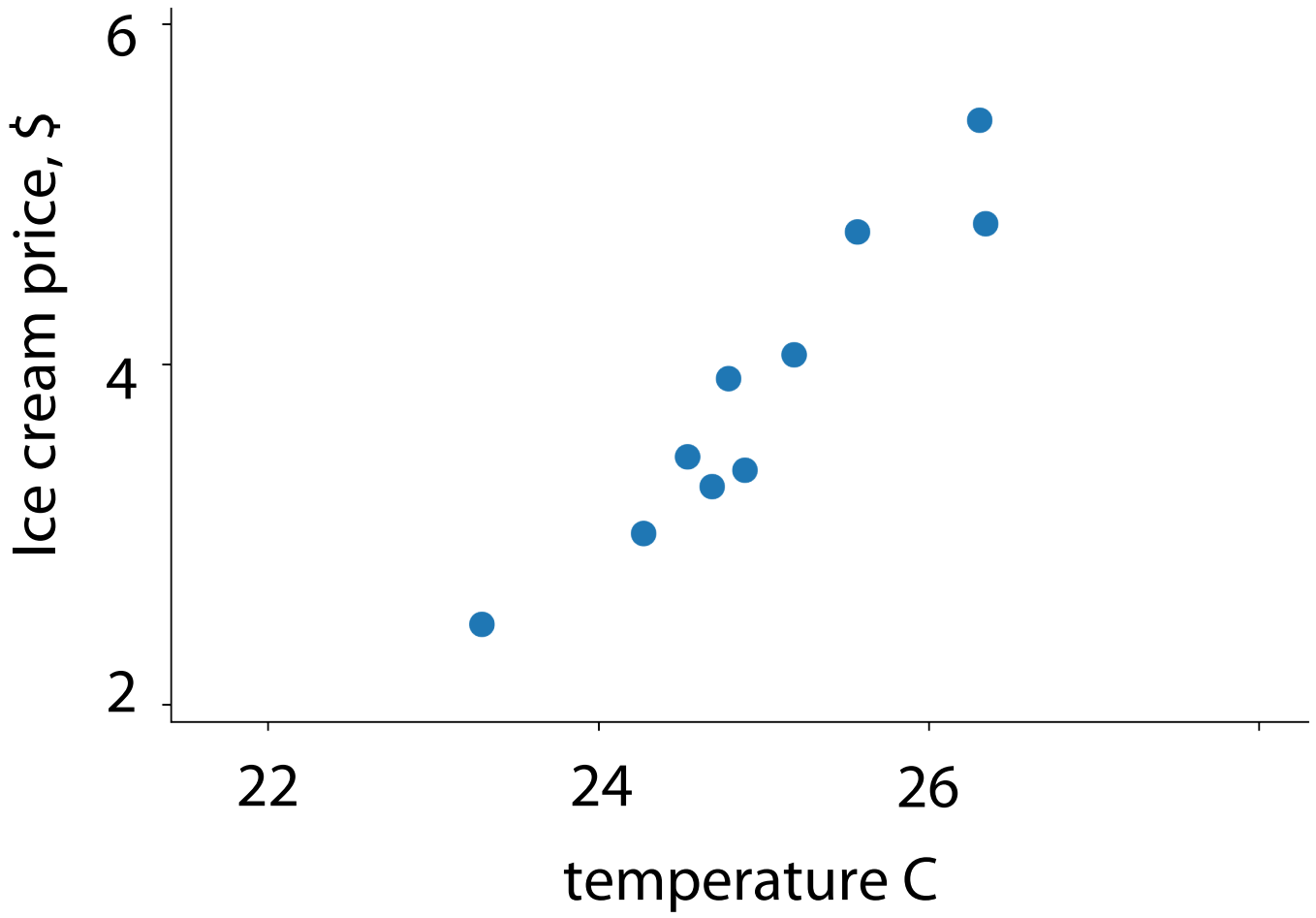
