Updated Simulation Results of the paper *Binary Response Forecasting under a Factor-Augmented Framework*

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1 Simulation

In this section, we conduct simulations to examine the finite-sample performance of our proposed estimator. Specifically, we consider the following data generating process (DGP):

$$y_{t+1} = \begin{cases} 1, \text{ if } \beta_0 + \beta_f^\top f_t + \beta_w^\top w_t - \epsilon_{t+1} \geq 0 \\ 0, \text{ otherwise} \end{cases}$$

where $\beta_0 = -2$, $\beta_f = (1,1)^\top$ and $\beta_w = (1,1)^\top$. The observable factors $w_t = (w_{1t}, w_{2t})'$ are generated from uniform distributions $U(0,2)$ and $U(-3,3)$, respectively, and both the unobservable factors $f_t = (f_{1t}, f_{2t})^\top$ follow AR(1) processes:

$$f_{1t} = \rho_1 f_{1t-1} + (1-\rho_1^2)^{1/2}\kappa_{1t},$$
$$f_{2t} = \rho_2 f_{2t-1} + (1-\rho_2^2)^{1/2}\kappa_{2t},$$

for $t \geq 1$, with $\rho_i = 0.8^i$ for $i = 1,2$ and $(\kappa_{1t}, \kappa_{2t})$ are independently generated from the standard normal distribution. The initial values f_{10} and f_{20} are independently drawn from the uniform distribution $U(0,2)$.

Additionally, the DGP for observable predictors $\{x_{it}\}$ is given by

$$x_{it} = \lambda_{i1} f_{1t} + \lambda_{i2} f_{2t} + \gamma_{it},$$

where the loadings are independently generated from the uniform distribution $U(0,6)$, and γ_{it} is following the standard normal distribution. We denote the factor estimators as $(\tilde{f}_{1t}, \tilde{f}_{2t})^\top$, where $t = 1, 2, ..., T$.

We consider two distributions for the innovations ϵ_{t+1}, corresponding to light-tailed and heavy-tailed distributions, respectively. In each example, we further vary the time-series correlation of the error term, covering the i.i.d. and autocorrelation scenarios to examine the robustness of the proposed method.

Example 1. Three normal distributions are considered in this example:

- **DGP1:** The error term ϵ_t follows the i.i.d. standard normal distribution.

- **DGP2:** The error term ϵ_t is generated from the following autoregressive process:

$$\epsilon_t = \rho_\epsilon \cdot \epsilon_{t-1} + \sqrt{1-\rho_\epsilon^2} \cdot \nu_t,$$

  where $\rho_\epsilon = 0.3$ and ν_t follows the i.i.d. standard normal distribution.

- **DGP3:** ϵ_t follows the same autoregressive process as DGP2 with $\rho_\epsilon = 0.7$.

Example 2. We adopt DGPs analogous to those in **Example 1**, replacing every normal distribution with the standard logistic distribution.

Example 3. In this example, we follow the DGP processes in **Example 2**. However, we adopt a Quasi-maximum likelihood estimation (QMLE) idea, that is, we use the standard normal distribution's likelihood function to estimate the coefficients.

For each simulation dataset, we employ the method outlined in Section 2 to construct the estimators and predicted probabilities. We vary the values of N and T from $\{100, 200, 300\}$ and $\{100, 200, 400\}$, respectively. After repeating the procedure for $R = 500$ times, we compute the root mean squared errors (RMSEs) to evaluate the finite-sample estimation accuracy of the proposed method:

$$\text{RMSE}_{all} = \sqrt{\frac{1}{R}\sum_{i=1}^{R} \|\widehat{\beta}_i - \beta^\circ\|^2},$$

where $\widehat{\beta}_i$ denotes the estimated coefficient vector in the ith replication, and β° is the rotated coefficeints as defined in Section 2.2. Additionally, We also report RMSE values for each individual coefficient, including $\text{RMSE}_{\text{cons}}$, RMSE_{f_1}, RMSE_{f_2}, RMSE_{w_1}, and RMSE_{w_2}.

We compute the area under the receiver operating characteristic curve (AUC) to assess predictive performance. The AUC measures the model's ability to distinguish between the two outcome classes (0 vs 1) across different threshold values. The receiver operating characteristic curve (ROC) curve maps the true positive rate

$$\text{TP}(\xi) = P_{t-h}\left(p_t > \xi \mid y_t = 1\right)$$

against the false positive rate

$$\text{FP}(\xi) = P_{t-h}\left(p_t > \xi \mid y_t = 0\right)$$

across all thresholds $0 \le \xi \le 1$, forming a function over the unit square $[0,1] \times [0,1]$. Here, p_t denotes the predicted conditional probability of the target variable. And a higher AUC indicates stronger predictive discrimination. Specifically, for each replication, we calculate the AUC based on the estimated probabilities and the true binary outcomes. The simulation results are then summarized by reporting the mean, median, and standard deviation of the AUC across all replications.

