

[Early Warning Signals Toolbox](#)

A User's Guide for Detecting Critical Transitions in Time series and Spatial data

- [Home](#)
- [Theory](#)
  - [What is a critical transition?](#)
  - [Why should we expect Early Warnings before Critical Transitions?](#)
  - [What are Early Warning Signals?](#)
- [Perturbation experiments](#)
- [Time series methods](#)
  - [Metric based methods](#)
    - [General Steps for Rolling Window Metrics](#)
      - [Ensemble example for rolling window metrics](#)
    - [Autocorrelation and Spectral properties](#)
    - [Detrended fluctuation analysis](#)
    - [Variance](#)
    - [Skewness and Kurtosis](#)
    - [Conditional heteroskedasticity](#)
    - [BDS test](#)
  - [Model based methods](#)
    - [Nonparametric drift-diffusion-jump models](#)
    - [Time-varying AR\(p\) models](#)
    - [Threshold AR\(p\) models](#)
    - [Potential analysis](#)
    - [Generalized models](#)
- [Spatial indicators](#)
  - [Slowing-down based indicators](#)
    - [Spatial correlation at lag 1](#)
    - [Discrete Fourier Transform \(DFT\)](#)
  - [Variability based indicators](#)
  - [Pattern-based indicators](#)
  - [Step-by-step analysis for spatial indicators](#)
    - [Step 1: Distinguishing periodic from non-periodic patterns](#)
    - [Step 2a: Probing spatial early warnings for non-periodic patterns](#)
    - [Step 2b: Probing spatial early warnings for periodic patterns](#)
- [Case studies](#)
  - [Transition to Extinction due to Photo-inhibition](#)
  - [Critical transition in a lake foodweb](#)
  - [Paleoclimatic transitions](#)
- [Resources](#)
  - [Glossary](#)
  - [Bibliography](#)
    - [Theory: Reviews and General reading](#)
    - [Methods](#)
    - [Case studies](#)
    - [Other reading](#)
  - [Code](#)
- [About the toolbox](#)
  - [Aim](#)
  - [Who we are](#)
  - [Contributors](#)

-- Nonparametric drift-diffusion-jump models



## Nonparametric drift-diffusion-jump models

### Basics

Often we do not know the underlying processes that generate the time series that we are analyzing for early warnings. Nonparametric drift-diffusion-jump (DDJ) models address this problem by fitting a general model that can approximate a wide range of nonlinear processes without the need to specify an explicit equation. Drift measures the local rate of change. Diffusion measures relatively small shocks that occur at each time step. Jumps are large intermittent shocks. Total variance combines the contributions of diffusion and jumps. The approach is to estimate terms of a drift-diffusion-jump model as a surrogate for the unknown data generating process.

From a time series we can estimate drift, diffusion and jump statistics that may serve as leading indicators of the transition. We do this by assuming that high-frequency observations of the system can be approximated by fitting a drift-diffusion-jump model:

$$dx(t) = f(x(t), \theta(t))dt + g(x(t), \theta(t))dW + dJ(t)$$

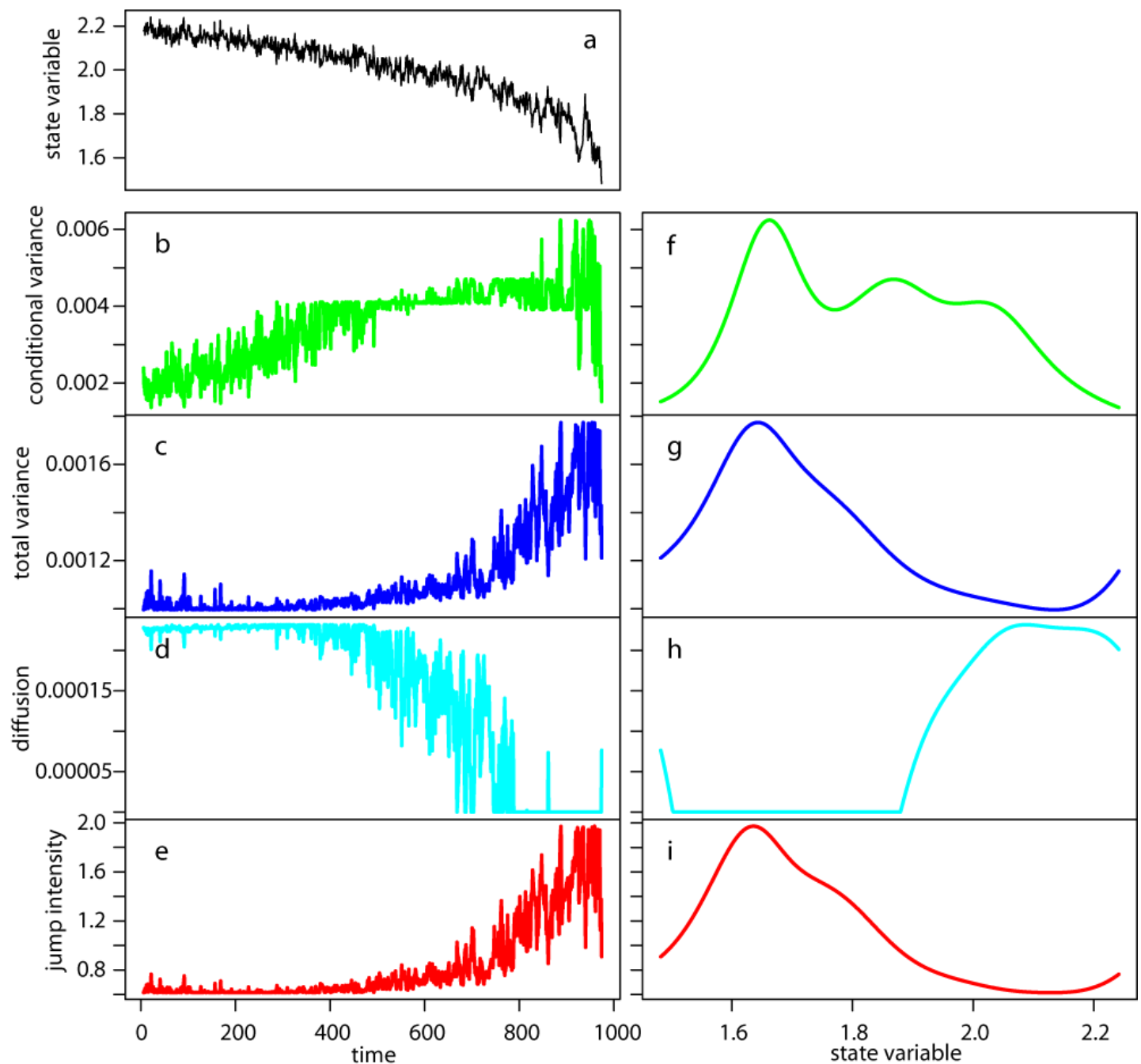
In this fitted model, the drift function  $f(\cdot)$  measures the instantaneous deterministic change in the time series, the diffusion function  $g(\cdot)$  measures the standard deviation of relatively small shocks that occur at each time step, whereas jumps  $J$ , the last term, are relatively large shocks that occur intermittently. In practice, the drift, diffusion, and jump functions are estimated using nonparametric regression. The regression yields estimates of drift, total variance, jump intensity, and diffusion variance. In addition, we can estimate conditional variance of  $x$  using standard nonparametric regression techniques. This conditional variance rises to infinity at the critical transition.