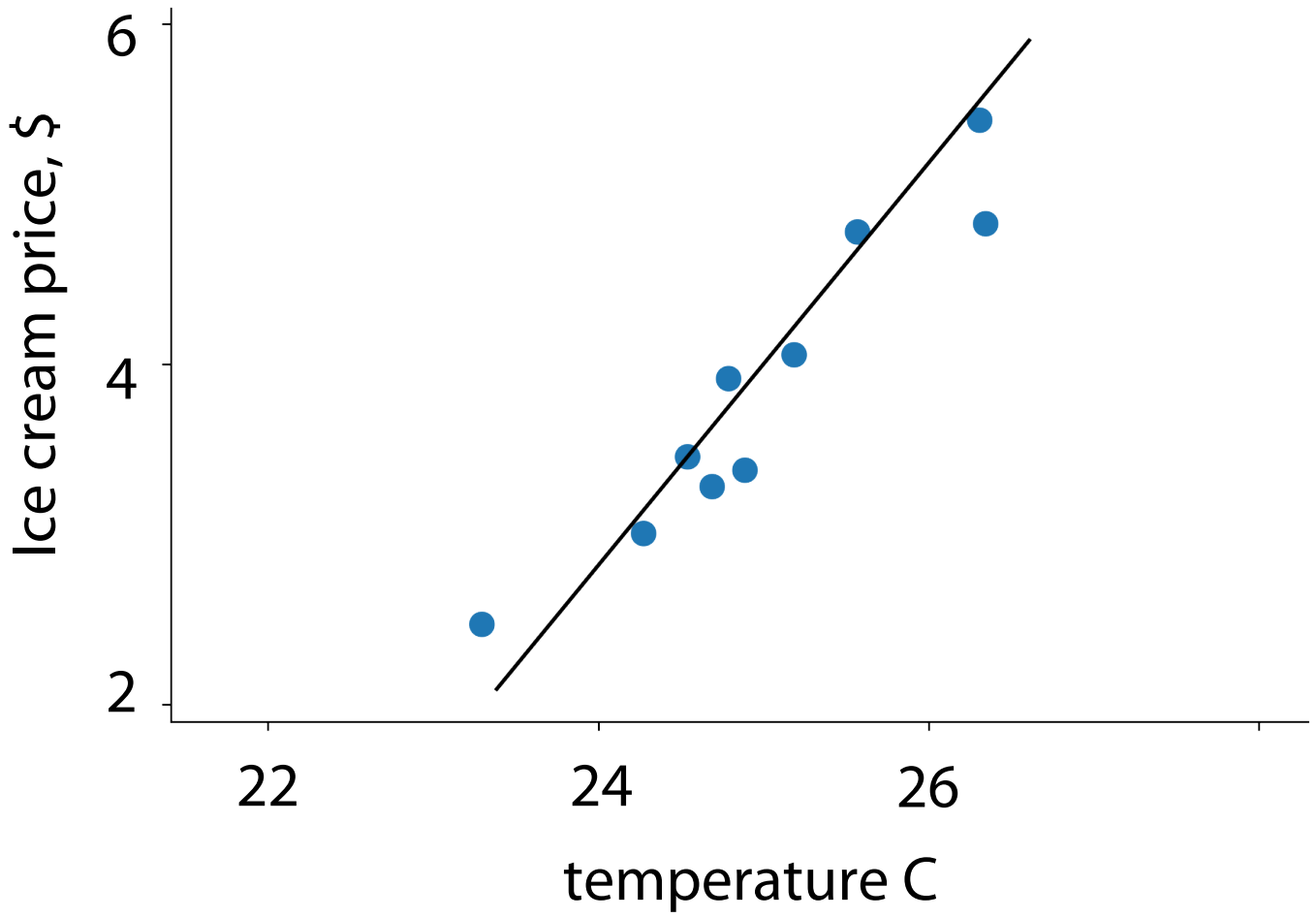
Ice Cream conspiracy

