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1. (a) Ways from Presint 4 to Presint 5 = 3 car routes + 4 walking routes
= 7 routes

Ways from Presint 5 to Presint 6 = 4 car routes + 5 walking routes
= 9 routes

∴ Total ways going from Presint 4 to Presint 6
= 7 routes × 9 routes
= 63 routes ☆

(b) (i) Order is important ⇒ Permutation

$$P(8,8) = \frac{8!}{(8-8)!}$$
$$= 8!$$
$$= 40320 \text{ ☆}$$

∴ Ways can arrange all the letters
= 40320 ways

(ii) Order is important ⇒ Permutation

$$P(8,5) = \frac{8!}{(8-5)!}$$
$$= \frac{8!}{3!}$$
$$= 6720 \text{ ☆}$$

∴ Ways can arrange 5 strings of letters
= 6720 ways

(iii) S E
1 × 6 × 5 × 4 × 3 × 2 × 1 × 1

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$
$= 720$

∴ Ways can arrange the strings that always start with S and end with E = 720 ways

1. (c) (i) Men = (5-1)!

= 4!

= 24

Women = 5!

= 120

∴ Ways that 5 men and 5 women set around a circular table so that the gender alternate = 120 × 24

= 2880 ✓

(ii) [_ _] _1 _2 _3 _4 _5 _6 _7 _8 _9

(9-1)! × 2! = 80640

∴ Ways that among the 10 people, 2 are couple and need to be seat next to each other

= 80640 ✓

(iii) [M M M M M] [W W W W W]

2! × 5! × 5! = 14400

∴ Ways that people were seated in groups between the men and women

= ~~28800~~ 14400

2. (a) Select ⇒ Order is not important

⇒ Combination

Case 1 : 3 women and 2 men

C(6,3) × C(8,2)

$= \frac{6!}{3!(6-3)!} \times \frac{8!}{2!(8-2)!}$

= 20 × 28

= 560

Case 2 : 4 women and 1 men

C(6,4) × C(8,1)

$= \frac{6!}{4!(6-4)!} \times \frac{8!}{1!(8-1)!}$

= 15 × 8

= 120

Case 3 : 5 women and 0 men

C(6,5) × C(8,0)

$= \frac{6!}{5!(6-5)!} \times \frac{8!}{0!(8-0)!}$

= 6 × 1

= 6

∴ Ways forming a committee that at least 3 women are on the committee

= case 1 + case 2 + case 3

= 560 + 120 + 6

= 686

20 students are there.

One group = 4 students.

Half of it is girls → 10 girls, 10 boys.

Case select 4 students with no restriction:

$$C(20,4) = \frac{20!}{4!(20-4)!} = 4845$$

Case that the group consists of girls only.

$$C(10,4) = \frac{10!}{4!(10-4)!} = 210$$

∴ Ways that a group be selected if at least one boy must be there in the team

= Case select 4 students with no restriction − Case that the group consists of girls only

= 4845 − 210

= 4635 ways

3. (a) (i) $(5-1)! = 4! = 24$ ∴ ways for these people to be seated around the table = 24 ways

(ii) C V V

1 2 3

$(3-1)! \times 3! = 2! \times 3!$

$= 12$

∴ ways if the captain and both vice-captains should be seated next to each other = 12 ways

(b) Case for allocating the beds with no restriction : $5! = 120$ ways

Case for allocating the beds with the head of camp sit next to the assistant :

_ _

1 2 3 4

$4! \times 2! = 48$ ways

∴ Case that the head of camp cannot sit next to the assistant

= case for allocating the bed with no restriction − case for allocating the bead with the head of camp sit next to the assistant

= 120 − 48

= 72 ways

(c) Half a dozen = 6

(c)(i) Choose 6 chocolates ⇒ Order is not important

⇒ Combination

Same types of chocolates is allowed ⇒ Combination with repetition is allowed

$$\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

$n = 10, r = 6$

$$\binom{10+6-1}{6} = \frac{(10+6-1)!}{6!(10-1)!}$$

$$= \frac{15!}{6!9!}$$

$$= 5005$$

∴ ways for buying half a dozen chocolate with no restriction = 5005 ways

(c)(ii) Case 1 : 4 hazelnut flavoured chocolate and 2 the other types of chocolate

Types of chocolate = 9 (excluding hazelnut flavored chocolate)

$n = 9, r = 2$

$$\binom{9+2-1}{2} = \frac{(9+2-1)!}{2!(9-1)!} = \frac{10!}{2!8!} = 45 \text{ ways}$$

Case 2 : 5 hazelnut flavored chocolate and 1 other types of chocolate

$n = 9, r = 1$

$$\binom{9+1-1}{1} = \frac{(9+1-1)!}{1!(9-1)!} = \frac{9!}{1!8!} = 9 \text{ ways}$$

