CF969-7-SU-CO

Big-Data for Computational Finance

Academic Year: 2023/24

Assignment 1

Name:	
ID:	
Date:	

Contents

Abstract

This report presents the application of the Markowitz model for portfolio optimization using quadratic programming. We address four given tasks involving different constraints and investment scenarios, providing theoretical insights, problems faced and practical results. The study examines the effects of varying expected returns, permitting partial capital investment, adjusting return constraints, and allowing short selling. The results are analysed and compared to understand the implications of these modifications on portfolio risk and return, aiming to enhance investment strategy development by offering a deeper understanding of risk-return trade-offs under different constraints.

1. Introduction

The Markowitz model, introduced by Harry Markowitz in 1952, is a foundational framework in modern portfolio theory, also known as mean-variance model. It aims to optimise the allocation of capital among various assets and helps the investors to construct a portfolio with the maximum possible return for a given level of risk, or conversely, the minimum level of risk for a given return. This report formulates the Markowitz model as a quadratic optimization problem and solves it using CVXPY(an open-source Python-embedded modelling language for convex optimization problems). We explore different scenarios by modifying constraints and analyse their impact on portfolio optimization.

2. Theoretical Background

The Markowitz model assumes that

- Investors are risk-averse, preferring portfolios that offer higher returns with lower risk.
- The risk of a portfolio is based on the variability of its returns.

The model uses the following parameters:

$\boldsymbol{\mu}$: vector of expected returns for n assets

C: covariance matrix of asset returns

x: vector of portfolio weights

The optimization problem is defined as:

$$\begin{aligned} &\text{minimise} && \boldsymbol{x}^T \boldsymbol{C} \boldsymbol{x} \\ &\text{subject to} && \boldsymbol{\mu}^T \boldsymbol{x} = r \\ & && \boldsymbol{e}^T \boldsymbol{x} = 1, \text{ where } e = (1, \ldots, 1)^T \\ & && \boldsymbol{x} \geq \mathbf{0}. \end{aligned}$$

where **e** is a vector of ones, ensuring the weights sum to one, and **r** is the desired portfolio return.

3. Data Generation

To generate the expected return vector **μ** and the covariance matrix **C**, we use a python **random module** that produces random values based on specific digits of the student's registration number, ensuring unique datasets.

4. Methodology

4.1. Quadratic Programming Solvers

We employ **CVXPY** to solve quadratic programming problems.

The tasks involve:

1. Standard Markowitz model
2. Allowing fractional capital investment ($\mathbf{e}^T \mathbf{x} \leq 1$)
3. Relaxed return constraint ($\boldsymbol{\mu}^T\mathbf{x} \geq \mathbf{r}$)
4. Permitting short selling (**x** unconstrained)

4.2. Optimization for Different Values of r

The optimization is performed for different values of **r** as given in the assignment, ranging from **2.00 to 9.00** with the increasing rate of **0.25**.

5. Challenges and Solutions

Task 1: Standard Markowitz Model

Challenges:

- Non-deterministic behaviour due to lack of seed in random number generation.
- The construction and population of the correlation matrix are unconventional and could be made more efficient using numpy's built-in functions

Solutions:

- Set a seed for random number generation to ensure reproducibility.
- Implement robust error handling for optimization results.

Task 2: Fractional Capital Investment

Challenges:

- Ensuring reproducibility and consistency in random number generation.
- Accurate matrix construction and indexing.
- Formulating the optimization problem correctly is critical.

Solutions:

- Use consistent random number generation methods.
- Carefully construct and manipulate matrices with proper indexing.

Task 3: Relaxed Return Constraint

Challenges:

- Managing matrix manipulation and indexing.
- Correct optimization problem formulation.

Solutions:

- Validate matrix construction and indexing logic.
- Define the optimization problem accurately with necessary constraints.

Task 4: Short Selling Allowed

Challenges:

- Formulating the optimization problem without short-selling constraints.
- Handling non-optimal solutions and ensuring proper data conversion for analysis.