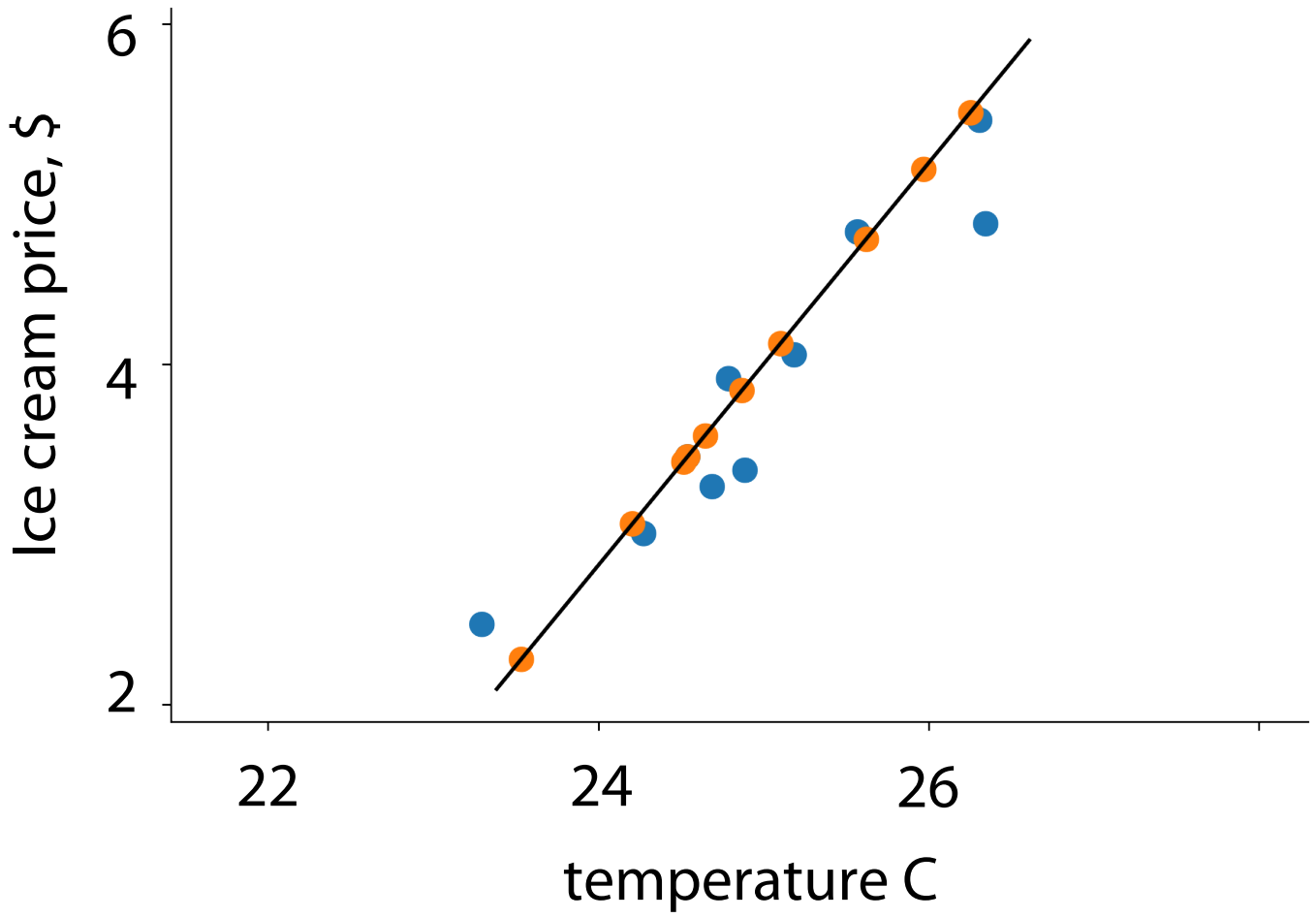
Principal Component Analysis



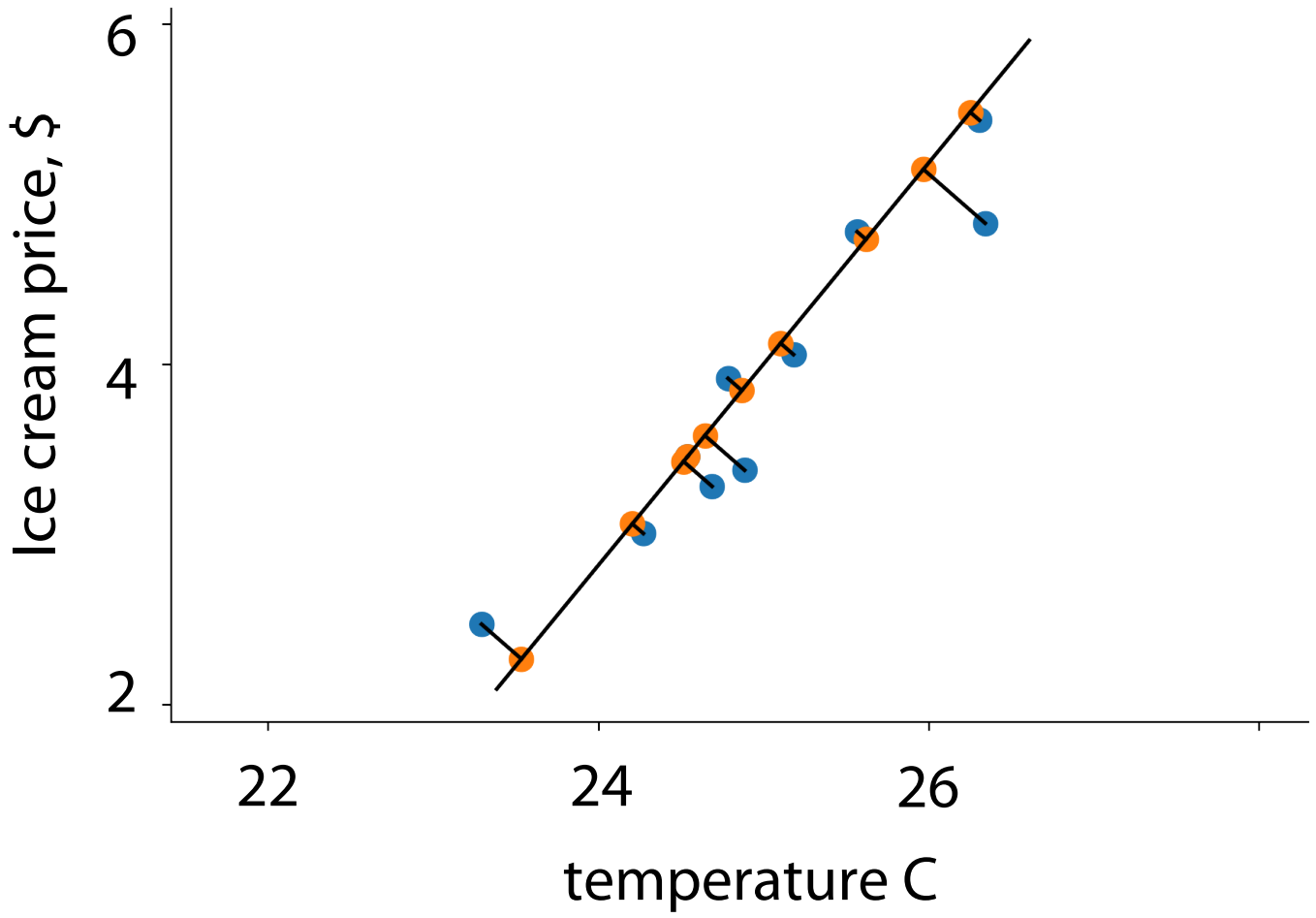
Principal Component Analysis



