Research on two-wheeled balance car based on improved LQR controller

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***Abstract*—The modern control model of the two-wheeled balancing vehicle is established by rational simplification and linearization and selection of appropriate state space variables. The state space expressions in modern control theory are used to make up for some deficiencies in the classical inverted pendulum model. By constructing the mathematical model of the LQR controller in MATLAB, using Simulink for model design and theoretical simulation analysis according to the actual application scenario, the results show that the improved LQR controller can be used in the autonomous balance control and anti-external interference of the two-wheeled self-balancing vehicle model. Has excellent performance.**

***Keywords*—Two-wheeled balance car; LQR controller; Matlab; mathematical modeling.**

I.PREFACE

As a hot application field of current research, mobile robot includes the comprehensive application of multi-disciplinary and multi-technology, and two-wheeled balance vehicle, as a branch of mobile robot, is a hot research topic of current research. The two-wheeled balance car can be abstracted as a first-order inverted pendulum model, which has the characteristics of flexibility, nonlinear instability, large disturbance, many control variables, and difficult control [1]. The discussion of such issues can help understand the control theory and its analysis methodology, and secondly, it can verify the performance of the control algorithm implementation, which can promote the integration and practical implementation of multidisciplinary technologies. In the practical application of the balance car design, due to the application of various control, electronic information and communication technologies, the research on the two-wheeled balance car is of great significance in various fields. At present, this type of mobile robot has a wide range of applications in the fields of security, industry and military [2]. Therefore, further in-depth research on the two-wheeled self-balancing robot can provide new design ideas and control methods for the development of self-balancing vehicles. Further expand the application field of two-wheeled self-balancing vehicles. The main content of this paper is to study the two-wheeled self-balancing vehicle based on the improved LQR controller. The main work includes using modern control theory to complete the establishment of the mathematical model of the trolley, realizing the design and improvement of the trolley controller algorithm, and using Simulink for simulation testing.

II. SYSTEM MODELING

Because this design is a system with strong coupling, dynamic variables, and nonlinearity, certain theoretical analysis and mathematical modeling analysis are required before performing specific operations on this system. In order to obtain a model that is both simplified and in line with the laws of motion control, it is necessary to idealize the PWM-encoded speed-regulating motor and the actual mechanical parts. The conventional treatment is to ignore the factors such as friction and relative elasticity inside the mechanical module, and the purpose of the controller design based on this design is to establish an ideal state space expression.

A.System stress analysis

1) Analysis of Wheel Movement State

The physical structure of the two-wheeled self-balancing vehicle is two coded motors with speed feedback as the power system. The installation method is to ensure that the center of gravity of the overall system is coaxially installed at the geometric center of the motor connection [6]. The motor parameters (mass, radius, deceleration, etc.) ratio and moment of inertia) are the same, and the schematic diagram is shown in Figure.1.

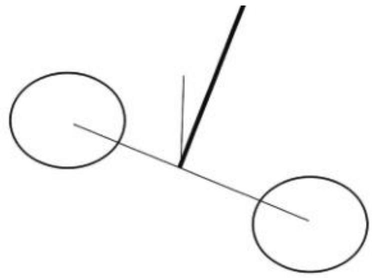

Figure .1. System structure diagram

First, the force analysis of the wheel is carried out, as shown in Figure.2.

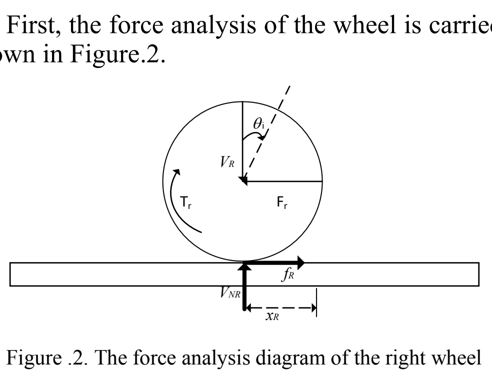

Figure .2. The force analysis diagram of the right wheel

The motion of the car can be decomposed into translation and rotation, then according to Newton's second law of motion, we can get:

$$m\ddot{x}_R = f_R - F_r \tag{1}$$

From the law of rigid body fixed axis rotation, we can get:

$$I\dot{\omega}_R = T_R - rf_R \tag{2}$$

Combine the above analysis results, eliminate , and get:

$$m\ddot{x}_R = \frac{T_R}{r} - \frac{I\dot{\omega}_R}{r} - F_r \tag{3}$$

Under ideal conditions, it can be obtained that the translational speed of the wheel is proportional to the rotational speed, as follows:

$$\begin{cases} \omega_R = \dfrac{\dot{x}_R}{r} \\ \dot{\omega}_R = \dfrac{\ddot{x}_R}{r} \end{cases} \tag{4}$$

After finishing the calculation process, the balance equation of the left and right wheels can be obtained as follows:

$$\left(m + \frac{I}{r^2}\right)\ddot{x}_R = \frac{T_R}{r} - F_r \tag{5}$$

In the same way, the force analysis of the left and right wheels is similar and will not be repeated here. The above symbols are explained in Table 1 below:

TABLE 1 DESCRIPTION OF SOME PARAMETERS OF THE SYSTEM MODEL

m	the quality of the wheels（kg）
r	the radius of the wheel（m）
F_r	The horizontal component of the external force on the wheel（N）
T_R	Right motor output torque（N*m）
I	moment of inertia of the wheel
x_R	Translational displacement of the right wheel（m）
f_R	The amount of friction on the right wheel（N）
ω_R	Right wheel angular velocity（rad/s）
V_R	The vertical pressure of the controller to the right wheel（N）

2) Analysis of vehicle body motion state

The motion state of the wheel is obtained by analyzing the force of the wheel. Now it is necessary to analyze the motion state of the vehicle body[4]. According to the motion requirements of the two-wheeled self-balancing vehicle, it can be known that the vehicle body can be divided into forward motion and steering motion during motion [3]. Before the analysis, the model of the car body is simplified, and the car is abstracted as a first-order inverted pendulum system, as shown in Figure.3.

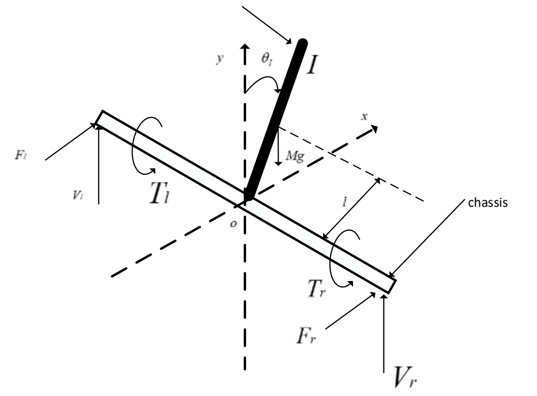

Figure .3. Force analysis of the forward motion of the trolley

The forward translation of the car body can be further decomposed into forward motion and relative rotation with the center of mass of the car body as the midpoint. First, some preconditions are analyzed:

a) Horizontal displacement of the geometric center of the trolley chassis:

$$x_o = 0.5\left(x_L + x_R\right) \tag{6}$$

b)Simultaneous wheel analysis and trolley displacement results show the relationship between vehicle body acceleration and torque force[7]:

$$\left(m + \frac{l}{r^2}\right)\ddot{x} = \frac{T_l + T_r}{2r} - \frac{F_l + F_r}{2} \tag{7}$$

Therefore, for the car body, the balance equations in the horizontal and vertical directions can be obtained according to Newton's second law[5].

