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Large-scale Fixed-wing UAV swarm system control with collision avoidance and formation maneuver

Jiacheng Li , Yangwang Fang , Haoyun Cheng , Zhikai Wang , Zihao Wu 

Abstract—In this paper, a distributed control algorithm for large-scale Fixed-wing UAV (Unmanned Aerial Vehicle) swarm system of collision avoidance and formation maneuver is proposed, which can meet the demands of a dense swarm with collaborative autonomy. First, a novel structure of a swarm is introduced, which can provide the reserved maneuverable space for UAVs during the flight. Besides, inspired by the movement of bird flocks, the LTM (Light transmission model) is proposed to improve the conventional potential field functions, which can realize the UAV swarm with the capable of repulsion in the short-range, speed and position cooperation in the middle range, and attraction in the remote range. Then, the distributed control architecture and the control algorithm of UAVs are designed, the stability of the swarm is proved correspondingly. In the end, several numerical simulation results show the effectiveness and superiority of the proposed method, which illustrates the distributed control architecture of the swarm system to be more adaptable, stable, and scalable. Therefore, the proposed control algorithm provides strong theoretical support for the actual application in a large-scale Fixed-wing UAV swarm system collaborative autonomy.

Index Terms—UAV, swarm system, Large-scale, Collision avoidance, Potential field functions.

I. INTRODUCTION

In recent years, there has been a surge of interest among multi-UAVs, which has wide applications in civilian and military. Compared with the Single-UAV, the multi-UAVs are more capable, which can cover a larger area and complete more tasks with less cost, higher flexibility, and stronger robustness [1]. Among them, the key issue for the multi-UAVs system study is to coordinate the obstacle avoidance and collision avoidance between UAVs [2].

There have been extensive studies of obstacle avoidance methods for multi-UAVs, where mathematical planning, graph theory, spatial decomposition, intelligent algorithm, random planning, machine learning, and potential field function are common methods [3], [4]. In addition, it was often needed for multi-UAVs to form a specific formation to perform some type of mission during the flight, which is not enough to rely solely on obstacle avoidance methods [5], [6]. It can be said that the coordinated control of multi-UAVs not only relies on obstacle

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avoidance methods, but also requires formation control as the basis. Virtual structure [7], behavior-based [8], and leader following [9] are common formations studied in the past. With the combination of the above methods, many effective applications have been achieved in multi-UAVs control. X. Yan reported an approach combining virtual structure with artificial potential field (APF), which can realize a high accuracy of formation keeping and transforming [10]. Yang proposed a formation control method with collision, obstacle avoidance, and connectivity maintenance based on Neural-network by the combination of the APF method and leader following structure [11]. It can be seen from the existing literature that the common methods have good control effects in the formation and obstacle avoidance control of a certain number of the multi-UAVs.

With the requirements of practical applications, demands for the number of UAVs in the multi-UAVs are also increased significantly, which gradually forms UAV swarm operation. The UAV swarm system can be considered as a form of collective behavior of a large number of interacting UAVs, which desires to achieve a common group goal. However, the computational complexity grows exponentially when the number of the UAVs increases continuously [12], especially in the large-scale UAV swarm system in which the number size of UAV is over hundreds [13], [14]. That is to say, the conventional formation and obstacle avoidance control methods are no longer suitable for such situations.

In the research of large-scale UAV swarm control field, we can divide it into two main research parts based on the autonomy degree of UAVs. One part is that many companies use UAVs in the light show, which is the automation systems devised for live performances that could control one to potentially thousands of UAVs [15], [16]. As yet, a Guinness World Record had been created by Intel because “The Most UAVs Airborne Simultaneously”, which has a swarm of 500 UAVs [17]. In addition, the largest UAV swarm had been developed by Ehang, where more than 1000 UAVs in their experiment [18]. However, these UAV swarm control methods are just tracking the predefined paths under ensuring the safety and reliability of each UAV, which the whole swarm will be affected accordingly if there is a fault problem with any individual UAVs. Therefore, it is not satisfied that the criteria of collaborative autonomy were used in such a case. And these control systems are often under a central control architecture that needs every UAV to communicate with the ground stations, which means a huge calculation amount and high hardware equipment requirements.

Another is that the collaborative autonomy UAV swarm with

the characteristics of collision avoidance, path planning, and decision making autonomously [19]–[21]. But the scales of UAVs in their theoretical researches are only about a hundred, where the numbers of UAVs in actual flying are only a few dozen. The military of the U.S. has done an experiment called Perdix with Fixed-wing drone swarms, which realized 103 autonomous UAVs adaptive formation flying. G. V. Srhelyi realized a swarm of 30 self-organized UAVs in field experiments, which was the first time in detail reported to achieve UAVs collaborative autonomy system in outdoor aerial with such number scale UAV swarm [23]. Wang and his team aimed at the formation control problem of large-scale Fixed-wing UAV swarm with a multi-layered group-based architecture, and carried out the numerical simulation of the flying process with 100 Fixed-wing UAVs swarm and realized 21 UAVs actual flying in field experiments [24] [25]. T. H. Chung completed the cooperative autonomous actual flight with 50 Fixed-wing UAVs in a low-cost and open-source component [26]. A. Kushleyev mentioned a useful control architecture and algorithms to coordinate a swarm of 20 quadrotors, which is at the expense of flexibility [27]. Unfortunately, the proposed control mechanisms and structures have effective effects in UAV swarm control, but it is uncertain whether their system can still steady working when the scale of the swarm continues increasing.

As for large-scale swarm control, there were some researches in multi-agent theory [28], especially in solving large-scale communication problems [29]. But unlike multi-agent under ideal model and quadrotors, Fixed-wing UAVs have limited turn radius and minimum flight speed, which determines it has less maneuverable and more strictly flying constraints [30], [31]. Some researchers focused on machine learning to solve these problems, which suffers from the drawbacks including only theory feasible, unable to prove the converge of the algorithm, and the requirement of high modeling accuracy with complicated calculation [32]. Therefore, it should be pointed out that a number of issues related to swarm control need to be resolved immediately. Especially, how to achieve cooperative control of large-scale swarm based on the suitable obstacle avoidance and formation control algorithms is still a challenge.

To solve this problem, a collision avoidance control method for large-scale Fixed-wing UAV swarm based on formation reserved maneuvering space is proposed, which has the main contributions as follows

(1) The light transmission model (LTM) is added to the conventional potential field functions to ensure the UAV flying decisions about collision avoidance by choosing the maneuvering space in a short time.

(2) A novel swarm structure is proposed to reserve the maneuver space for UAVs obstacle avoidance and collision avoidance, which can make every UAV detect sufficient light transmission. The capability of scalability and reconstruction for the swarm is improved under such structure, which ensures only a small part of the swarm will be influenced rather than the whole one, when the swarm runs into obstacles or has a collision with each other.

(3) A distributed control architecture divided the large-scale swarm into groups by makes full use of parallelism is

proposed, which reduces the complexity of task planning and the demand of communication capabilities.

The structure of this paper is organized as follows: Some basics about swarm control are introduced and the LTM is also proposed in section II. The potential field functions used in swarm cooperative control are designed in Section III. Section IV designs the control algorithm for UAV and proves the stability of the swarm system. In Section V, some numerical simulations are carried out to show the effectiveness and superiority of the proposed method. Section VI provides the conclusions of this study.

II. PRELIMINARIES AND MODELING

In this section, we will introduce the structure and topology of the swarm, the models of UAVs, and the light transmission models.

A. Structure of swarm

To control a large number of Fixed-wing UAV swarm, we divide the swarm into several groups. A reference point of the swarm structure is set to maintain the formation structure of the swarm. Inspired by the layout of the high-rise apartment building, where the capacity of the daylighting for each building in their design process is considered fully, a new formation structure for the swarm in Fig. 1 is proposed in this paper.

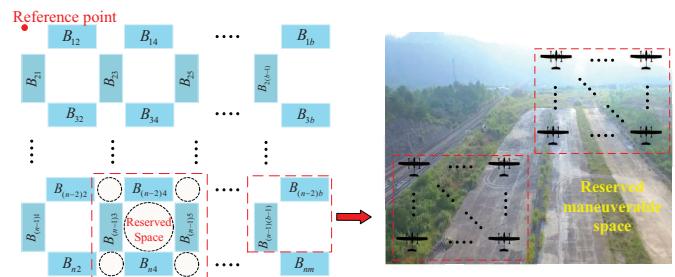


Fig. 1: The formation shape under ideal state.