Solutions:

- Implement error handling for optimization and ensure correct data handling.
- Handle data based on assumptions.

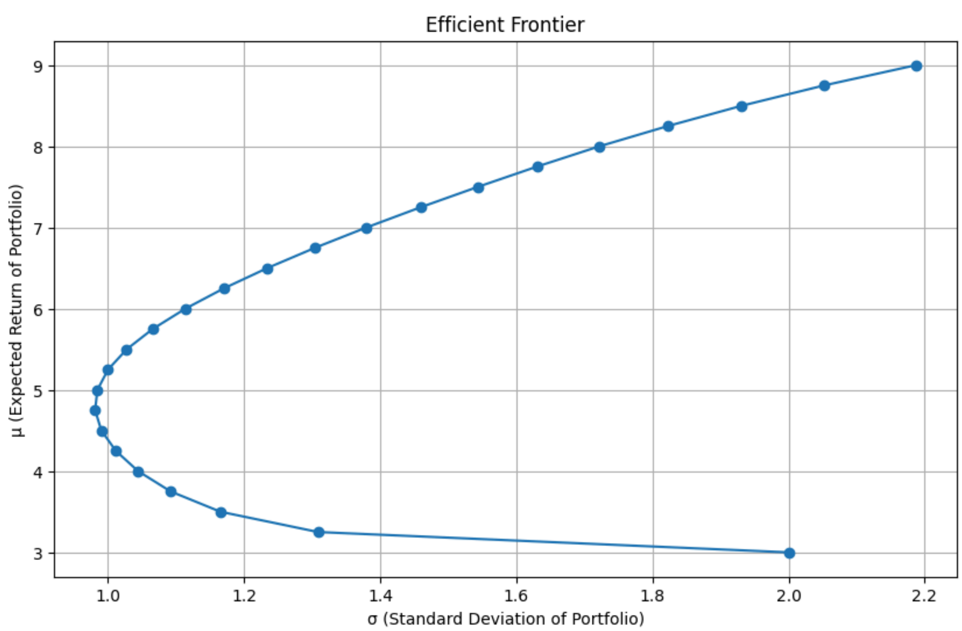
6. Results

6.1. Task 1:

Standard Markowitz Model

For each value of **r** from 2.00 to 9.00, the portfolio's risk **σ**(x) and return **μ**(x) are calculated and plotted.

Graphical Plot:



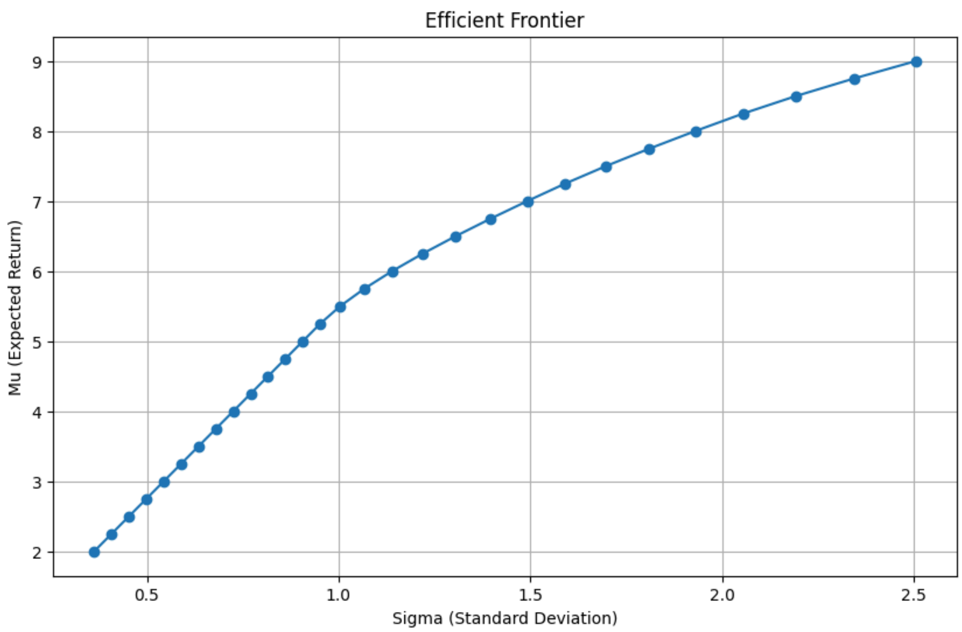
The results demonstrated the classic efficient frontier, illustrating the optimal portfolios for different levels of expected return (r). The analysis showed that as the desired return increased, the associated risk also rose, reaffirming the fundamental principles of the mean-variance optimization model.