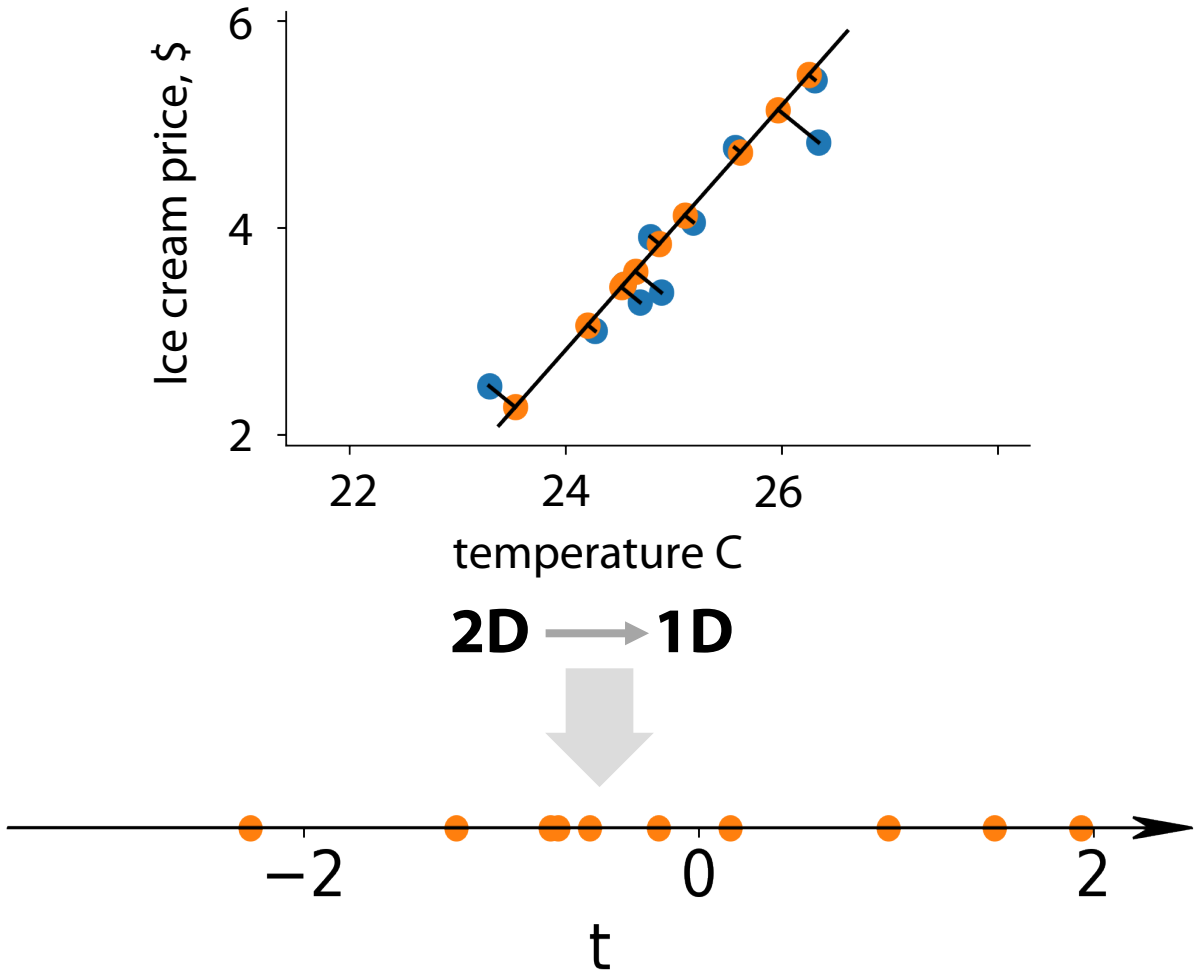
Principal Component Analysis



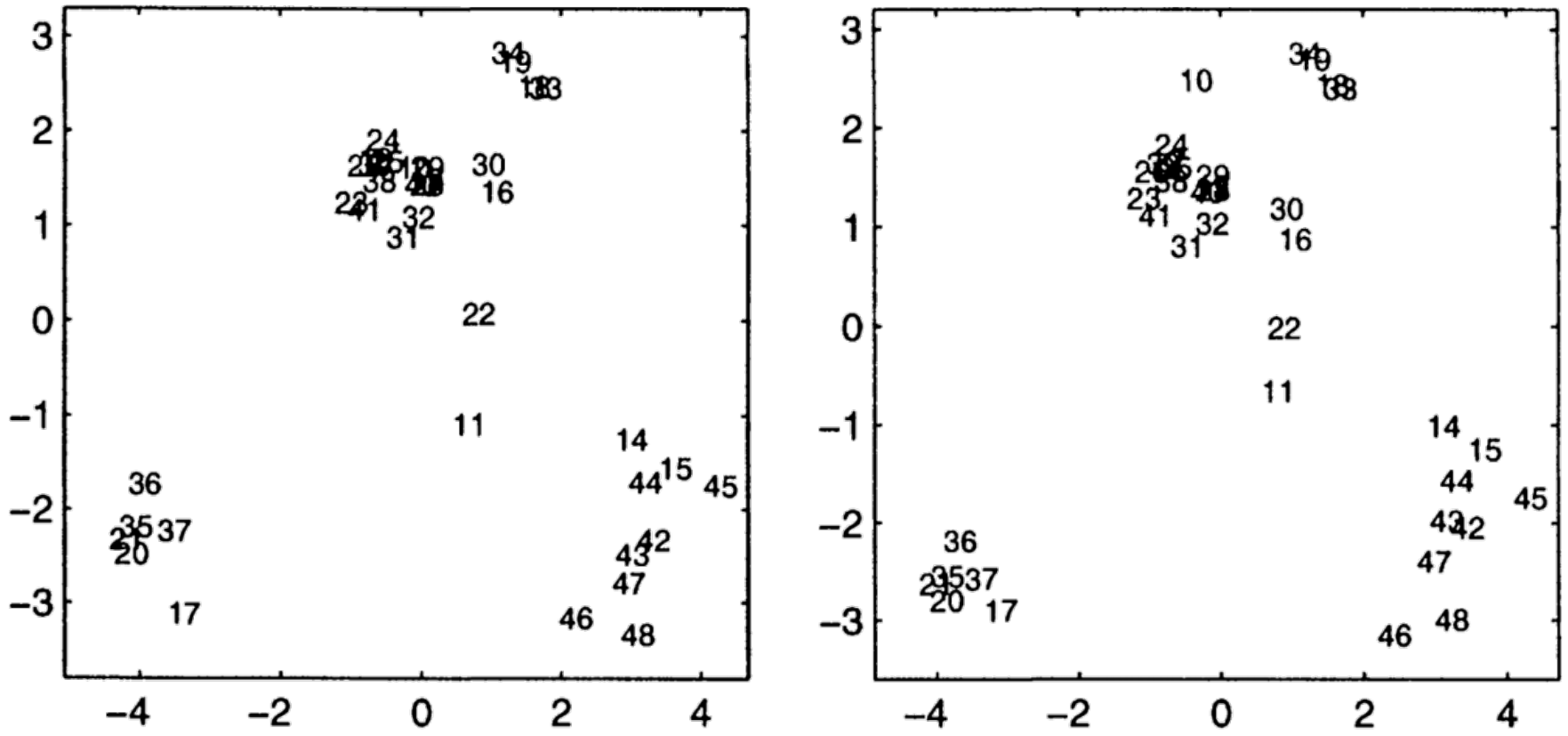
Principal Component Analysis



Principal Component Analysis