The maneuver space for UAVs is reserved under this structure through a certain regular arrangement and combination of each group position, which can provide enough light transmission and obstacle avoidance space for UAVs. Considering the scalability of the swarm, space is reserved on the right side of each group to ensure a new group to join at any time. The matrix $\Xi_{n \times m}$ is used to describe the structure of the swarm, which consists of a certain number of partitioned matrices B_{lk} . And $\Xi_{n \times m}$ can be specifically expressed as Eq. (1), where

$$n = n_1 + n_2 + \dots + n_a \quad (2)$$

$$m = m_1 + m_2 + \dots + m_b \quad (3)$$

$$B_{lk} = B_{lk(n_l \times m_k)} \quad (l = 1, \dots, a; k = 1, \dots, b) \quad (4)$$

Considering the reserved space of the swarm structure, the following elements in matrix $\Xi_{n \times m}$ are defined as the partitioned matrices composed of 0.

$$\mathbf{B}_{lk} = \begin{cases} k = 1, 3, \dots, b-1 & , l \text{ is odd} \\ k = 2, 4, \dots, b & , l \text{ is even} \end{cases} \quad (5)$$

In the matrix $\Xi_{n \times m}$, the elements of the partitioned matrix \mathbf{B}_{lk} are composed of each UAV element, which are specifically expressed as $\alpha_{ij}^{\mathbf{B}_{lk}} \in \mathbf{B}_{lk}$, where $\alpha_{ij}^{\mathbf{B}_{lk}}$ represents the UAV in the i -th row and j -th column of the group \mathbf{B}_{lk} and satisfies

$$\begin{cases} i = 1, \dots, n_1 & , l = 1 \\ i = n_1 + \dots + n_{l-1} + 1, \dots, n_1 + & \\ \dots + n_{l-1} + n_l & , 1 < l \leq a \end{cases} \quad (6)$$

$$\begin{cases} j = 1, \dots, m_1 & , k = 1 \\ j = m_1 + \dots + m_{k-1} + 1, \dots, m_1 + & \\ \dots + m_{k-1} + m_k & , 1 < k \leq b \end{cases} \quad (7)$$

For any UAV $\alpha_{ij}^{\mathbf{B}_{lk}}$, its coordinate position can be recorded as $\mathbf{q}_{ij}^{\alpha\mathbf{B}_{lk}} = [x_{ij}^{\alpha\mathbf{B}_{lk}}, y_{ij}^{\alpha\mathbf{B}_{lk}}]^T$. Assume that the coordinate space between every two UAVs are $\Delta \mathbf{q}_{ij}^{\alpha\mathbf{B}_{lk}} = [\Delta dx \ \Delta dy]^T$. And we also define the coordinate of the element in the first row and first column of the structure matrix as the coordinate of reference point of the swarm structure, that is $\mathbf{q}^r = [0 \ 0]^T$. Therefore, the relative position expression of the UAV $\alpha_{ij}^{\mathbf{B}_{lk}}$ is as

$$\mathbf{q}_{ij}^{\alpha\mathbf{B}_{lk}} - \mathbf{q}^r = \xi_{ij}^{\alpha\mathbf{B}_{lk}} \Delta \mathbf{q}_{ij}^{\alpha\mathbf{B}_{lk}} \quad (8)$$

where $\xi_{ij}^{\alpha\mathbf{B}_{lk}}$ is the coefficient matrix used to determine the distance between the reference point and UAV $\alpha_{ij}^{\mathbf{B}_{lk}}$ in the swarm, which can be expressed as

$$\xi_{ij}^{\alpha\mathbf{B}_{lk}} = \begin{bmatrix} \xi_{ij1}^{\alpha\mathbf{B}_{lk}} & 0 \\ 0 & \xi_{ij2}^{\alpha\mathbf{B}_{lk}} \end{bmatrix} \quad (9)$$

where,

$$\xi_{ij1}^{\alpha\mathbf{B}_{lk}} = \begin{cases} j-1 & , l = 1 \\ [(m_1 + \dots + m_{k-1}) + j] - 1 & , 1 < l \leq a \end{cases} \quad (10)$$

$$\xi_{ij2}^{\alpha\mathbf{B}_{lk}} = \begin{cases} i-1 & , k = 1 \\ [(n_1 + \dots + n_{l-1}) + i] - 1 & , 1 < k \leq b \end{cases} \quad (11)$$

B. Modeling of UAV

Suppose that the simplified dynamic model of UAV $\alpha_{ij}^{\mathbf{B}_{lk}}$ can be expressed as

$$\begin{cases} \dot{x}_{ij}^{\alpha\mathbf{B}_{lk}} = v_{ij}^{\alpha\mathbf{B}_{lk}} \cos \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \\ \dot{y}_{ij}^{\alpha\mathbf{B}_{lk}} = v_{ij}^{\alpha\mathbf{B}_{lk}} \sin \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \\ \dot{v}_{ij}^{\alpha\mathbf{B}_{lk}} = n_{ij}^{v\alpha\mathbf{B}_{lk}} \\ \dot{\psi}_{ij}^{\alpha\mathbf{B}_{lk}} = n_{ij}^{\varphi\alpha\mathbf{B}_{lk}} \end{cases} \quad (12)$$

where, $(x_{ij}^{\alpha\mathbf{B}_{lk}}, y_{ij}^{\alpha\mathbf{B}_{lk}})$ is the position of UAV $\alpha_{ij}^{\mathbf{B}_{lk}}$ in the inertial coordinate system; $\varphi_{ij}^{\alpha\mathbf{B}_{lk}}$ is the heading angle of UAV $\alpha_{ij}^{\mathbf{B}_{lk}}$; $n_{ij}^{v\alpha\mathbf{B}_{lk}}$ and $n_{ij}^{\varphi\alpha\mathbf{B}_{lk}}$ are the control inputs which represent the overload of the UAV $\alpha_{ij}^{\mathbf{B}_{lk}}$ in inertial coordinate system; $v_{ij}^{\alpha\mathbf{B}_{lk}}$ is the velocity of UAV $\alpha_{ij}^{\mathbf{B}_{lk}}$.

To simplify the UAV model, we need to linearize the Eq. (12). As mentioned above $\mathbf{q}_{ij}^{\alpha\mathbf{B}_{lk}} = [x_{ij}^{\alpha\mathbf{B}_{lk}} \ y_{ij}^{\alpha\mathbf{B}_{lk}}]^T$, and calculating its first time derivative, we have

$$\dot{\mathbf{p}}_{ij}^{\alpha\mathbf{B}_{lk}} = \dot{\mathbf{q}}_{ij}^{\alpha\mathbf{B}_{lk}} = \begin{bmatrix} \dot{x}_{ij}^{\alpha\mathbf{B}_{lk}} \\ \dot{y}_{ij}^{\alpha\mathbf{B}_{lk}} \end{bmatrix} = \begin{bmatrix} v_{ij}^{\alpha\mathbf{B}_{lk}} \cos \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \\ v_{ij}^{\alpha\mathbf{B}_{lk}} \sin \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \end{bmatrix} \quad (13)$$

Further, calculating the first derivative of formula Eq. (13), we have

$$\begin{aligned} \ddot{\mathbf{p}}_{ij}^{\alpha\mathbf{B}_{lk}} = \ddot{\mathbf{q}}_{ij}^{\alpha\mathbf{B}_{lk}} &= \begin{bmatrix} \ddot{x}_{ij}^{\alpha\mathbf{B}_{lk}} \\ \ddot{y}_{ij}^{\alpha\mathbf{B}_{lk}} \end{bmatrix} \\ &= \begin{bmatrix} \dot{v}_{ij}^{\alpha\mathbf{B}_{lk}} \cos \varphi_{ij}^{\alpha\mathbf{B}_{lk}} - v_{ij}^{\alpha\mathbf{B}_{lk}} \dot{\varphi}_{ij}^{\alpha\mathbf{B}_{lk}} \sin \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \\ \dot{v}_{ij}^{\alpha\mathbf{B}_{lk}} \sin \varphi_{ij}^{\alpha\mathbf{B}_{lk}} + v_{ij}^{\alpha\mathbf{B}_{lk}} \dot{\varphi}_{ij}^{\alpha\mathbf{B}_{lk}} \cos \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \end{bmatrix} \\ &= \begin{bmatrix} \cos \varphi_{ij}^{\alpha\mathbf{B}_{lk}} & -v_{ij}^{\alpha\mathbf{B}_{lk}} \sin \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \\ \sin \varphi_{ij}^{\alpha\mathbf{B}_{lk}} & v_{ij}^{\alpha\mathbf{B}_{lk}} \cos \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \end{bmatrix} \begin{bmatrix} n_{ij}^{v\alpha\mathbf{B}_{lk}} \\ n_{ij}^{\varphi\alpha\mathbf{B}_{lk}} \end{bmatrix} \end{aligned} \quad (14)$$