Horizontal direction:

$$M\frac{d\left(x + l\sin\left(\theta_l\right)\right)}{\left(dt\right)^2} = F_l + F_r \tag{8}$$

Vertically:

$$M\frac{d\left(l\cos\left(\theta_l\right)\right)}{\left(dt\right)^2} = V_l + V_r - Mg \tag{9}$$

Using the law of rigid body fixed axis rotation, the following results can be obtained:

$$J_I\ddot{\theta}_I=(V_l+V_r)l\sin(\theta_I)-(F_l+F_r)l\cos(\theta_I)-(T_l+T_r) \tag{10}$$

Combining the above formula with the intermediate results and the wheel motion analysis results, it can be concluded that:

$$\left(M+2m+\frac{2I}{r^2}\right)\ddot{x}-\frac{T_l+T_r}{r}+Ml\ddot{\theta}_I\cos(\theta_I)-Ml\dot{\theta}_I^2\sin(\theta_I)=0 \tag{11}$$

It can be seen from the above formula that the system has nonlinear characteristics and the control variables are more complicated[10]. In the research process, the system conditions can be linearized, and the specific processing is as follows:

$$\begin{cases}\cos(\theta_I)=1\\ \sin(\theta_I)=\theta_I\\ \dot{\theta}_I^2=0\end{cases} \tag{12}$$

The simplified result is:

$$\ddot{x}=\frac{T_l+T_r}{\left(M+2m+\frac{2l}{r^2}\right)r}-\frac{Ml}{\left(M+2m+\frac{2l}{r^2}\right)}\ddot{\theta}_I \tag{13}$$

The $\ddot{x}$ and $\ddot{\theta}_I$ in Equation 13 are then separated in two steps.

First, bring the balance motion equations in the horizontal and vertical directions into the vehicle body rotation law equation, we can get:

$$\left(\frac{J_I}{Ml}+l\right)\ddot{\theta}_I+\ddot{x}\cos(\theta_I)-g\sin(\theta_I)+\frac{T_l+T_r}{Ml}=0 \tag{14}$$

Then perform a similar linearization process on Equation 15, we can get:

$$\ddot{\theta}_I=\frac{M\lg}{(J_I+Ml^2)}\theta_I-\frac{Ml}{(J_I+Ml^2)}-\frac{T_l+T_r}{(J_I+Ml^2)} \tag{15}$$

Finally, through the above two intermediate results and the linearization processing results, and are eliminated respectively, the following motion analysis of positive translation can be obtained:

$$\begin{cases}\ddot{x}=\dfrac{J_I+Ml^2+Mlr}{Q_{eq}r}(T_l+T_r)-\dfrac{M^2l^2g}{Q_{eq}}\theta_I\\ \ddot{\theta}_I=\dfrac{M\lg\left(M+2m+\frac{2l}{r^2}\right)}{Q_{eq}}\theta_I-\dfrac{\left(\frac{Ml}{r}+M+2m+\frac{2l}{r^2}\right)}{Q_{eq}}(T_l+T_r)\end{cases}$$

Notes: $Q_{eq}=J_IM+(J_I+Ml^2)\left(2m+\frac{2l}{r^2}\right)$ (16)

Next, the motion equation of the system during the steering motion is obtained. At this time, a concept of yaw angle needs to be introduced[8]. that is, the angle between the projection of the vehicle body steering in the horizontal direction and the horizontal direction of the inertial navigation system.

The steering motion is generated because the driving forces of the left and right motors in the horizontal direction are inconsistent, but it is still a motion model in which a rigid body rotates around a fixed axis. Here, the right rotation motion is used as an example to obtain:

$$J_\alpha\ddot{\alpha}=\frac{d}{2}(F_l-F_r) \tag{17}$$

In the dynamic process of the car turning, the driving force of the left and right wheels is different. When turning right, the equations of motion of the two wheels can be subtracted to obtain:

$$\left(m+\frac{I}{r^2}\right)(\ddot{x}_L-\ddot{x}_R)=\frac{T_l-T_r}{r}-(F_l-F_r) \tag{18}$$

When the speeds of the left and right wheels are not equal, the car turns in the direction of low speed, and according to the steering dynamic analysis in Figure.4, it can be obtained:

$$\begin{cases}\dot{x}_L=\dot{\alpha}r_L\\ \dot{x}_R=\dot{\alpha}r_R\\ r_L=r_R+d\end{cases} \tag{19}$$