6.2. Task 2:

Fractional Capital Investment

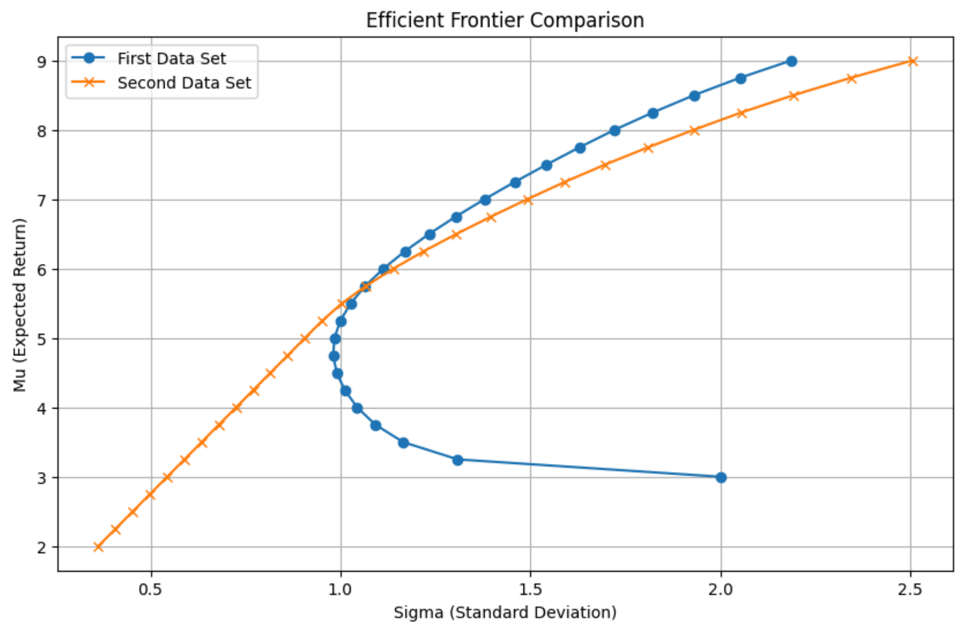
The constraint $\boldsymbol{e}^T\boldsymbol{x} = 1$ is relaxed to $\mathbf{e}^{\mathrm{T}}\,\mathbf{x} \leq 1$. This scenario is justified by recognizing that uninvested capital can be saved without risk or return.

Graphical Plot:



When allowing fractional capital investment (eT x ≤ 1), the efficient frontier broadened, suggesting that not fully investing the available capital can yield a wider range of risk-return combinations. This flexibility enables investors to maintain liquidity while achieving optimal returns, offering a strategic advantage in uncertain market conditions.