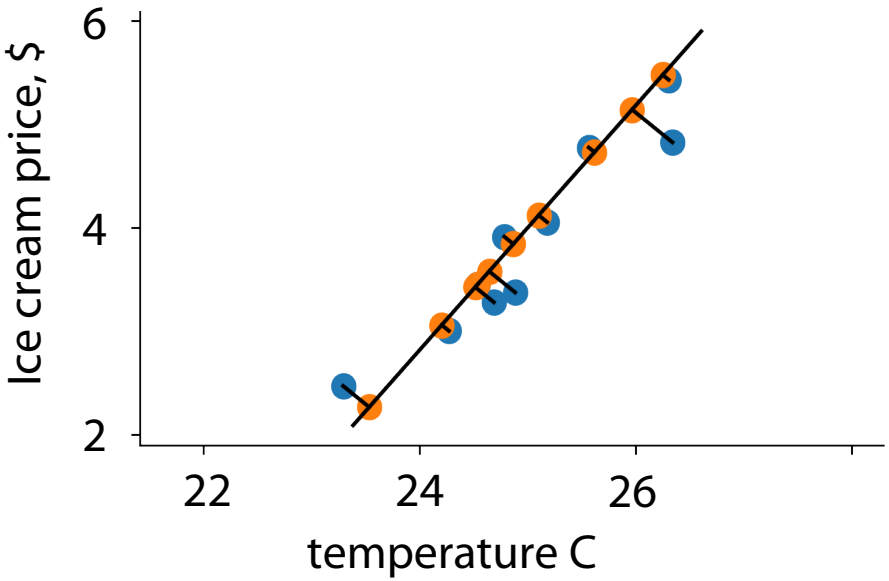
Principal Component Analysis



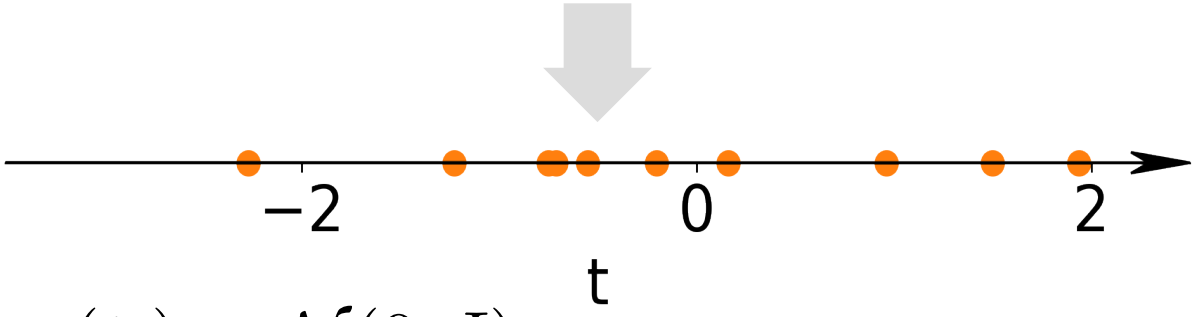
Projection of the Tobamovirus data by using PCA on the full data set and PPCA with 136 missing values

[source: Tipping, M. E., & Bishop, C. M. (1999). Probabilistic principal component analysis]