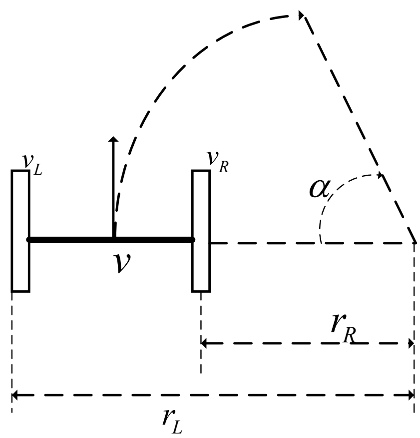

Figure .4. Dynamic analysis diagram of the car turning right

The description of the dynamic motion process of the car takes the angular velocity and angular acceleration during steering as the key variables, we can get:

$$\frac{(\ddot{x}_L-\ddot{x}_R)}{d}=\ddot{\alpha} \tag{20}$$

Bring in the car parameters to get:

$$\frac{\left(T_l - T_r\right)}{r\left(md + \frac{Id}{r^2} + \frac{2J_\alpha}{d}\right)} = \ddot{\alpha} \tag{21}$$

Select the displacement of the car, the forward motion speed, the inclination angle of the car body, the angular velocity of the car body, the steering angle of the car body and the steering angular velocity as the state variables of the mathematical model, we can get:

$$\left(x \ \dot{x} \ \theta_I \ \dot{\theta}_I \ \alpha \ \dot{\alpha}\right)^T \tag{22}$$

According to the calculation results of the steering motion and forward motion of the car, the state space expression of the system can be obtained:

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta}_I \\ \ddot{\theta}_I \\ \dot{\alpha} \\ \ddot{\alpha} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \\ \theta_I \\ \dot{\theta}_I \\ \alpha \\ \dot{\alpha} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ B1 & B2 \\ 0 & 0 \\ B3 & B4 \\ 0 & 0 \\ B5 & B6 \end{pmatrix} \begin{pmatrix} \frac{rT_l}{I} \\ \frac{rT_r}{I} \end{pmatrix} \\ y = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \\ \theta_I \\ \dot{\theta}_I \\ \alpha \\ \dot{\alpha} \end{pmatrix} \end{cases} \tag{23}$$

The specific parameters in the matrix are as follows:

$$\begin{cases} A1 = -\frac{M^2 l^2 g}{Q_{eq}} \\ A2 = \frac{Mlg\left(M + 2m + \frac{2I}{r^2}\right)}{Q_{eq}} \\ B1 = B2 = \frac{\left(J_I + Ml^2 + Mlr\right)I}{Q_{eq} r^2} \\ B3 = B4 = -\frac{I\left(\frac{Ml}{r} + M + 2m + \frac{2I}{r^2}\right)}{Q_{eq} r} \\ B5 = -B6 = \frac{I}{r^2\left(md + \frac{Id}{r^2} + \frac{2J_\alpha}{d}\right)} \end{cases} \tag{24}$$

B. LQR algorithm implementation

No matter how the controller is designed, the final actuator that achieves the effect is the DC motor. Therefore, the implementation of the LQR algorithm needs to combine the controller and the voltage control principle of the DC motor to achieve two-wheel autonomous upright balance and steering. action[11]. In terms of DC motor drive, it is mainly realized by changing the duty cycle of the input PWM wave and the rotation polarity of the motor.

According to the established vehicle body state space expression in Section II, when feedback control needs to be implemented for a system in modern control theory, a closed-loop control system as shown in Figure.5 can generally be obtained by setting the feedback controller K.

