instantiation from (1)
3.	$c(y)$	Simplification from (2)
4.	$f(y)$	Simplification from (2)
5.	$\forall x(f(x) \rightarrow l(x))$	Premise 2
6.	$f(z) \rightarrow l(z)$	for every $z$ Universal instantiation from (5)
7.	$l(z)$	Modus ponens from (4) and (6)
8.	$c(y) \wedge l(z)$	Conjunction from (3) and (7)
9.	$\exists x(c(x) \wedge l(x))$	Existential generalization from (8)

## Chapter 1-7

28. Prove: If  $n$  is a positive integer, then  $n$  is even if and only if  $7n + 4$  is even.

We need to prove:  $\begin{cases} 1. & n \text{ is even} \rightarrow 7n + 4 \text{ is even} \\ 2. & 7n + 4 \text{ is even} \rightarrow n \text{ is even} \end{cases}$

1. If  $n$  is even  $\implies$  let  $n = 2k$ ,  $k = 1, 2, 3, \dots$

$$\text{then } 7n + 4 = 7(2k) + 4 = 14k + 4 = 2(7k + 2)$$

$\therefore 7n + 4$  is even

2. Suppose  $n$  is odd, then  $7n + 4$  is odd  $\implies$  let  $n = 2k + 1$ ,  $k = 0, 1, 2, \dots$

$$\text{then } 7n + 4 = 7(2k + 1) + 4$$

$$= 14k + 11$$

$$= 2(7k + 5) + 1$$

$\therefore 7n + 4$  is odd

$\therefore n \text{ is odd} \rightarrow 7n + 4 \text{ is odd} \implies \text{True}$

$\therefore 7n + 4 \text{ is even} \rightarrow n \text{ is even} \implies \text{True (Contrapositive)}$

By (1) and (2),  $n \text{ is even} \leftrightarrow 7n + 4 \text{ is even}$  ■