Principal Component Analysis

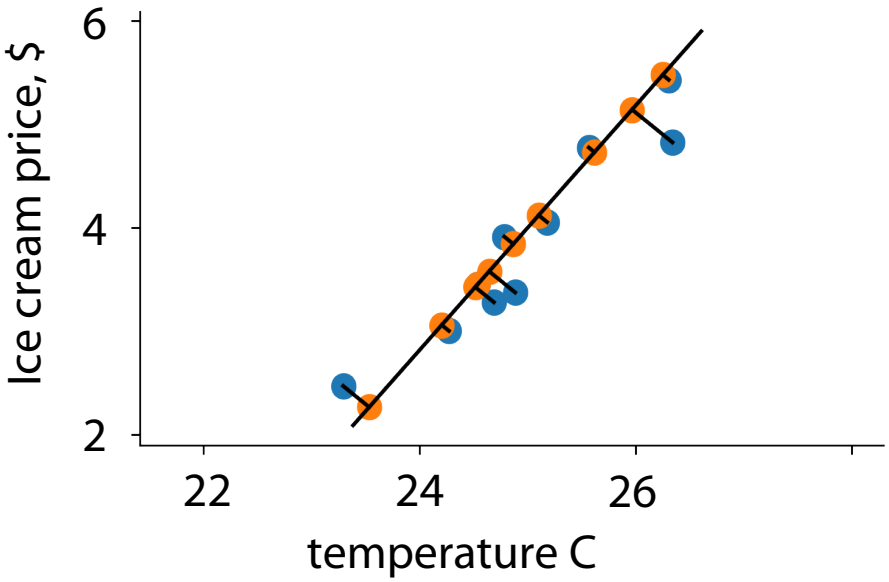


2D → 1D

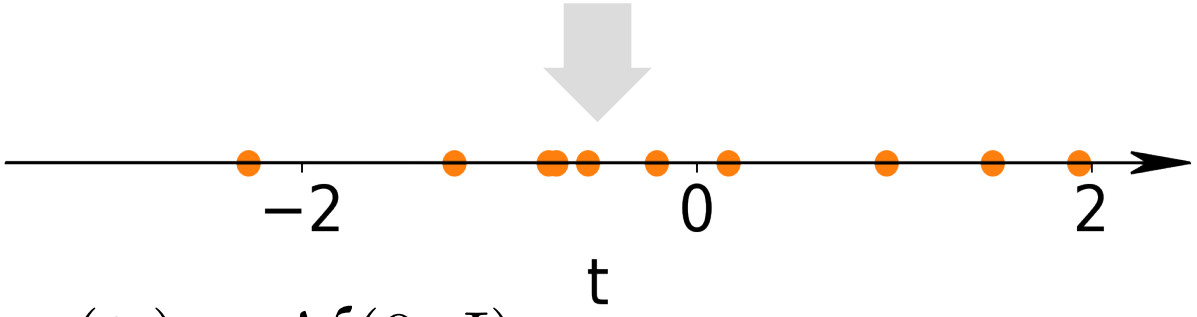


$$p(t_i) = \mathcal{N}(0, I)$$

Principal Component Analysis



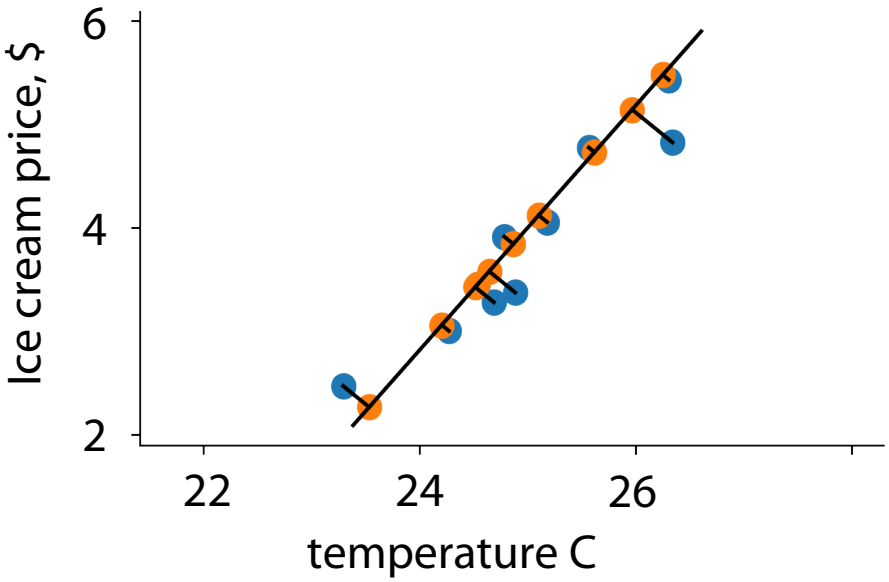
2D → 1D



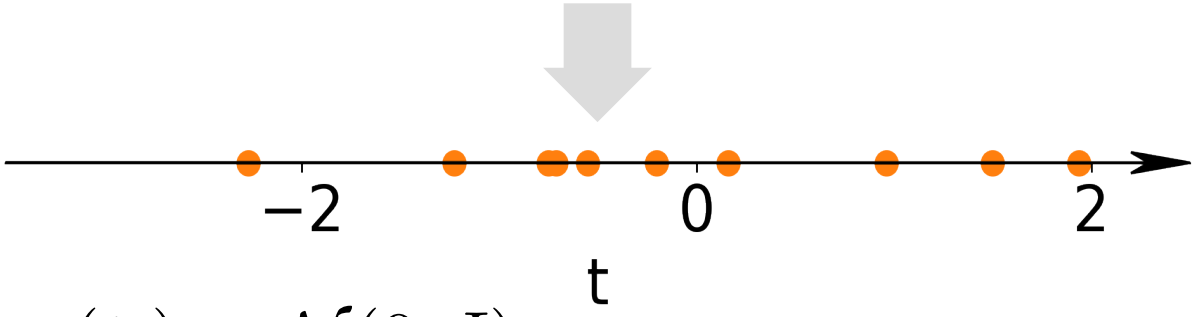
$$p(t_i) = \mathcal{N}(0, I)$$

$$x_i = (1, 1)t_i + (25, 4)$$

Principal Component Analysis



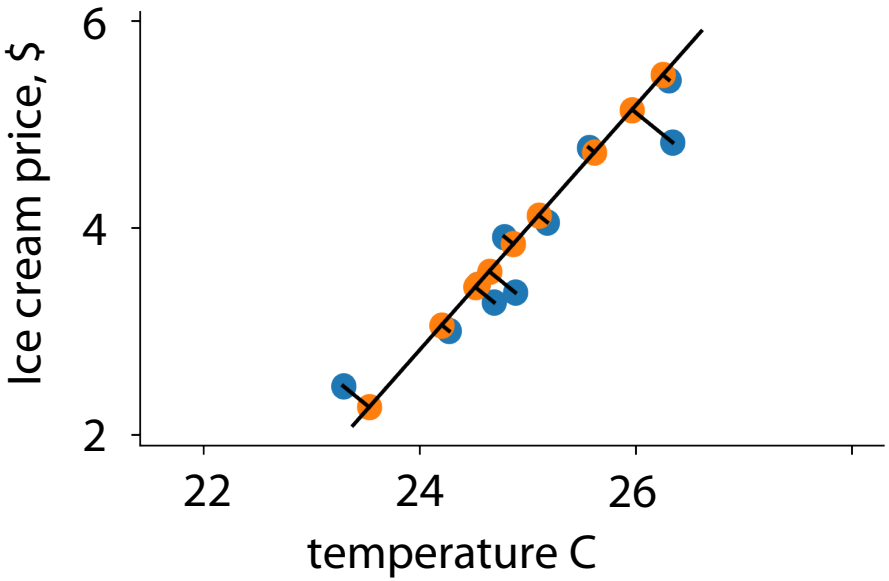
2D → 1D



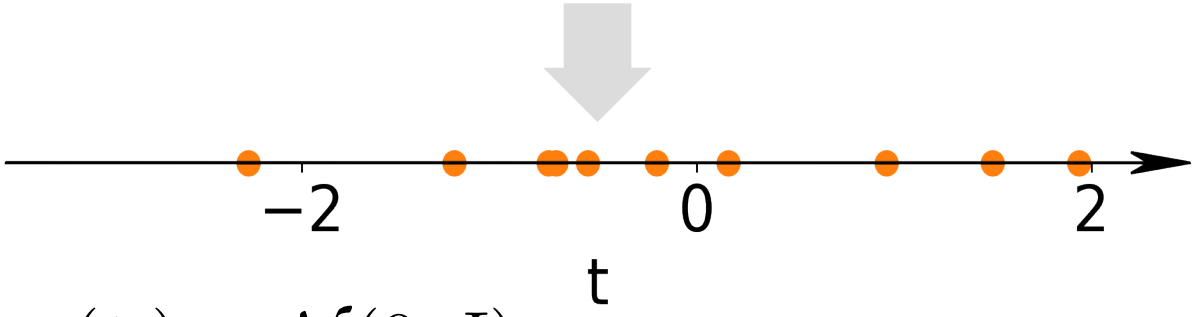
$$p(t_i) = \mathcal{N}(0, I)$$

$$x_i = W t_i + b$$

Principal Component Analysis



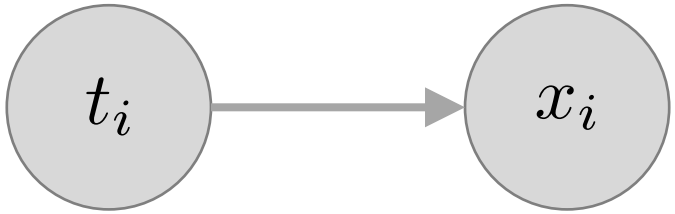