Then, define

$$\mathbf{H}_{ij}^{\alpha\mathbf{B}_{lk}} = \begin{bmatrix} \cos \varphi_{ij}^{\alpha\mathbf{B}_{lk}} & -v_{ij}^{\alpha\mathbf{B}_{lk}} \sin \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \\ \sin \varphi_{ij}^{\alpha\mathbf{B}_{lk}} & v_{ij}^{\alpha\mathbf{B}_{lk}} \cos \varphi_{ij}^{\alpha\mathbf{B}_{lk}} \end{bmatrix} \quad (15)$$

$$\mathbf{n}_{ij}^{\alpha\mathbf{B}_{lk}} = \begin{bmatrix} n_{ij}^{v\alpha\mathbf{B}_{lk}} & n_{ij}^{\varphi\alpha\mathbf{B}_{lk}} \end{bmatrix}^T \quad (16)$$

$$\mathbf{u}_{ij}^{\alpha\mathbf{B}_{lk}} = \begin{bmatrix} u_{ij}^{v\alpha\mathbf{B}_{lk}} & u_{ij}^{\varphi\alpha\mathbf{B}_{lk}} \end{bmatrix}^T \quad (17)$$

It follows that

$$\mathbf{u}_{ij}^{\alpha\mathbf{B}_{lk}} = \mathbf{H}_{ij}^{\alpha\mathbf{B}_{lk}} \mathbf{n}_{ij}^{\alpha\mathbf{B}_{lk}} \quad (18)$$

$$\Xi_{n \times m} = \begin{bmatrix} 0 & \cdots & 0 & \alpha_{1,m_1+1} & \cdots & \alpha_{1,m_1+m_2} & \cdots & \alpha_{1,m-m_b+1} & \cdots & \alpha_{1,m} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \alpha_{n_1,m_1+1} & \cdots & \alpha_{n_1,m_1+m_2} & \cdots & \alpha_{n_1,m-m_b+1} & \cdots & \alpha_{n_1,m} \\ \alpha_{n_1+1,1} & \cdots & \alpha_{n_1+1,m_1} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \vdots \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \alpha_{n_1+n_2,1} & \cdots & \alpha_{n_1+n_2,m_1} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \vdots \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \alpha_{n-n_a+1,1} & \cdots & \alpha_{n-n_a+1,m_1} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \vdots \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \alpha_{n,1} & \cdots & \alpha_{n,m_1} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \vdots \end{bmatrix} \quad (1)$$

Therefore, the relationship between the original control input $n_{ij}^{\alpha B_{lk}}$ and the new input $u_{ij}^{\alpha B_{lk}}$ is as follows

$$n_{ij}^{\alpha B_{lk}} = H_{ij}^{\alpha B_{lk}-1} u_{ij}^{\alpha B_{lk}} \quad (19)$$

Therefore, Eq. (13) and Eq. (14) can be transformed into the following second-order integral system

$$\begin{cases} \dot{q}_{ij}^{\alpha B_{lk}} = p_{ij}^{\alpha B_{lk}} \\ \dot{p}_{ij}^{\alpha B_{lk}} = u_{ij}^{\alpha B_{lk}} \end{cases} \quad (i=1, \dots, n_l; j=1, \dots, m_k) \quad (20)$$

where, $p_{ij}^{\alpha B_{lk}}$ represents the speed of UAV $\alpha_{ij}^{B_{lk}}$; $u_{ij}^{\alpha B_{lk}}$ is the system control input.

Considering the maneuverability of Fixed-wing UAV, UAV $\alpha_{ij}^{B_{lk}}$ should satisfy the following formula of constraint conditions

$$\begin{cases} 0 < v_{min}^{\alpha B_{lk}} \leq v_{ij}^{\alpha B_{lk}} \leq v_{max}^{\alpha B_{lk}} \\ |w_{ij}^{\alpha B_{lk}}| \leq w_{max}^{\alpha B_{lk}} \\ R_t \geq R_{min} \end{cases} \quad (21)$$

where, $v_{min}^{\alpha B_{lk}}$ and $v_{max}^{\alpha B_{lk}}$ represent the minimum and maximum airspeed allowed by all UAVs in fixed altitude flight, respectively; $w_{ij}^{\alpha B_{lk}}$ is the heading angular rate of UAV $\alpha_{ij}^{B_{lk}}$; $w_{max}^{\alpha B_{lk}}$ is the maximum allowable heading angular rate of all UAVs; R_t is the turning radius of all UAVs; n_{max} is the maximum overload of all UAVs, and the minimum turning radius can be expressed as

$$R_{min} = \frac{(v_{ij}^{\alpha B_{lk}})^2}{n_{max}} \quad (22)$$

C. Topology of Swarm

Let $G = (v, \epsilon)$ represents a graph with n nodes, where $v = \{v_1, v_2, \dots, v_n\}$ is the set of nodes and $\epsilon \subseteq v \times v$ is the set of edges. Assuming that there is the information interaction between node ij and node pq , then there are edges that connect the two nodes. If $(ij, pq) \in \epsilon \iff (pq, ij) \in \epsilon$, the graph G is called undirected. If there is a path between any two nodes, the undirected graph G is called connected. To describe the relationship between nodes and edges, for an undirected graph G with n nodes, we define the adjacency matrix $A = [a_{ij,pq}] (i, j, p, q = 1, \dots, n)$ as

$$a_{ij,pq} = \begin{cases} 1 & , (v_{ij}, v_{pq}) \in \epsilon \\ 0 & , \text{otherwise} \end{cases} \quad (23)$$

L is the Laplace matrix of graph G , which can be defined as

$$L = D - A \quad (24)$$

where D is the degree matrix of graph G .

In this paper, the topology of the swarm is generated at time $t = 0$, which will remain unchanged when $t > 0$. The set of adjacent UAVs of UAV $\alpha_{ij}^{B_{lk}}$ at time $t = 0$ is defined as

$$N_{pq}^{\alpha B_{lk}}(t) = \left\{ \begin{array}{l} \left\| q_{ij}^{\alpha B_{lk}} - q_{pq}^{\alpha B_{lk}} \right\| < r_\alpha, \\ (i, p = 1, \dots, n_l; j, q = 1, \dots, m_k) \end{array} \right\} \quad (25)$$

where, $\|\cdot\|$ represents the Euclidean norm in \mathbb{R}^n ; $q_{pq}^{\alpha B_{lk}}$ is the position of adjacent UAV $\alpha_{ij}^{B_{lk}}$; r_α is the interaction radius of two UAVs in the same group.

To achieve large-scale swarm control and reduce communication costs, it is defined that members of the same group can communicate with each other. Namely, the UAVs in the partitioned matrix B_{lk} can communicate with each other, where other group members are treated as obstacles. Therefore, the set of obstacles detected by UAV $\alpha_{ij}^{B_{lk}}$ is time-varying. At time t , it can be expressed as

$$N_{ij}^o(t) = \left\{ \left\| q_{ij}^{\alpha B_{lk}} - q_\nu^o \right\| < r_o, \nu = 1, 2, \dots, N_o \right\} \quad (26)$$

where, r_o is the repulsion radius of UAV $\alpha_{ij}^{B_{lk}}$ to obstacles; N_o is the number of obstacles; q_ν^o is the position of obstacles. Here, the obstacle set includes actual obstacles and nearby UAVs of other groups.

D. The light transmission model (LTM)

Suppose that birds have an ability to feel the intensity of the light around themselves, they can feel strong light when there is no obstacle around. In this way, they can fly in a direction with less threat of collisions by judging the strength of light, and eventually form a stable distribution. Therefore, this property of the bird swarm is defined as the light transmission model in this paper [33].

We can also think that UAVs in the border and in the middle of the swarm have different abilities to feel the threat of collision, which is conducive to maintain the structure of the swarm. As the UAVs move closer to the middle of the swarm, the brightness that they feel is gradually getting weaken, and the light transmission is also getting weaken.

Considering the characteristics of Fixed-wing UAVs that cannot fly backward in time, the field angle of any UAV is defined in $[-\frac{\pi}{3}, \frac{4\pi}{3}]$. Moreover, to simplify the calculation and improve the accuracy of the light transmission judgment, we divide the field of view evenly into five parts with $\frac{\pi}{3}$ as the interval, which is shown in Fig. 2.

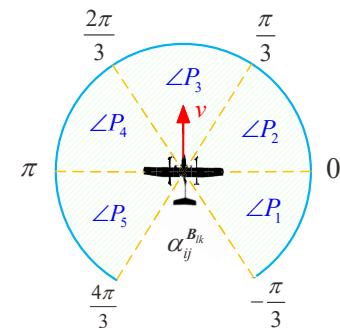


Fig. 2: The light transmission model of UAV.

In the actual judgment process, the light transmission in each field angle is judged separately. Then, the field angle direction with the strongest light transmission will be selected for a UAV to fly. Specially, the field angle closest to the UAV speed direction will be selected to fly, when there are multiple field angles with the same value of the light transmission. The light transmission can be considered as the probability of light transmission, that is, the probability of UAVs falling into the free space. Therefore, the candidate LTM can be defined as

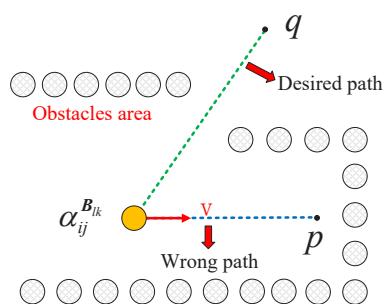


Fig. 3: The situation of partial transparency.

$$P'(\alpha_{ij}^{B_{lk}}) = \frac{3A(\alpha_{ij}^{B_{lk}})}{5\pi} \quad (27)$$

where $A(\alpha_{ij}^{B_{lk}})$ is the angular of the selected field angle direction, which is the sum of the adjacent field angles with the same light transmission, and can be expressed as

$$A(\alpha_{ij}^{B_{lk}}) = \dots + \angle P_{i-1} + \angle P_i + \angle P_{i+1} + \dots \quad (28)$$

where, $\angle P_{i-1}$, $\angle P_i$, $\angle P_{i+1}$, ... are the adjacent field angles with the same value of light transmission.