Tables 1, 2, and 3 present the simulation results for the three Examples. The findings indicate that our method delivers robust performance under both light-tailed and heavy-tailed error distributions in the MLE setting, as well as in the QMLE setting. The RMSE values decrease steadily as the sample size T increases. This pattern supports the $\sqrt{T}$-consistency of the augmented maximum likelihood estimators established in Theorem 2.

Tables 4, 5, and 6 summarize the mean, median, and standard deviation of the resulting AUCs. Overall, the proposed model exhibits high predictive accuracy, with both the mean and median AUC values exceeding 0.89 across simulations under various settings. Comparisons between Tables 4, 5 and 6 reveal that variations in the distribution of idiosyncratic error terms have limited influence on the AUC results.

Table 1: RMSEs for Example 1.

	RMSE_{all}			$\text{RMSE}_{\text{cons}}$			RMSE_{f_1}			RMSE_{f_2}			RMSE_{w_1}			RMSE_{w_2}		
N/T	100	200	400	100	200	400	100	200	400	100	200	400	100	200	400	100	200	400
	Panel A: DGP1																	
100	1.063	0.632	0.423	0.772	0.463	0.310	0.460	0.266	0.175	0.263	0.151	0.112	0.399	0.247	0.163	0.307	0.173	0.114
200	1.051	0.625	0.406	0.755	0.448	0.289	0.474	0.273	0.180	0.239	0.158	0.110	0.399	0.239	0.155	0.305	0.184	0.112
300	1.054	0.620	0.398	0.763	0.449	0.288	0.460	0.272	0.164	0.248	0.157	0.106	0.399	0.234	0.157	0.311	0.172	0.113
	Panel B: DGP2																	
100	1.038	0.704	0.424	0.727	0.512	0.304	0.472	0.309	0.188	0.286	0.172	0.115	0.383	0.262	0.154	0.314	0.196	0.121
200	1.088	0.645	0.428	0.784	0.465	0.313	0.493	0.271	0.177	0.252	0.169	0.118	0.404	0.251	0.166	0.314	0.188	0.113
300	1.115	0.649	0.437	0.810	0.461	0.322	0.497	0.294	0.182	0.266	0.171	0.113	0.414	0.244	0.166	0.315	0.183	0.120
	Panel C: DGP3																	
100	1.227	0.744	0.474	0.887	0.549	0.341	0.550	0.319	0.215	0.316	0.197	0.131	0.442	0.266	0.169	0.353	0.202	0.126
200	1.169	0.757	0.461	0.842	0.542	0.338	0.524	0.346	0.192	0.304	0.203	0.133	0.411	0.271	0.169	0.350	0.213	0.124
300	1.259	0.740	0.473	0.918	0.532	0.342	0.567	0.335	0.205	0.319	0.202	0.132	0.437	0.264	0.170	0.359	0.205	0.134

Table 2: RMSEs for Example 2.

	RMSE_{all}			$\text{RMSE}_{\text{cons}}$			RMSE_{f_1}			RMSE_{f_2}			RMSE_{w_1}			RMSE_{w_2}		
N/T	100	200	400	100	200	400	100	200	400	100	200	400	100	200	400	100	200	400
	Panel A: DGP1																	
100	1.178	0.693	0.482	0.858	0.499	0.352	0.474	0.285	0.189	0.331	0.220	0.151	0.471	0.264	0.192	0.309	0.177	0.114
200	1.163	0.720	0.489	0.825	0.521	0.349	0.504	0.294	0.202	0.362	0.214	0.151	0.439	0.283	0.196	0.311	0.187	0.122
300	1.155	0.688	0.462	0.836	0.499	0.331	0.497	0.283	0.191	0.325	0.214	0.147	0.444	0.258	0.185	0.292	0.178	0.108
	Panel B: DGP2																	
100	1.196	0.717	0.475	0.824	0.512	0.338	0.563	0.300	0.198	0.366	0.250	0.165	0.448	0.260	0.179	0.315	0.176	0.113
200	1.272	0.714	0.485	0.906	0.517	0.347	0.599	0.287	0.201	0.339	0.230	0.166	0.460	0.278	0.186	0.334	0.172	0.109
300	1.167	0.747	0.520	0.823	0.532	0.381	0.517	0.313	0.202	0.380	0.254	0.168	0.439	0.273	0.201	0.284	0.195	0.124
	Panel C: DGP3																	
100	1.376	0.858	0.534	0.982	0.622	0.379	0.641	0.366	0.236	0.436	0.311	0.194	0.466	0.284	0.179	0.333	0.193	0.124
200	1.452	0.830	0.560	1.033	0.605	0.408	0.690	0.364	0.238	0.443	0.271	0.194	0.498	0.285	0.196	0.346	0.191	0.120
300	1.342	0.910	0.577	0.942	0.676	0.425	0.624	0.388	0.242	0.438	0.300	0.200	0.465	0.294	0.194	0.339	0.209	0.126

Table 3: RMSEs for Example 3.

