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(see week 2)

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But if x is image, why not $\mu(t) = \mathrm{CNN}_1(t)$

$$\Sigma(t) = \mathrm{CNN}_2(t)$$

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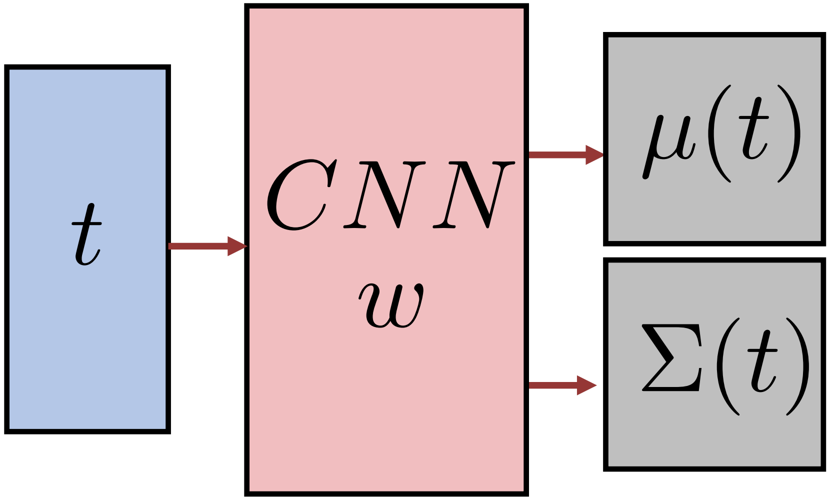

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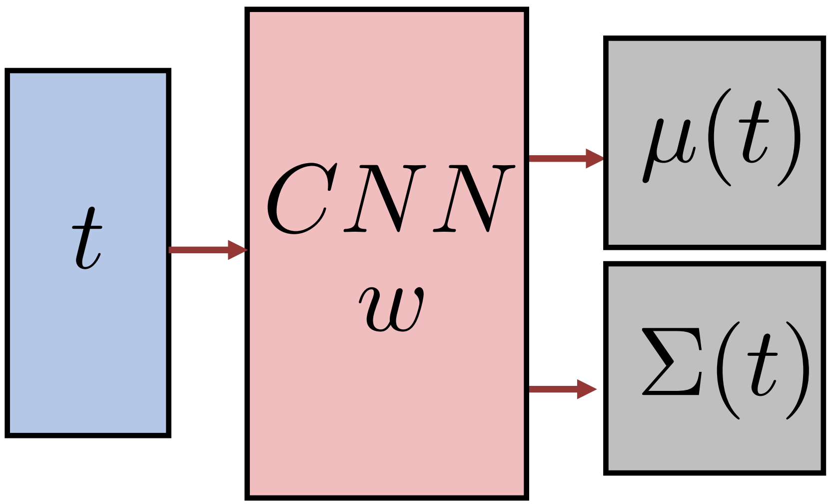

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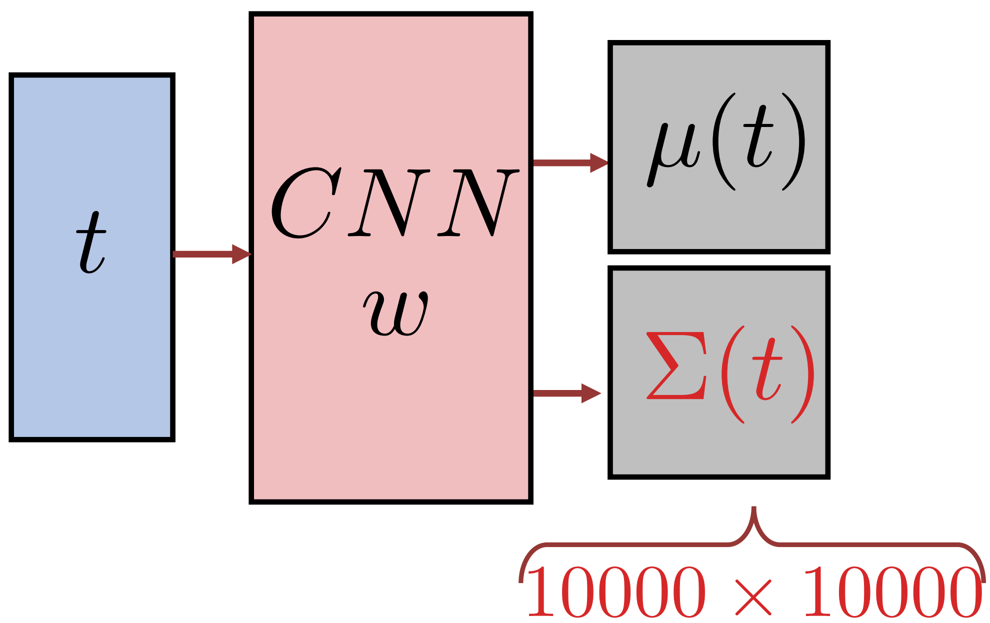

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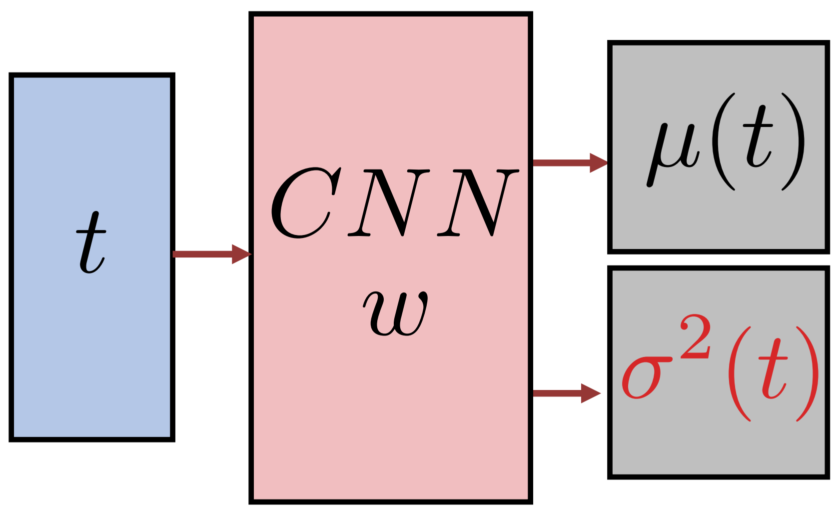

Scaling up Expectation Maximization

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Need to compute $p(T \mid X, w)$

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