Considering the LTM defined in Eq. (27) is only suitable for the situation of partial transparency as shown in Fig. 3, which probably leads UAV $\alpha_{ij}^{B_{lk}}$ to a wrong path called dead-end channel (arrive at point P). Here, partial transparency can be understood as the UAV can detect the feasible routes with certain light transmission, when the obstacles are in front of it. And the flyable space of the UAV decreases as the distance between the UAV and obstacle decreases. When the flyable space of the UAV approaches zero, it can be considered that the UAV has entered the dead-end channel. Therefore, a distance factor must be added to Eq. (27) to help the UAVs avoid falling into the dead-end channel, which can be defined as

$$\lambda = \begin{cases} e^{\delta_M \cdot \|q_{ij}^{\alpha B_{lk}} - q_\nu^o\|} & \text{if } \alpha_{ij}^{B_{lk}} \notin B \\ e^{\delta_B \cdot \|q_{ij}^{\alpha B_{lk}} - q_\nu^o\|} & \text{if } \alpha_{ij}^{B_{lk}} \in B \end{cases} \quad (29)$$

where, δ_M and δ_B are the weight coefficients of distance factor ($\delta_M > 0, \delta_B > 0$) for the UAV located in the swarm middle and boundary, respectively. The larger value of δ_C and δ_B are, the greater influence of the distance factor on the LTM is, and vice versa; B is a set of UAVs located at the boundary of formation.

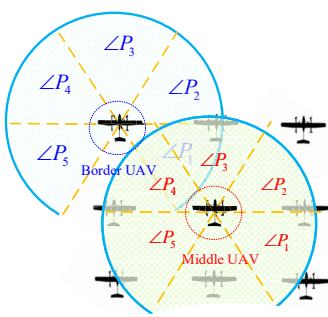


Fig. 4: Middle UAV and border UAV.

In a largely dense swarm, it is defined that the UAV in the middle is surrounded by its neighbors, whose all-view field will be occlusive. On the contrary, when the UAV located at the boundary, there are at least one interval of the five field angles as the maneuverable space. In this way, the UAV can judge its position in the swarm. The characteristics of the UAVs located at the middle and boundary of the swarm are shown in Fig. 4. In this way, we can get the final LTM expression as

$$P(\alpha_{ij}^{B_{lk}}) = \frac{3A(\alpha_{ij}^{B_{lk}})}{5\pi} \cdot \lambda \quad (30)$$

$$= \begin{cases} \frac{3A(\alpha_{ij}^{B_{lk}})}{5\pi} \cdot e^{\delta_M \cdot \|q_{ij}^{\alpha B_{lk}} - q_\nu^o\|} & \text{if } \alpha_{ij}^{B_{lk}} \notin B \\ \frac{3A(\alpha_{ij}^{B_{lk}})}{5\pi} \cdot e^{\delta_B \cdot \|q_{ij}^{\alpha B_{lk}} - q_\nu^o\|} & \text{if } \alpha_{ij}^{B_{lk}} \in B \end{cases}$$

Fig. 5 shows the numerical values of light transmission in a UAV formation, which are indicated by the red text.

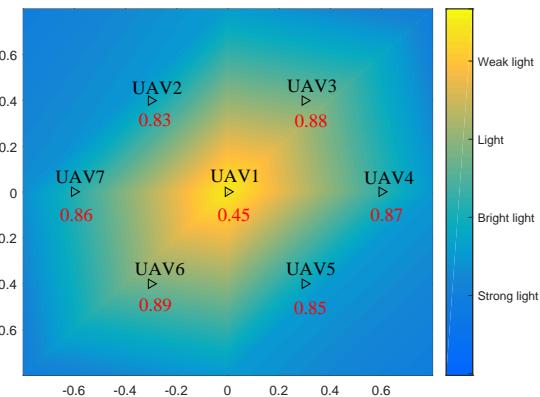


Fig. 5: The light transmission in a UAV formation.

III. IMPROVED POTENTIAL FIELD FUNCTIONS DESIGN

In this section, the improved potential field functions are designed based on the interaction between UAVs, obstacle collision avoidance, and swarm structure maintenance, respectively. To overcome the inherent shortcomings of potential field function with non-reachable and local minima, the LTM is introduced to improve the functions, which can achieve a better control effect in the swarm control.

To begin, considering that the Euclidean norm is not differentiable at zero, the σ -norm is adopted to construct a non-negative smooth potential function between two UAVs [34]. The σ -norm is defined as

$$\|z\|_\sigma = \frac{1}{\epsilon} \left[\sqrt{1 + \epsilon \|z\|^2} - 1 \right] \quad (31)$$

where, the σ -norm of a vector is a map of $\mathbb{R}^n \rightarrow \mathbb{R}^+$; parameters $\epsilon > 0$. And a gradient $\sigma_\epsilon(z)$ can be expressed as

$$\sigma_\epsilon(z) = \nabla \|z\|_\sigma = \frac{z}{\sqrt{1 + \epsilon \|z\|^2}} \quad (32)$$

1 A. Interaction between UAVs

2 The derivable potential field function between UAVs can be
 3 defined as

$$5 \quad \psi^\alpha(\|q_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma) = C_1 \int_{\|d\|_\sigma}^{\|q_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma} \rho_\eta^\alpha\left(\frac{s}{\|r_\alpha\|_\sigma}\right) w_1(s) ds \quad (33)$$

6 where,

$$7 \quad q_{ij,pq}^{\alpha\alpha B_{lk}} = q_{ij}^{\alpha B_{lk}} - q_{pq}^{\alpha B_{lk}} \quad (ij \neq pq) \quad (34)$$

8 and $\|d\|_\sigma$ is the desired distance between two UAVs. The value
 9 of potential field function will tend to the globally unique
 10 minimum, when the two UAVs maintain the desired distance
 11 $\|d\|_\sigma$. The term C_1 can be expressed as

$$12 \quad C_1 = 1 + \tau_1 P(\alpha_{ij}^{B_{lk}}) \quad (35)$$

13 where, τ_1 is the correlation coefficient of LTM; $P(\alpha_{ij}^{B_{lk}})$ is
 14 the LTM. The term $w_1(s)$ can be denoted as

$$15 \quad w_1(s) = \frac{s - \|d\|_\sigma}{\sqrt{1 - (s - \|d\|_\sigma)^2}} \quad (36)$$

16 The collision function $\rho_\eta^\alpha(s)$ between UAVs can be written as

$$17 \quad \rho_\eta^\alpha(s) = \begin{cases} 1 & , s \in [0, \eta] \\ \frac{1}{2}[1 + \cos(\pi \frac{s-\eta}{1-\eta})] & , s \in [\eta, 1] \\ 0 & , \text{otherwise} \end{cases} \quad (37)$$

18 where $\eta \in (0, 1)$.

19 The potential field function designed in Eq. (33) satisfies
 20 the following conditions:

21 (1) When $\|q_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma \geq \|r_\alpha\|_\sigma$, this potential field function
 22 remains a constant;

23 (2) When $\|q_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma < \|d\|_\sigma$, UAV $\alpha_{ij}^{B_{lk}}$ excludes UAV
 24 $\alpha_{pq}^{B_{lk}}$ in the same group;

25 (3) When $\|d\|_\sigma < \|q_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma < \|r_\alpha\|_\sigma$, UAV $\alpha_{ij}^{B_{lk}}$ attracts
 26 UAV $\alpha_{pq}^{B_{lk}}$.