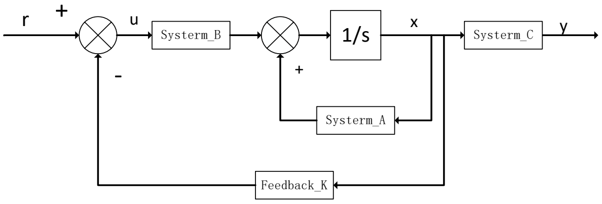

Figure .5. State closed-loop feedback control system

At this point, a stable system can be obtained:

$$\dot{x} = \left(A - BK\right)x = A_c x \tag{25}$$

At this time, the LQR linear quadratic regulator begins to play its role, and the optimal control law of linear feedback can be obtained through LQR and form closed-loop control. A cost function needs to be introduced in the LQR controller design [14]:

$$J = 0.5\left(\int_0^\infty x^T Q x + u^T R u dt\right) \tag{26}$$

When designing a controller, the main consideration is that the system reaches a steady state as quickly as possible. with minimal deviation. At the same time, the amount of control in the controller should be small, so as to avoid the introduction of the controller to have an additional impact on the system, that is, the minimum cost should be used to achieve the control purpose [9]. The design of the LQR cost function can achieve the above goals. Now the remaining problem is how to solve the K matrix. The solving steps are as follows:

1) Combining the state feedback matrix with the cost function, we can get:

$$J = 0.5\left(\int_0^\infty x^T \left(Q + K^T R K\right) x dt\right) \tag{27}$$

2) To find K, make reasonable assumptions about a constant matrix P:

$$\frac{d}{dt}\left(x^T P x\right) = -x^T \left(Q + K^T R K\right) x \tag{28}$$

3) Therefore, the cost function can be expressed as follows:

$$J = -0.5\left(\int_0^\infty \left(x^T P x\right)\right) = 0.5 x^T\left(0\right) P x\left(0\right) \tag{29}$$

4) According to the controller design, the system is stable at this time, so the joint stable system

expression can be obtained:

$$x^T\left(A_C{}^T P + PA_C + Q + K^T RK\right)x = 0 \qquad (30)$$

By the above equations simultaneously, we can get:

$$K = R^{-1}B^T P \qquad (31)$$

Finally, according to the actual parameters of the motor, the final result can be obtained by calculating the state feedback K through MATLAB.

III. MATLAB SIMULATION OF LQR ALGORITHM

Through the previous calculation and analysis, the design of the controller has been completed, and then the simulation research of the control system is carried out by using the tool Simulink in MATLAB. First, the Simulink simulation model needs to be established according to the system state space expression model as shown in Figure.6. At this time, the initial conditions of the system are that the velocity, angular velocity, and yaw angle are all zero[12]. The main test contents of the simulation include the system starting to work to maintain the balance, the disturbance simulation of the system equilibrium state, the dynamic simulation of the steering situation, and the speed tracking simulation.

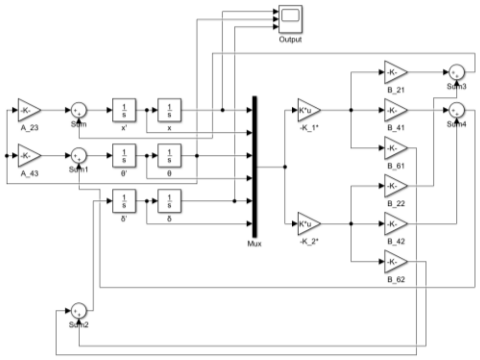

Figure .6. Simulink simulation diagram

1) Simulation of static autonomous balance state

The initial parameters of the system are set to all 0, which means a static balance without interference balance scene. The following results can be obtained in Figure .7.

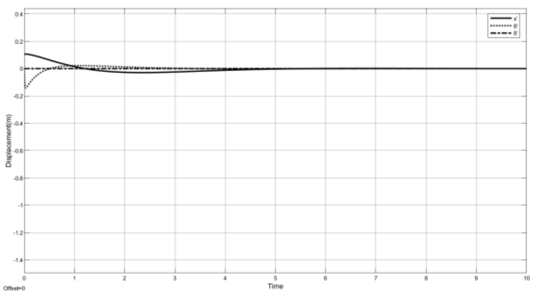

Figure .7. LQR autonomous balance simulation test results

Analysis of the results shows that the speed of the system has tended to 0 after 1s, and the system can be considered to be in a state of equilibrium at this time. However, generally in practical applications[15], due to the influence of different hardware and different gravitational acceleration environments, the system will generate a deviation of the mechanical zero angle, which will cause a certain delay in reaching stability.