An interesting feature of the drift-diffusion-jump model is that conditional variance and diffusion estimates may be useful for distinguishing bifurcations that occur in the drift from bifurcations that occur in the diffusion (so-called noise-induced transitions: an abrupt shift in the shape of the stationary distribution). A bifurcation in the drift only may be indicated in advance by conditional variance but not diffusion. A bifurcation in the diffusion may be indicated by increases in both conditional variance and diffusion.

### Example

We present the DDJ method applied to a simulated time series in which a critical transition is approaching. The nonparametric DDJ model was not applied on rolling windows, but to the entire time series after log-transforming the data (panel a). We found

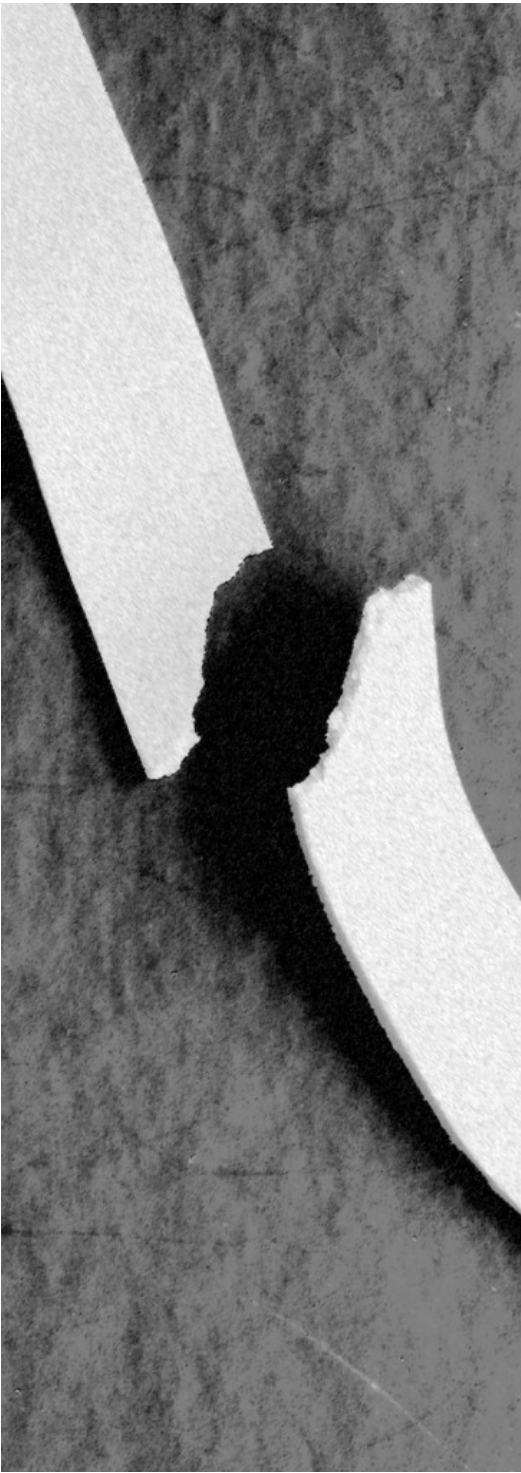
an increase in conditional and total variance as well as in jump intensity in the time series (panels b, c, e) and a decrease in the diffusion term (panel d). The trends were noisy, but they became very clear when plotted against biomass values (due to smoothing) (panels f-i). For log-transformed values between 1.6 and 1.8, the indicators started to signal the upcoming transition.



Posted in Tagged with [critical slowing down](#), [flickering](#)

General search

Search for:



#### latest news

- [Spatial methods paper and update in EWS toolbox](#)
- [EWS package gets into WICI Data challenge finalists!](#)
- [Special issue on early-warnings and regime shifts in journal of Theoretical Ecology](#)
- [early warnings in space in experimental and field data](#)
- [earlywarnings package is now an R library!](#)

Supported by SPARCS  
Synergy Program for Analyzing Resilience and Critical transitions

email: [info@early-warning-signals.org](mailto:info@early-warning-signals.org)