1. $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$B = \{10, 12, 14, 16, 18, 20\}$

$C = \{1, 3, 5, 7, 9\}$

(a) $A \cap C \cup B = \{2,3,5,7,11,13,17,19\} \cap \{1,3,5,7,9\} \cup \{10,12,14,16,18,20\}$

$= \{3,5,7,10,12,14,16,18,20\}$

(b) $A \cap B \cup C = \{2,3,5,7,11,13,17,19\} \cap \{10,12,14,16,18,20\} \cup \{1,3,5,7,9\}$

$= \{1,3,5,7,9\}$

$P(A \cap B \cup C) = \{\phi, \{1\}, \{3\}, \{5\}, \{7\}, \{9\}, \{1,3\}, \{1,5\}, \{1,7\}, \{1,9\},$
$\{3,5\}, \{3,7\}, \{3,9\}, \{5,3\}, \{5,7\}, \{5,9\}, \{7,9\}, \{1,3,5\},$
$\{1,3,7\}, \{1,3,9\}, \{1,5,7\}, \{1,5,9\}, \{1,7,9\}, \{3,5,7\}, \{3,5,9\},$
$\{3,7,9\}, \{5,7,9\}, \{1,3,5,7\}, \{1,3,5,9\}, \{1,5,7,9\}, \{3,5,7,9\},$
$\{1,3,5,7,9\}\}$

(c) $A - C = \{2, 11, 13, 17, 19\}$

(d) $|A| = 8$

$|B| = 6$

$|C| = 5$

(e) $A \cap C = \{3,5,7\}$

$|P(A \cap C)| = 2^3 = 8$

(f) $B = \{10,12,14,16,18,20\}$

$C' = \{2,4,6,8,10,11,12,13,14,15,16,17,18,19,20\}$

$\therefore B \subset C'$ ~~False~~ True

(g) True

2. (a) $(A - C') \cup (B - C) = (A \cap C'') \cup (B \cap C')$ Set difference law

$= (A \cap C) \cup (B \cap C')$

$\therefore (A - C') \cup (B - C)$ is not equal to $A \cup B$.

(b) $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$ Set difference law

$= A \cup (B \cap B')$ Distributive law

$= A \cup \phi$ Complement law

$= A$ Identity law (shown)

$\therefore (A \cap B) \cup (A - B)$ is equal to A

3. (a) $S = \{a, b, c, d, e, f, g\}$

$T = \{h, j, k, l, m, n, p, q\}$

$E = \{p, q, r, s, t, v, w, y, z\}$

(b) $T \cap E = \{p, q\}$

$S \times (T \cap E) = \{a, b, c, d, e, f, g\} \times \{p, q\}$

$= \{(a,p), (a,q), (b,p), (b,q), (c,p), (c,q), (d,p), (d,q), (e,p), (e,q), (f,p), (f,q), (g,p), (g,q)\}$

4. (a) TRUE

(b) TRUE

5. (a)

p	q	r	$\neg r$	$p \wedge r$	$q \vee \neg r$	$p \vee q$	$Q = (p \wedge r) \vee (q \vee \neg r)$	$R = (p \vee q) \vee \neg r$
T	T	T	F	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	T
T	F	F	T	F	T	T	T	T
F	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	F	F	F	F	F	F
F	F	F	T	F	T	F	T	T

$\therefore$ Since the truth values of Q and truth values of R are the same, $Q \equiv R$.

(b)

p	q	r	$\neg q$	$p \wedge r$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$q \vee r$	$Q = (p \wedge r) \vee \neg(p \wedge \neg q)$	$R = (p \wedge r) \to (q \vee r)$
T	T	T	F	T	F	T	T	T	T
T	T	F	F	F	F	T	T	T	T
T	F	T	T	T	T	F	T	T	F
T	F	F	T	F	T	F	F	F	T
F	T	T	F	F	F	T	T	T	T
F	T	F	F	F	F	T	T	T	T
F	F	T	T	F	F	T	T	T	T
F	F	F	T	F	F	T	F	T	T

$\therefore$ Since the truth values of Q and R are different, $Q \not\equiv R$.

6. (a) Not true. counterexample is 1.
1 is an odd number and less than 7. must have all counterexample

(b) Not true. counterexample is 9.
9 is odd number and greater than 7.

7. $\exists x(P(x) \wedge Q(x))$

8. a is odd.
Let $a = 2n+1$, where $n \in \mathbb{Z}$.

$$
\begin{aligned}
a^2-3a &= (2n+1)^2 - 3(2n+1) \\
&= 4n^2+4n+1-(6n+3) \\
&= 4n^2+4n+1-6n-3 \\
&= 4n^2-2n-2 \\
&= 2(2n^2-n-1) \\
&= 2k \text{ (proven)}
\end{aligned}
$$

$\therefore$ There exists an integer k, such that $a^2-3a = 2k$. Therefore, a^2-3a is even.

9. Contradiction: Suppose n^2 is an odd integer, then n is not odd. Hence, n is an even.

$n = 2a$, where $a \in \mathbb{Z}$

$$
\begin{aligned}
n^2 &= (2a)^2 \\
&= 4a^2 \\
&= 2(2a^2) \text{ where } 2a^2 \text{ is an integer and } k = 2a^2 \\
&= 2k
\end{aligned}
$$