2) The system equilibrium state is disturbed simulation

First, set the initial parameters of the system to all 0. After reaching the equilibrium state, input a step signal θ from the outside to simulate the scene where the system is unbalanced by an external force. The test results are shown in Figure.8.

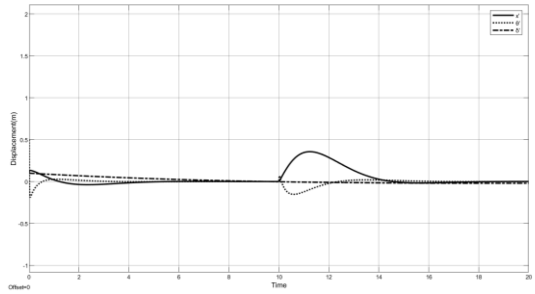

Fig .8. LQR balance disturbance simulation results

As can be seen from the above figure, after an external force is applied to break the equilibrium state, the change of the system displacement is relatively large, but the system can return to the autonomous equilibrium state in a short time[13], and the overshoot of the system is also within the allowable range. It can be seen that The LQR controller can meet the design requirements under this condition.

3) Steering dynamic simulation

The control of the steering motion is also an important indicator to measure the effect of the controller. The simulation of this part is mainly by giving an initial value of the yaw angle of the system, and by measuring the time for the system to return to the autonomous balance state, the result is shown in Figure.9.

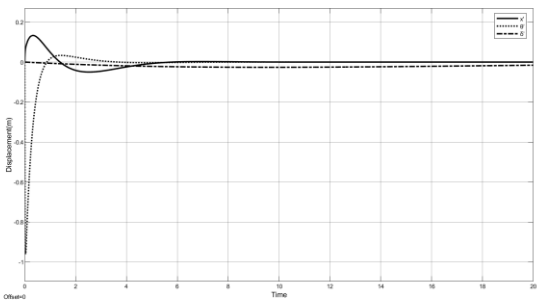

Figure. 9. LQR steering simulation test results

It can be seen from the simulation results that by observing the change of the yaw angular velocity, it can be seen that the system can quickly provide the steering angular velocity to meet the demand when the yaw angle has an initial value, and reach an autonomous balance state in about 2s. It can be seen from the above that the designed LQR controller can meet the design requirements in the overall function, and the controller has good performance in the adjustment range and system response time.

4) Speed tracking simulation

Finally, set all state variables to 0, use the constant module as the initial speed setting, and observe the change of each variable from 0 to the predetermined speed. The results are shown in Figure.10.

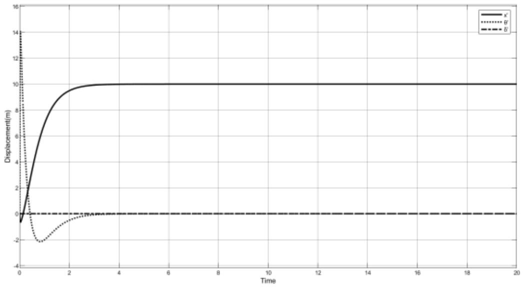

Figure .10. LQR speed tracking simulation test results

According to the simulation results, the theoretical situation of the system can accelerate from 0 to the preset speed in about 2s, and at the same time reach a stable state, and there is no obvious error after reaching the stable state. It can be seen that the controller still has fast and stable characteristics for speed adjustment.

IV. SUMMARY AND OUTLOOK

In this paper, a two-wheel self-balancing vehicle model with complex structure and poor stability is studied through analysis, the mechanical model of the two-wheel self-balancing vehicle is analyzed, and the specific realization principle of LQR controller in balance control is determined. At the same time, in the system In the process of model establishment, the state space expression in modern control theory is used to make up for some deficiencies in the classical inverted pendulum model, such as poor control accuracy, system integrity, and single control strategy. Finally, the theoretical simulation analysis of the system is carried out using MATLAB. The results show that the improved LQR controller in this paper has excellent performance in the autonomous balance control and anti-external interference of the two-wheeled self-balancing vehicle model.

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