<u>Comparison Plot with Task1</u>:



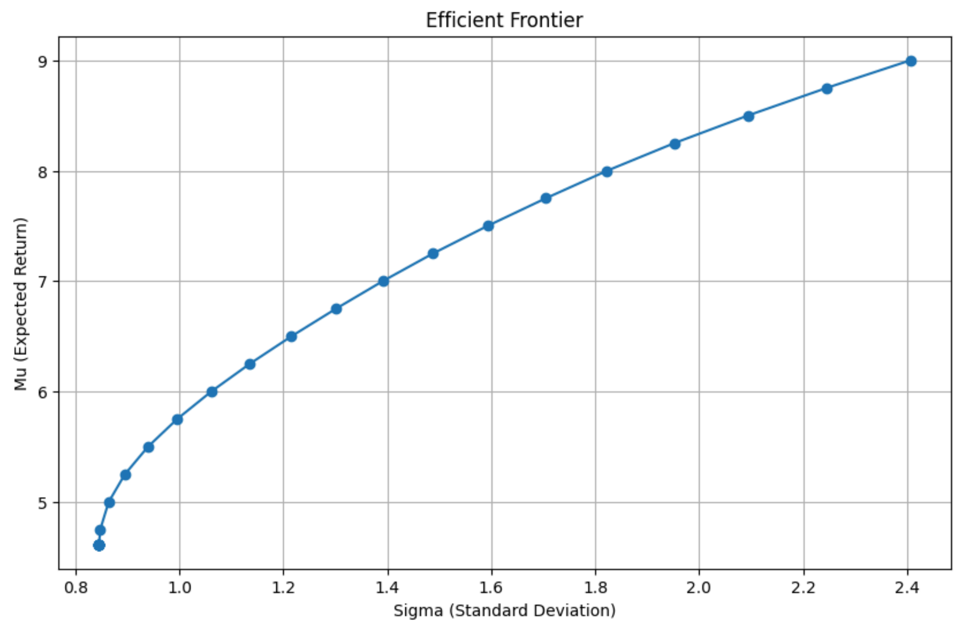
This Plot compares the efficient frontiers of two Plots: the blue line for the first dataset and the orange line for the second. The blue frontier is broader and more curved, indicating a wider range of expected returns (μ) and standard deviations (σ), spanning σ = 0.5 to σ = 2.0 and μ = 2 to 9. The orange frontier is more linear and constrained, with σ from σ = 0.5 to σ = 2.0 and μ = 2 to μ = 8. They intersect around σ = 0.75 and μ = 4, offering similar risk-return combinations. Beyond this point, the blue frontier provides higher returns for the same risk, particularly at higher risk levels. Thus, the first data set suggests more aggressive, higher-return investments, while the second data set offers more stable, conservative options.

6.3. Task 3:

<u>Relaxed Return Constraint</u>

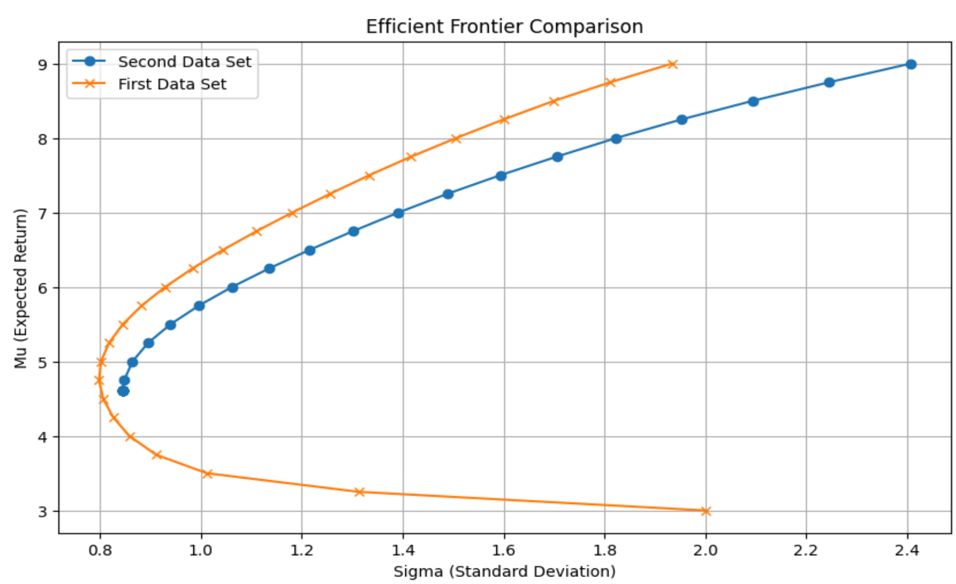
The constraint $\boldsymbol{\mu}^T\boldsymbol{x} = r$ is changed to $\boldsymbol{\mu}^T\boldsymbol{x} \geq r$. This modification implies a minimum required return, offering more flexibility in portfolio selection.