38 B. Obstacle collision avoidance

39 The derivable potential field function between an UAV and
 40 an obstacle can be defined as

$$41 \quad \psi^o(\|q_{ij,\nu}^{\alpha\alpha B_{lk}}\|_\sigma) = C_2 \int_{\|r_o\|_\sigma}^{\|q_{ij,\nu}^{\alpha\alpha B_{lk}}\|_\sigma} \rho^o\left(\frac{s}{\|r_o\|_\sigma}\right) w_2(s) ds \quad (38)$$

42 where,

$$43 \quad q_{ij,t}^{\alpha\alpha B_{lk}} = q_{ij}^{\alpha B_{lk}} - q_o^o \quad (39)$$

44 The value of this potential field function will tend to 0, when
 45 the obstacle is outside the repulsion radius $\|r_o\|_\sigma$. The term
 46 C_2 can be expressed as

$$47 \quad C_2 = 1 + \frac{\tau_2}{P(\alpha_{ij}^{B_{lk}})} \quad (40)$$

48 where τ_2 is the correlation coefficient of LTM. The term $w_2(s)$
 49 can be denoted as

$$50 \quad w_2(s) = \frac{s - \|r_o\|_\sigma}{s^3} \quad (41)$$

51 The collision function of the obstacle to the UAV can be
 52 written as

$$53 \quad \rho^o(s) = \begin{cases} 1 & , s \in [0, 1] \\ 0 & , \text{otherwise} \end{cases} \quad (42)$$

54 When $\|q_{ij,\nu}^{\alpha\alpha B_{lk}}\|_\sigma \rightarrow 0$, this potential field function tends to
 55 ∞ , where $\|q_{ij,\nu}^{\alpha\alpha B_{lk}}\|_\sigma \in (0, \|r_o\|_\sigma]$. That is to say, UAV $\alpha_{ij}^{B_{lk}}$
 56 stay away from the obstacle under the action of repulsive force
 57 when it approaches obstacles.

58 C. Swarm structure maintenance

59 The derivable potential field function between an UAV and
 60 a reference point of swarm structure can be defined as

$$61 \quad \psi^r(\|q_{ij}^{\alpha\alpha B_{lk}}\|_\sigma) = \int_0^{\|q_{ij}^{\alpha\alpha B_{lk}}\|_\sigma} s ds \quad (43)$$

62 where,

$$63 \quad \hat{q}_{ij}^{\alpha\alpha B_{lk}} = q_{ij}^{\alpha B_{lk}} - q^r - \xi_{ij}^{\alpha B_{lk}} \Delta q_{ij}^{\alpha B_{lk}} \quad (44)$$

64 When $\|q_{ij}^{\alpha\alpha B_{lk}} - q^r - \xi_{ij}^{\alpha B_{lk}} \Delta q_{ij}^{\alpha B_{lk}}\|_\sigma \rightarrow 0$, this potential
 65 field function tends to 0.

IV. SWARM ALGORITHMS

A. Preparation

66 As mentioned above, a architecture of group control is
 67 proposed in this paper. Fig. 6 shows the control structure of
 68 the swarm. For a clearer presentation, combine Eq. (34), Eq.
 69 (39), and Eq. (44), we can define the following

$$70 \quad \begin{cases} \hat{q}_{ij,pq}^{\alpha\alpha B_{lk}} = q_{ij,pq}^{\alpha\alpha B_{lk}} \\ \hat{q}_{ij,\nu}^{\alpha\alpha B_{lk}} = q_{ij,\nu}^{\alpha\alpha B_{lk}} \end{cases} \quad (i = 1, \dots, n_l; j = 1, \dots, m_k) \quad (45)$$

71 It follows that the second-order integral system model of UAV
 72 $\alpha_{ij}^{B_{lk}}$ in Eq. (20) can be rewritten as

$$73 \quad \begin{cases} \dot{\hat{q}}_{ij}^{\alpha\alpha B_{lk}} = \hat{p}_{ij}^{\alpha\alpha B_{lk}} \\ \dot{\hat{p}}_{ij}^{\alpha\alpha B_{lk}} = \hat{u}_{ij}^{\alpha\alpha B_{lk}} \end{cases} \quad (i = 1, \dots, n_l; j = 1, \dots, m_k) \quad (46)$$

74 where

$$75 \quad \hat{p}_{ij}^{\alpha\alpha B_{lk}} = p_{ij}^{\alpha B_{lk}} - p^r \quad (47)$$

76 **Lemma 1:** [34] $G = (\mathbf{v}, \varepsilon)$ is an undirected graph of order
 77 n with a nonnegative adjacency matrix $\mathbf{A} = \mathbf{A}^T$. Then, \mathbf{L}
 78 is a positive semidefinite matrix that satisfies the following
 79 sum-of-squares (SOS) property

$$80 \quad (\hat{\mathbf{p}}^\alpha)^T (\mathbf{L}(t) \otimes I_n) (\hat{\mathbf{p}}^\alpha) = \frac{1}{2} \sum_{\Gamma} a_{ij,pq}(t) (\hat{p}_{ij,pq}^{\alpha\alpha B_{lk}})^T \hat{p}_{ij,pq}^{\alpha\alpha B_{lk}} \quad (48)$$

81 where $\Gamma = \{(\alpha_{ij}^{B_{lk}}, \alpha_{pq}^{B_{lk}}) \in \varepsilon(t)\}$.

82 **Lemma 2:** [35](LaSalle's principle) Let $L: \mathbb{R}^n \rightarrow \mathbb{R}$ be a
 83 locally positive definite function such that on the compact set
 84 $\Omega_c = \{x \in \mathbb{R}^n : L(x) \leq C\}$ (C is a constant greater than 0).
 85 If $\dot{L}(x) \leq 0$ is satisfied, then where

$$86 \quad S = \{x \in \Omega_c : \dot{L}(x) = 0\} \quad (49)$$

87 As $t \rightarrow \infty$, the trajectory tends to the largest invariant set
 88 inside S . In particular, if S contains no invariant sets other
 89 than $x = 0$, then 0 is an only equilibrium point.

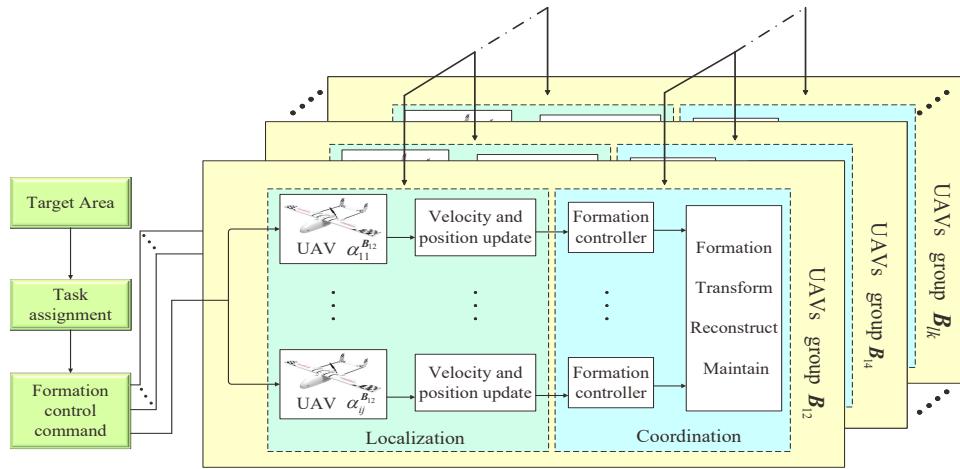


Fig. 6: Diagram for control system.

B. Control algorithm of swarm

To achieve collaborative autonomy in the swarm, the swarm control algorithm for each UAV is designed, in which the swarm structure maintenance and speed consistency control are considered.

$$\begin{aligned} \hat{u}_{ij}^{\alpha B_{lk}} = & - \sum_{N_{pq}^{\alpha B_{lk}}(t)} \nabla(\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}}) \psi^\alpha(\|\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma) \\ & + \sum_{N_{pq}^{\alpha B_{lk}}(t)} a_{ij,pq}(t) \hat{p}_{ij,pq}^{\alpha\alpha B_{lk}} \\ & - \sum_{N_{ij}^o(t)} \nabla(\hat{q}_{ij,\nu}^{\alpha o B_{lk}}) \psi^o(\|\hat{q}_{ij,\nu}^{\alpha o B_{lk}}\|_\sigma) \\ & - \nabla(\hat{q}_{ij}^{\alpha B_{lk}}) \psi^r(\|\hat{q}_{ij}^{\alpha B_{lk}}\|_\sigma) - c_4 \hat{p}_{ij}^{\alpha B_{lk}} \end{aligned} \quad (50)$$

C. Stability analysis of swarm

Theorem 1: Consider a UAV swarm consisting of $a \times b$ groups like B_{lk} in Eq. (5) including $n_l \times m_k$ UAVs whose model can be described as Eq. (46). Then, under the swarm control algorithm in Eq. (50), the following statements hold.

(1) (*Convergence*) The energy of the swarm system is non-increasing, which means the control algorithm proposed in Eq. (50) is convergent. And the total energy of the swarm system is not exceed the initial energy L_0 ($L_0 = C$).

(2) (*Consistency*) The speeds of the UAVs in each group tends the same value p^r , when the swarm system tends to be stable.

(3) (*No collision*) There is no collision between any two UAVs, and between any UAV and any obstacle.