	RMSE_{all}			$\text{RMSE}_{\text{cons}}$			RMSE_{f_1}			RMSE_{f_2}			RMSE_{w_1}			RMSE_{w_2}		
N/T	100	200	400	100	200	400	100	200	400	100	200	400	100	200	400	100	200	400
	Panel A: DGP1																	
100	1.214	1.223	1.210	0.869	0.871	0.852	0.569	0.599	0.609	0.192	0.127	0.089	0.454	0.435	0.428	0.391	0.416	0.421
200	1.194	1.218	1.195	0.849	0.866	0.844	0.567	0.596	0.594	0.206	0.125	0.089	0.431	0.437	0.425	0.392	0.414	0.417
300	1.223	1.199	1.221	0.874	0.848	0.869	0.582	0.596	0.602	0.186	0.125	0.085	0.447	0.427	0.437	0.400	0.408	0.418
	Panel B: DGP2																	
100	1.241	1.193	1.218	0.886	0.839	0.862	0.589	0.593	0.603	0.207	0.145	0.097	0.450	0.423	0.434	0.404	0.408	0.424
200	1.195	1.221	1.217	0.843	0.859	0.858	0.579	0.611	0.609	0.197	0.134	0.097	0.428	0.433	0.434	0.401	0.418	0.421
300	1.187	1.217	1.219	0.840	0.863	0.867	0.568	0.597	0.601	0.217	0.148	0.096	0.428	0.438	0.437	0.386	0.406	0.417
	Panel C: DGP3																	
100	1.263	1.209	1.232	0.912	0.858	0.872	0.590	0.586	0.610	0.242	0.178	0.115	0.446	0.428	0.434	0.397	0.409	0.428
200	1.213	1.231	1.225	0.859	0.876	0.862	0.580	0.604	0.616	0.252	0.157	0.113	0.428	0.434	0.433	0.389	0.412	0.423
300	1.206	1.221	1.241	0.866	0.865	0.884	0.563	0.600	0.609	0.249	0.174	0.115	0.427	0.433	0.445	0.378	0.405	0.422

Table 4: AUCs for Example 1.

	AUC_{mean}			AUC_{median}			AUC_{std}		
N/T	100	200	400	100	200	400	100	200	400
	Panel A: DGP1								
100	0.963	0.960	0.958	0.965	0.961	0.958	0.017	0.011	0.009
200	0.963	0.959	0.959	0.965	0.960	0.959	0.017	0.013	0.008
300	0.961	0.960	0.958	0.963	0.961	0.959	0.018	0.012	0.008
	Panel B: DGP2								
100	0.962	0.961	0.958	0.965	0.962	0.959	0.016	0.012	0.009
200	0.963	0.961	0.959	0.965	0.961	0.959	0.016	0.012	0.009
300	0.963	0.960	0.958	0.964	0.961	0.958	0.018	0.012	0.009
	Panel C: DGP3								
100	0.904	0.900	0.894	0.907	0.901	0.894	0.034	0.023	0.018
200	0.906	0.898	0.896	0.910	0.899	0.897	0.035	0.026	0.017
300	0.907	0.901	0.895	0.911	0.901	0.895	0.034	0.023	0.018

Table 5: AUCs for Example 2.

	AUC_{mean}			AUC_{median}			AUC_{std}		
N/T	100	200	400	100	200	400	100	200	400
	Panel A: DGP1								
100	0.963	0.960	0.958	0.965	0.961	0.958	0.017	0.011	0.009
200	0.963	0.959	0.959	0.965	0.960	0.959	0.017	0.013	0.008
300	0.961	0.960	0.958	0.963	0.961	0.959	0.018	0.012	0.008
	Panel B: DGP2								
100	0.962	0.961	0.958	0.965	0.962	0.959	0.016	0.012	0.009
200	0.963	0.961	0.959	0.965	0.961	0.959	0.016	0.012	0.009
300	0.963	0.960	0.958	0.964	0.961	0.958	0.018	0.012	0.009
	Panel C: DGP3								
100	0.904	0.900	0.894	0.907	0.901	0.894	0.034	0.023	0.018
200	0.906	0.898	0.896	0.910	0.899	0.897	0.035	0.026	0.017
300	0.907	0.901	0.895	0.911	0.901	0.895	0.034	0.024	0.018

Table 6: AUCs for Example 3.

	AUC_{mean}			$\text{AUC}_{\text{median}}$			AUC_{std}		
N/T	100	200	400	100	200	400	100	200	400
	Panel A: DGP1								
100	0.904	0.898	0.898	0.905	0.898	0.898	0.030	0.021	0.015
200	0.905	0.899	0.898	0.909	0.900	0.899	0.031	0.024	0.016
300	0.903	0.898	0.898	0.906	0.899	0.899	0.032	0.022	0.014
	Panel B: DGP2								
100	0.901	0.900	0.896	0.903	0.899	0.896	0.033	0.020	0.015
200	0.903	0.898	0.897	0.906	0.898	0.897	0.032	0.022	0.015
300	0.905	0.901	0.897	0.908	0.901	0.899	0.031	0.023	0.017
	Panel C: DGP3								
100	0.904	0.900	0.894	0.907	0.901	0.894	0.034	0.023	0.018
200	0.906	0.898	0.896	0.910	0.899	0.897	0.035	0.026	0.017
300	0.907	0.901	0.895	0.911	0.901	0.895	0.035	0.024	0.018