Graphical Plot:



Relaxing the return constraint (μT x ≥ r) provided more flexibility in portfolio selection, resulting in an efficient frontier that allowed for higher potential implies that portfolios can meet or exceed a minimum required return, catering to investors with specific return thresholds while maintaining an optimal risk balance.

Comparison Plot with Task1



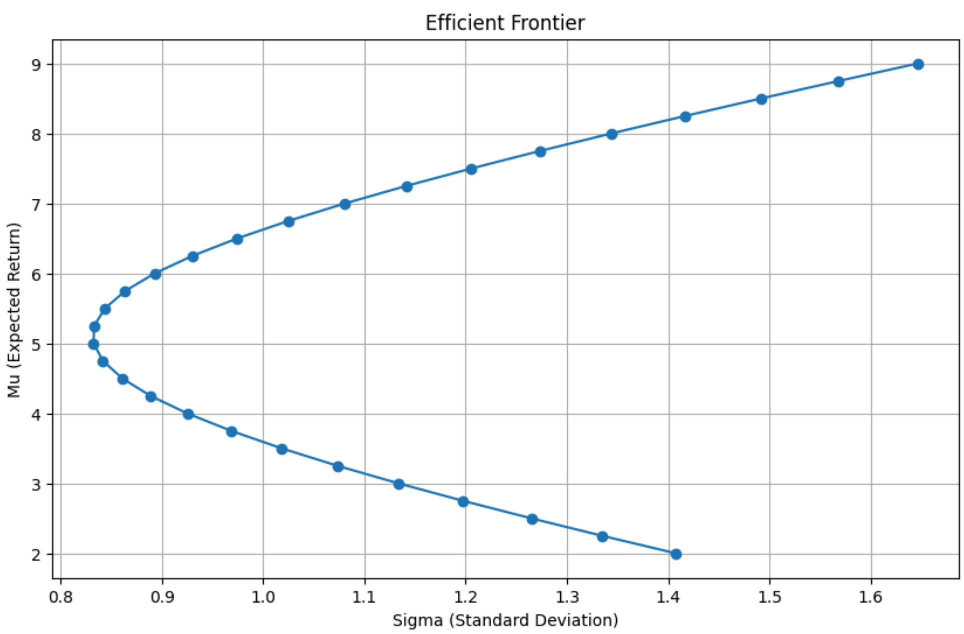
This Plot compares the efficient frontiers of two Plots, with the first dataset in orange and the second in blue. The orange line spans σ from 0.75 to 2.0 and μ from 3 to 8, showing a more pronounced curve. The blue line spans σ from 0.75 to 1.75 and μ from 5 to nearly 9, showing a more linear progression. They intersect around $\sigma = 0.85$ and $\mu = 5$. Beyond this, the blue frontier offers higher returns for the same risk until about $\sigma = 1.5$, after which the orange frontier declines. This suggests the first data set (orange) offers higher but riskier returns, while the second (blue) provides more consistent and higher returns at moderate risk levels.