Proof:

1) *Proof of Convergence:* The energy function of the swarm system can be defined as

$$\begin{aligned} L(\hat{q}^\alpha, \hat{p}^\alpha) = & \sum_{l=1}^a \sum_{k=1}^b \left\{ \sum_{i=1}^{n_l} \sum_{j=1}^{m_k} \left[\sum_{p=1}^{n_l} \sum_{q=1}^{m_k} \frac{1}{2C_1} \psi^\alpha(\|\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma) \right. \right. \\ & \left. \left. + \sum_{t=1}^{N_o} \frac{1}{C_2} \psi^o(\|\hat{q}_{ij,\nu}^{\alpha o B_{lk}}\|_\sigma) \right] \right\} \end{aligned} \quad (51)$$

$$\begin{aligned} & + \sum_{i=1}^{n_l} \sum_{j=1}^{m_k} [\psi^r(\|\hat{q}_{ij}^{\alpha B_{lk}}\|_\sigma) \\ & + \frac{1}{2} (\hat{p}_{ij}^{\alpha B_{lk}})^T \hat{p}_{ij}^{\alpha B_{lk}}] \} \end{aligned}$$

According to the symmetry of the potential field function ψ^α designed in Eq. (33) between the two UAVs, we can get

$$\begin{aligned} \frac{\partial \psi^\alpha(\|\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma)}{\partial(\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}})} & = \frac{\partial \psi^\alpha(\|\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma)}{\partial(\hat{q}_{ij}^{\alpha B_{lk}})} \\ & = \frac{\partial \psi^\alpha(\|\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma)}{\partial(\hat{q}_{pq}^{\alpha B_{lk}})} \end{aligned} \quad (52)$$

It follows from Eq. (52) that the time derivative of the potential field function ψ^α can be calculated as

$$\begin{aligned} \frac{d\psi^\alpha(\|\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma)}{dt} = & 2C_1 (\hat{p}_{ij}^{\alpha B_{lk}})^T \nabla(\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}}) \\ & \cdot \psi^\alpha(\|\hat{q}_{ij,pq}^{\alpha\alpha B_{lk}}\|_\sigma) \end{aligned} \quad (53)$$

Similarly, the time derivative of the potential field function ψ^o in Eq. (38), and ψ^r in Eq. (43) can be calculated as

$$\begin{aligned} \frac{d\psi^o(\|\hat{q}_{ij,\nu}^{\alpha o B_{lk}}\|_\sigma)}{dt} = & C_2 \frac{d(\hat{q}_{ij}^{\alpha B_{lk}} - q_t^o)}{dt} \nabla(\hat{q}_{ij,\nu}^{\alpha o B_{lk}}) \\ & \cdot \psi^o(\|\hat{q}_{ij,\nu}^{\alpha o B_{lk}}\|_\sigma) \\ = & C_2 (\hat{p}_{ij}^{\alpha B_{lk}})^T \nabla(\hat{q}_{ij,\nu}^{\alpha o B_{lk}}) \\ & \cdot \psi^o(\|\hat{q}_{ij,\nu}^{\alpha o B_{lk}}\|_\sigma) \end{aligned} \quad (54)$$

$$\begin{aligned} \frac{d\psi^r(\|\hat{q}_{ij}^{\alpha B_{lk}}\|_\sigma)}{dt} = & \frac{d(\hat{q}_{ij}^{\alpha B_{lk}})^T}{dt} \nabla(\hat{q}_{ij}^{\alpha B_{lk}}) \psi^r(\|\hat{q}_{ij}^{\alpha B_{lk}}\|_\sigma) \\ = & (\hat{p}_{ij}^{\alpha B_{lk}})^T \nabla(\hat{q}_{ij}^{\alpha B_{lk}}) \psi^r(\|\hat{q}_{ij}^{\alpha B_{lk}}\|_\sigma) \end{aligned} \quad (55)$$

If we plug Eq. (53), Eq. (54), and Eq. (55) back into the first order time derivative of Eq. (51), the time derivative of the swarm energy function can be obtained as follows

$$\begin{aligned}
\dot{L}(\hat{\mathbf{q}}^\alpha, \hat{\mathbf{p}}^\alpha) = & \sum_{l=1}^a \sum_{k=1}^b \left\{ \sum_{i=1}^{n_l} \sum_{j=1}^{m_k} \sum_{N_{pq}^{\alpha B_{lk}}(t)} (\hat{\mathbf{p}}_{ij}^{\alpha B_{lk}})^T \right. \\
& \cdot \nabla(\hat{\mathbf{q}}_{ij,pq}^{\alpha \alpha B_{lk}}) \psi^\alpha \left(\left\| \hat{\mathbf{q}}_{ij,pq}^{\alpha \alpha B_{lk}} \right\|_\sigma \right) + \sum_{i=1}^{n_l} \sum_{j=1}^{m_k} \sum_{N_{ij}^o(t)} \\
& \cdot (\hat{\mathbf{p}}_{ij}^{\alpha B_{lk}})^T \nabla(\hat{\mathbf{q}}_{ij,\nu}^{\alpha o B_{lk}}) \psi^o \left(\left\| \hat{\mathbf{q}}_{ij,\nu}^{\alpha o B_{lk}} \right\|_\sigma \right) \\
& + \sum_{i=1}^{n_l} \sum_{j=1}^{m_k} [(\hat{\mathbf{p}}_{ij}^{\alpha B_{lk}})^T \nabla(\hat{\mathbf{q}}_{ij}^{\alpha B_{lk}}) \\
& \cdot \psi^r \left(\left\| \hat{\mathbf{q}}_{ij}^{\alpha B_{lk}} \right\|_\sigma \right) + (\hat{\mathbf{p}}_{ij}^{\alpha B_{lk}})^T \dot{\hat{\mathbf{p}}}_{ij}^{\alpha B_{lk}}] \}
\end{aligned} \tag{56}$$

Substituting Eq. (50) into Eq. (56), we have

$$\begin{aligned}
\dot{L}(\hat{\mathbf{q}}^\alpha, \hat{\mathbf{p}}^\alpha) = & \sum_{l=1}^a \sum_{k=1}^b \left\{ - \sum_{i=1}^{n_l} \sum_{j=1}^{m_k} (\hat{\mathbf{p}}_{ij}^{\alpha B_{lk}})^T \left[\sum_{N_{pq}^{\alpha B_{lk}}(t)} a_{ij,pq}(t) \right. \right. \\
& \cdot \hat{\mathbf{p}}_{ij,pq}^{\alpha \alpha B_{lk}} + c_4 \hat{\mathbf{p}}_{ij}^{\alpha B_{lk}}] \}
\end{aligned} \tag{57}$$

Using Lemma 1, Eq. (57) can be rewritten as

$$\dot{L}(\hat{\mathbf{q}}^\alpha, \hat{\mathbf{p}}^\alpha) = -(\hat{\mathbf{p}}^\alpha)^T [(\mathbf{L}(t) + c_4 I'_n) \otimes I_n] (\hat{\mathbf{p}}^\alpha) \tag{58}$$

where, I'_n and I_n are identity matrices, \otimes is the Kronecker product.

Considering \mathbf{L} to be a positive semidefinite matrix, thus, we have

$$-(\hat{\mathbf{p}}^\alpha)^T [(\mathbf{L}(t) + c_4 I'_n) \otimes I_n] (\hat{\mathbf{p}}^\alpha) \leq 0 \tag{59}$$

It follows from Eq. (59), we have

$$\dot{L}(\hat{\mathbf{q}}^\alpha, \hat{\mathbf{p}}^\alpha) \leq 0 \tag{60}$$

Considering that the energy of swarm system is limited, it is easy to see

$$L(\hat{\mathbf{q}}^\alpha, \hat{\mathbf{p}}^\alpha) \leq L_0 \tag{61}$$

where L_0 is the initial energy of the swarm system, and $L_0 = C$.

Hence the convergence of swarm system in Theorem 1 is proved.

2) *Proof of speed consistency:* We first show that the speeds of UAVs in the same group B_{lk} will eventually converge.

Assuming that graph G is connected, there must be a connected path between the nodes ij and pq in graph G , which length is not exceed $(n_l \times m_k - 1)$. In this way, we can obtain that the distance $\left\| \hat{\mathbf{q}}_{ij,pq}^{\alpha \alpha B_{lk}} \right\|_\sigma$ between UAV $\alpha_{ij}^{B_{lk}}$ and UAV $\alpha_{pq}^{B_{lk}}$ is bounded; the distance $\left\| \hat{\mathbf{q}}_{ij}^{\alpha B_{lk}} \right\|_\sigma$ between UAV $\alpha_{ij}^{B_{lk}}$ and the reference point of swarm structure is bounded, and their relative velocity in time $[t_d, t_{d+1}]$, $(d = 0, 1, \dots)$ is also bounded. It follows that the point set $(\hat{\mathbf{q}}_{ij}^{\alpha B_{lk}}, \hat{\mathbf{p}}_{ij}^{\alpha B_{lk}})$ of UAV $\alpha_{ij}^{B_{lk}}$ is a compact set.