Case 3 : 6 hazelnut flavored chocolate and no other types of chocolate

$n = 9, r = 0$

$$\binom{9+0-1}{0} = \frac{(9+0-1)!}{0!(9-1)!} = \frac{8!}{0!8!} = 1 \text{ ways}$$

∴ ways for buying half a dozen chocolate with at least 4 hazelnut flavored chocolate = Case1 + Case 2 + Case 3

$$= 45 + 9 + 1$$

$$= 55 \text{ ways}$$

(iii) No two chocolates for the same type ⇒ Repetition of types of chocolate is not allowed

$$C(10,6) = \frac{10!}{6!(10-6)!}$$

$$= 210 \text{ ways}$$

∴ Ways for buying half a dozen chocolate with no two chocolates of the same type = 210

(a) (i) Select 11 player ⇒ Combination

$$CC(13,11) = \frac{13!}{11!(13-11)!}$$

$$= 78 \text{ ways}$$

∴ Ways are there to chose 11 players to take start the game = 78 ways

(ii) Assign 11 positions ⇒ Order is important ⇒ Permutation

$$P(13,11) = \frac{13!}{(13-11)!}$$

$$= \frac{13!}{2!}$$

$$= 3113510400$$

∴ Ways are there to assign 11 positions from the pool of 13 players = 3113510400 ways.

(iii) Case 1 : 1 woman and 10 men

$$C(3,1) \times C(10,10) = \frac{3!}{1!(3-1)!} \times \frac{10!}{10!(10-10)!}$$

$$= 3 \times 1$$

$$= 3 \text{ ways}$$

Case 2 : 2 women and 9 men

$$C(3,2) \times C(10,9) = \frac{3!}{2!(3-2)!} \times \frac{10!}{9!(10-9)!}$$

$$= 3 \times 10$$

$$= 30 \text{ ways}$$

Case 3 : 3 women and 8 men

$$C(3,3) \times C(10,8) = \frac{3!}{3!(3-3)!} \times \frac{10!}{8!(10-8)!}$$

$$= 1 \times 45$$

$$= 45 \text{ ways}$$

∴ Ways to chose 11 players to start the game must be a women = case 1 + case 2 + case 3

= 3 + 30 + 45

= 78 ways ✓

4. (a) To get two balls of the same colour from 3 colour balls, at least 4 balls must be taken.

∴ At least 4 balls

(b) pigeon = total number of cheesecakes (n = 80)
pigeonholes = people which included thirty students and two teachers (m = 32)

$k = \frac{n}{m}$, where k is the minimum number of cheese cake per people.

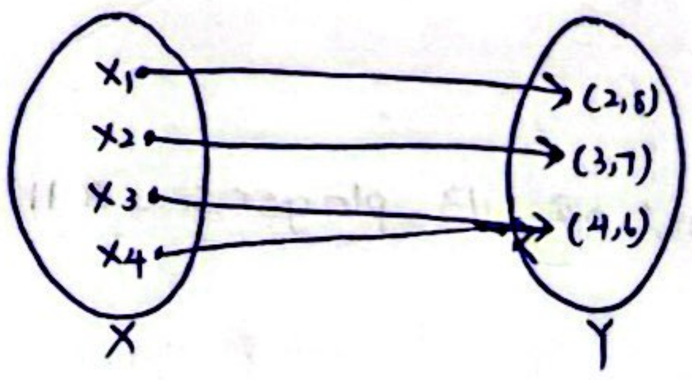
$= \frac{80}{32}$

$= 2.5$

$k = 3$ ✓ (shown)

∴ Each people can have at least three pieces of cheesecake.

(c)



The pairs that can has sum of 10 is the pigeonholes, Y.

$Y = \{(2,8), (3,7), (4,6)\}$

$|Y| = 3$

The integers that we choose from the set X is the pigeon, Sthe there may occurs when we choose 3 integers from set X that has no sum of 10, which is $\{2, 3, 4\}$.
So that X should be at least 4.

$|X| = 4$

∴ Since $|X| > |Y|$, at least one pair has a sum of 10 if at least 4 integers be choose.

(d) By using the third form of Pigeonhole Principle,

$k = \frac{n}{m}$, where k = minimum number of student per same grade (k = 6)
n = number of student (pigeon)
m = number of grades (pigeonholes, m = 5)

$6 = \frac{n}{5}$, for the $\frac{n}{5}$, it must be > 5 (which can be round up)

Here,

$\frac{n}{5} > 5$

$n > 5 \times 5$

$n > 25$

$n = 26$

∴ There is minimum 26 of students required in a discrete mathematics class to be sure that at least six will receive the same grade.

Pigeon : number of computers ($n=6$)

The computer can connect to the other computers, which 0 or more. But it cannot connected to iteself.

Pigeonhole : number of computer connected ($m=5$)

$$k = \frac{n}{m}$$

$$= \frac{6}{5}$$

$$= 1.2$$

$k = 2$ ✱ (shaun)