6.4.Task 4:

<u>Short Selling Allowed</u>

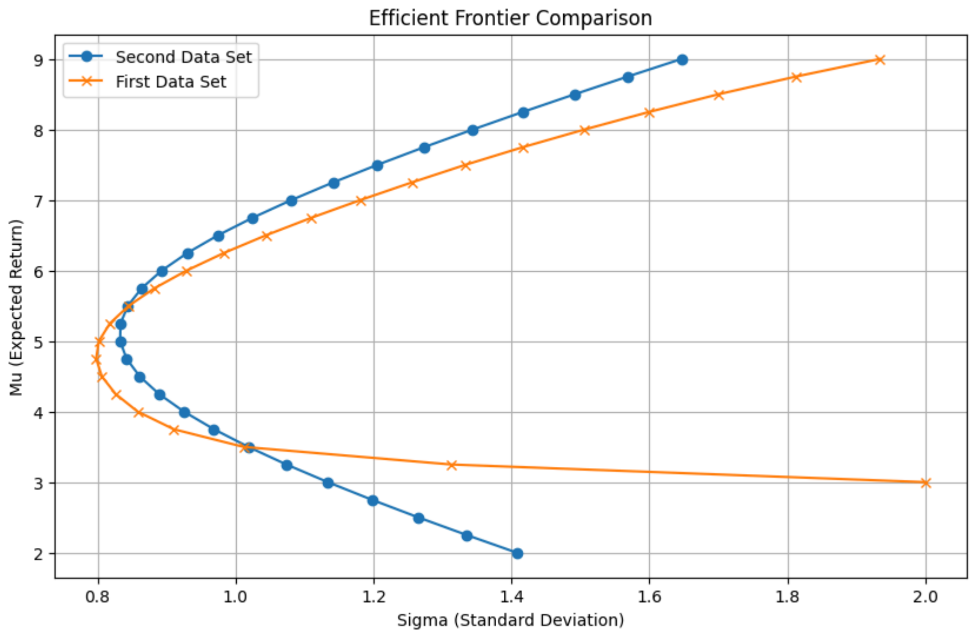
Removing the non-negativity constraint $\boldsymbol{x} \geq \mathbf{0}$ allows short selling. This scenario is motivated by the ability to borrow and sell assets, increasing potential returns but also risk.

<u>Graphical Plot</u>:



Permitting short selling (removing the $x \geq 0$ constraint) significantly impacted the efficient frontier, indicating higher potential returns at increased risk levels. This scenario highlights the aggressive nature of short selling, where borrowing and selling assets can amplify returns but also heighten risk.

Comparison Plot with Task1



The figure compares the efficient frontiers of two data sets, with the first data set in orange and the second in blue. The orange frontier spans σ from 0.75 to 2.0 and μ from 3 to 8, showing a pronounced curve. The blue frontier spans σ from 0.8 to 1.6 and μ from 2 to 7, also curving but with a different pattern. They intersect around σ = 1.0 and μ = 5. Beyond this point, the orange frontier offers higher returns for increasing risk levels up to σ 1.6, while the blue frontier declines. This suggests that the first data set (orange) offers higher returns for a wider range of risk, while the second (blue) declines beyond a certain risk, indicating a more conservative approach.

7. Limitations and Assumptions

- Assumes ideal market conditions without transaction costs or liquidity issues.
- Assumptions about risk-free savings for uninvested capital may not reflect real options, as even 'risk-free' investments carry some risk.
- Adjusting constraints for flexibility can introduce complexity and instability in optimization results.
- Model sensitivity to input parameters poses challenges in volatile markets.
- Does not consider practical and regulatory constraints, such as restrictions on short selling.
- Ignore behavioural factors like risk aversion and market sentiment, which influence investment decisions significantly.

8. Conclusion

This report explored the application of the Markowitz model for portfolio optimization using quadratic programming in Python. By addressing four distinct tasks with varying constraints and scenarios, we provided a comprehensive analysis of how these modifications impact portfolio risk and return. Key findings include the broadening of the efficient frontier with fractional capital investment, increased flexibility and higher potential returns with relaxed return constraints, and the significant impact of allowing short selling. Challenges faced during the implementation, such as ensuring reproducibility, accurate matrix construction, and robust optimization problem formulation, were addressed with appropriate solutions. Overall, this study enhances the understanding of risk-return trade-offs and offers valuable insights for developing tailored investment strategies.