Combining Lemma 2 with Eq. (61), we can get that all UAVs in the same group will converge to the maximum invariant set, when the group system reaches a steady state, namely

$$S = \{(\hat{\mathbf{q}}^\alpha, \hat{\mathbf{p}}^\alpha) : -(\hat{\mathbf{p}}^\alpha)^T [(\mathbf{L}(t) + c_4 I'_n) \otimes I_n] (\hat{\mathbf{p}}^\alpha) = 0\} \tag{62}$$

According to the property of the sum of squares of vectors, we can get

$$\hat{\mathbf{p}}^\alpha = 0 \tag{63}$$

Taking Eq. (47) into consideration, we get the speed of each UAV in one group as follows

$$\hat{\mathbf{p}}_{ij}^{\alpha B_{lk}} = \mathbf{p}^r \tag{64}$$

The proof of speed consistency in one group is completed. Proceeding as in the proof of this, we get the speeds of other groups of UAVs will eventually converge to the value of the maximum invariant set, namely

$$\hat{\mathbf{p}}_{ij}^{\alpha B_{12}} = \hat{\mathbf{p}}_{ij}^{\alpha B_{21}} = \dots = \hat{\mathbf{p}}_{ij}^{\alpha B_{lk}} = \mathbf{p}^r \quad (l = 1, \dots, a; k = 1, \dots, b) \tag{65}$$

This completes the proof of the consistency of swarm system in Theorem 1.

3) *Proof of no collision:* Next, contradiction is used to prove that the UAVs in the swarm system will not collide with obstacles at any time. Suppose at least one UAV $\alpha_{ij}^{B_{lk}}$ colliding with an obstacle at time t_1 ($t_1 > 0$) in the swarm system. Combining the definition of potential field function ψ^o in Eq. (38), we can obtain that the value of potential field ψ^o for UAV $\alpha_{ij}^{B_{lk}}$ tends to the infinite, namely

$$\psi^o \left(\left\| \hat{\mathbf{q}}_{ij,\nu}^{\alpha o B_{lk}} \right\|_\sigma \right) \rightarrow \infty \tag{66}$$

Substituting Eq. (66) to Eq. (51), we can obtain

$$L(\hat{\mathbf{q}}^\alpha, \hat{\mathbf{p}}^\alpha) \rightarrow \infty \tag{67}$$

which leads to a contradiction with Eq. (61).

Therefore, the above assumption does not hold, which implies that there exists no collision for UAV $\alpha_{ij}^{B_{lk}}$ with the obstacles during the flying process. As before, we identify the other group UAVs as obstacles. Hence the proof of no collision between UAVs is similar to the above proof, which is omitted here. Finally, we obtain that under the control law in Eq. (50), the swarm system can achieve obstacle avoidance and collision avoidance control.

This completes the proof of Theorem 1. ■

V. SIMULATION

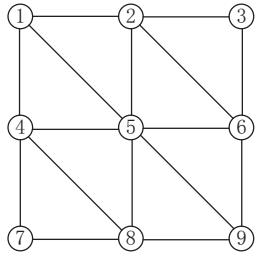
In this section, we provide simulation results to illustrate the effectiveness of the proposed control algorithm and compare it with the conventional control method of collision avoidance designed in the existing literatures.

A. Preliminaries of Simulation

Consider an UAV swarm consisting of 360 fixed wing UAVs, which was divided into 40 groups. The structure of this UAV swarm is described as matrix $\Xi_{n \times m}$ in Eq. (1), where the partitioned matrices B_{lk} consists of 9 UAVs.

1
2 1) *Communication graph:* For the UAVs in partitioned
3 matrices B_{lk} , the following communication topology was
4 designed in Fig. 7, whose Laplacian matrix is
5

$$L = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & -1 & 0 \\ -1 & -1 & 0 & -1 & 6 & -1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 & 3 \end{bmatrix} \quad (68)$$



23 Fig. 7: Structure of communication topology
24

25 2) *Parameters:* For the initial configuration, we set the
26 spacing interval between UAVs in one group to $1.5m$, which
27 is because of the movement restriction about the minimum
28 turning radius of each UAV, namely $dx = dy = 1.5m$. It is
29 desired that the UAV swarm ultimately achieves the fly task
30 from area A to area B of the x - y plane while keeping the
31 swarm structure and without collision during the flight process,
32 where the areas are
33

$$A := \{(x_1, y_1) \in A | 0 \leq x_1 \leq 50, -30 \leq y_1 \leq 20\} \quad (69)$$

$$B := \{(x_2, y_2) \in B | 90 \leq x_2 \leq 140, 90 \leq y_2 \leq 140\} \quad (70)$$

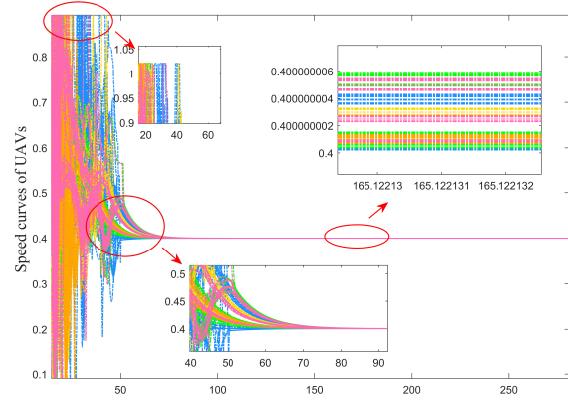
34 The initial positions of the UAVs were randomly generated
35 within a certain position interval. And the flight constraints of
36 the Fixed-wing UAV were designed as follows: the minimum
37 speed $v_{min}^{\alpha B_{lk}}$ and maximum speed $v_{max}^{\alpha B_{lk}}$ of UAV $\alpha_{ij}^{B_{lk}}$ are
38 $0.2m/s$ and $1m/s$, respectively; the minimum turning radius
39 is $0.5m$; and the maximum overload is $10G$. For the weight
40 coefficients of LTM distance factor in Eq. (30), we set $\delta_C = 0.4$
41 and $\delta_B = 0.8$; the correlation coefficient of LTM $c_0 = 0.4$,
42 $c_1 = 0.25$. And some distance settings in the simulation are
43 $\|r_\alpha\|_\sigma = 10m$ and $\|r_o\|_\sigma = 1.2m$, respectively.
44

45 B. Simulation result analysis 46

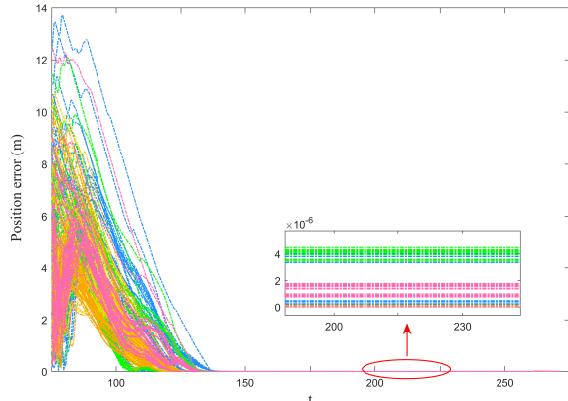
47 1) *Verification simulation:* In this subsection, several nu-
48 matical simulations were conducted to illustrate the rationality
49 of theoretical results and the validity of the control algorithm
50 in UAV swarm.
51

52 (a) Formation action without obstacles

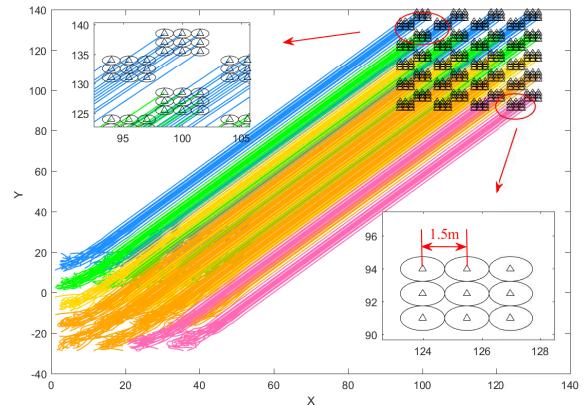
53 At first, the stability and consistency of the flight process
54 for the UAV swarm without the influence of obstacles were
55 verified. Fig. 7(a) is the speed curves of all UAVs in the
56 swarm, which shows the speed of 360 UAVs can eventually
57 converge to the p^r ($p^r = 0.4m/s$) and the maximum speed
58 do not exceed $1m/s$. That is, the swarm system has good
59



(a) Speed curves of UAVs.



(b) Position error between UAVs and reference point.



(c) Flight path of the UAV swarm without obstacles.

Fig. 7: Simulation results of the UAV swarm without obstacles.

performance in speed consistency with the proposed control algorithm. Fig. 7(b) are the position errors between all UAVs and the reference point of the swarm structure, which shows the errors will eventually converge to 0. That is, the swarm can eventually converge to the desired structure, and the UAVs can keep the desired distance during the flight. The flight path of the UAV swarm is shown in Fig. 7(c), which shows the swarm

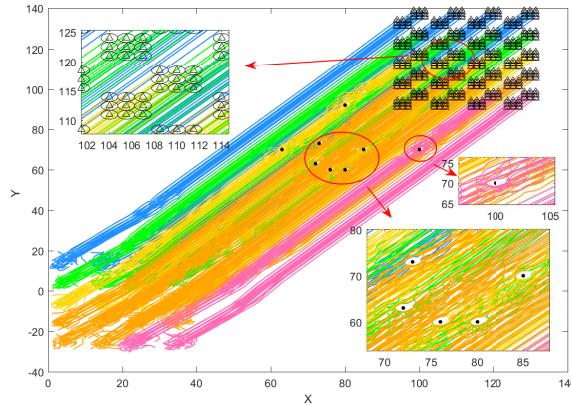


Fig. 8: Flight path of the UAV swarm with obstacles.

can reach the destination in the desired structure without UAV collision.

(b) Formation action with several obstacles

In this part, the obstacle avoidance performance of the UAV swarm were verified by setting some random obstacles, which are also used to verify the reconstruction and maintenance performance of the swarm structure.

Fig. 8 is the flight path of the UAV swarm under the obstacle environment, which shows that the UAV swarm can bypass the obstacles under the action of the LTM and the force of obstacle repulsion, in the meanwhile, there is no collision in the UAV swarm. In addition, only part of the swarm rather the whole are affected by obstacles when meeting the obstacles, where as we have reserved the maneuver space for avoidance collision in advance. The maximum, minimum, and average position errors of the swarm avoidance obstacle process are about 17.3m, 0m, and 1.2m, respectively. In other words, the designed models and swarm control algorithms have a better ability to cope with the collision and avoidance of UAV swarm, especially in large-scale. The partial moving processes of UAVs for obstacle avoidance are shown in Fig. 9.

(c) Break down of several UAVs

To further test the reliable ability of obstacle avoidance and reconstruction capabilities of the swarm, the case that several UAV groups are broken down in the flight process was set.

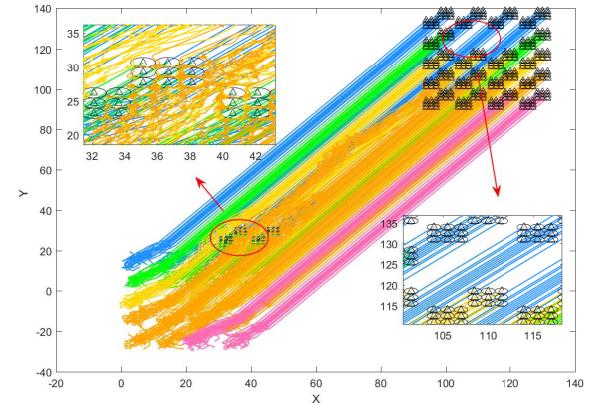


Fig. 10: Flight path of the UAV swarm with some breakdown.

Suppose that four UAV groups in the middle of the swarm are broken down and cannot receive the swarm commands normally at $t=150$, which leads to the groups being detained in the air and go straight east with the minimum speed. In this case, the failed groups will be considered as obstacles for other UAVs. Fig. 10 is the flight path of the UAV swarm with some breakdown, which shows that UAVs without malfunction can bypass the failed UAVs and reach the target area smoothly with the desired formation. The partial moving processes of serval breakdown UAVs are shown in Fig. 11.

As can be seen, there is a good agreement between the present result and the theoretical calculation result, which means the swarm system has good consistency and coordination in formation maneuver and collision avoidance.

2) Simulation for comparison: To verify the superiority of the proposed collision avoidance control method, we compared the flight performance of the UAV swarm under the conventional potential field function and the proposed control architecture in this paper.

Fig. 12 is the result for the conventional method, where the potential field function method in [34] is the most commonly used method in UAV swarm obstacle avoidance. Besides, this method is also the most similar control algorithm to that in this paper, whereas other methods, such as MPC (Model Predictive Control), speed barrier method, and Markov decision method

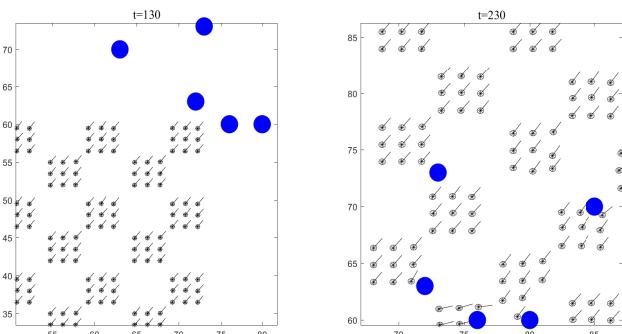
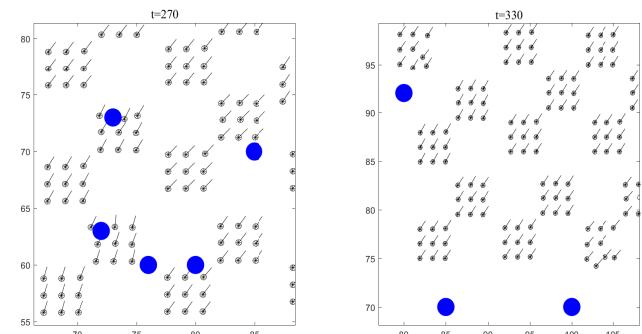


Fig. 9: Partial moving process of UAVs obstacle avoidance.



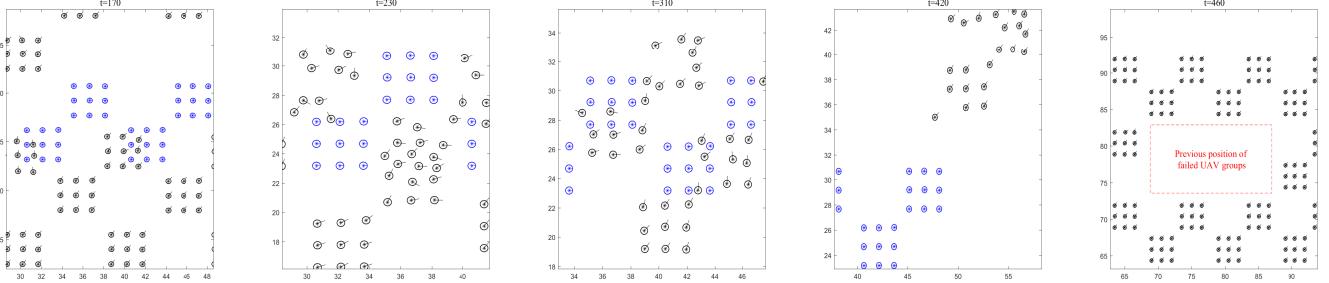


Fig. 11: Partial moving process of several UAVs breakdown.

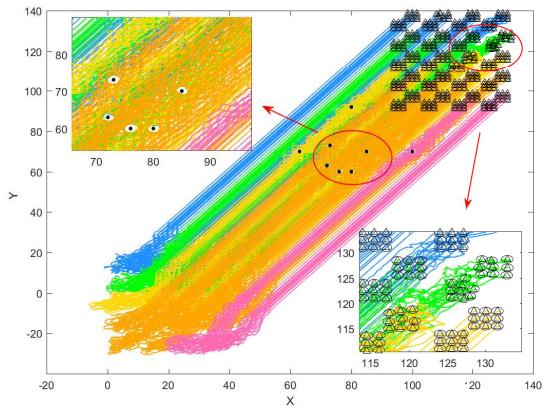


Fig. 12: Flight path of UAV swarm with conventional method.

are all not suitable for large-scale swarm control, which have been introduced in the introduction.

It can be seen from Fig. 12, the flight path of the UAV swarm is bumpy, the trajectory cost is high, and the desired structure cannot be formed completely. During the obstacle avoidance process, the UAVs are bouncing away from the obstacle instead of smoothly avoiding obstacles. That is, the conventional method has extremely high requirements for UAVs overload, which is even not suitable for Fixed-wing UAV obstacle avoidance. From the performance comparison in TABLE I, it can be seen intuitively that the methods we designed have bigger performance improvements compared to the existing method.

TABLE I: Comparison of simulation results.

Performance	Method	This paper	Method in [34]
Collision avoidance	No collision	No collision	
Calculation quantity	Low	High	
Consistency	High	Normal	
Flight path	Smooth	Bumpy	
Stability	High	Low	
Robustness	High	Normal	

VI. CONCLUSION

In this paper, a collision avoidance and formation control algorithm for a large-scale Fixed-wing UAV swarm based

on reserved maneuvering space has been designed, which can guarantee the safety and survivability of UAVs via effectively avoiding obstacles while maintaining the stability of the swarm. Under the distributed control architecture, a new swarm structure has been proposed, which cooperates with the LTM to find a feasible flying route for UAVs in time. Furthermore, the improved potential field functions and the swarm control algorithms for UAVs have been designed. Finally, the simulation results show that the control algorithms have good advantages in collision, obstacle avoidance, system stability maintenance, and rapid formation reconstruction, and also have a better performance than the methods in the existing literature. In future research, we will proceed from the actual flight verification, and the rapid turn of the swarm based on such structure, respectively.

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