2D → 1D



$$p(t_i) = \mathcal{N}(0, I)$$

$$x_i = W t_i + b + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \Sigma)$$

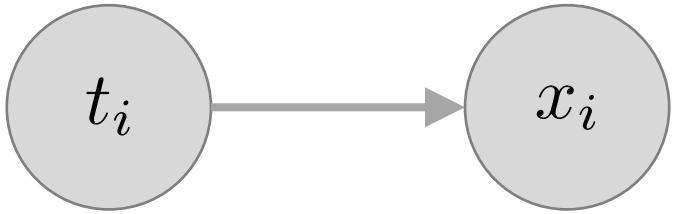
Principal Component Analysis



$$p(t_i) = \mathcal{N}(0, I)$$

$$p(x_i \mid t_i, \theta) = \mathcal{N}(Wt_i + b, \Sigma)$$

Principal Component Analysis

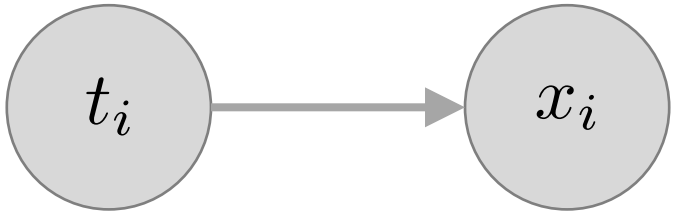


$$p(t_i) = \mathcal{N}(0, I)$$

$$p(x_i \mid t_i, \theta) = \mathcal{N}(Wt_i + b, \Sigma)$$

$$\max_{\theta} p(X \mid \theta)$$

Principal Component Analysis

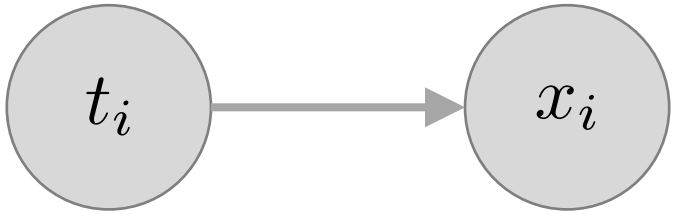


$$p(t_i) = \mathcal{N}(0, I)$$

$$p(x_i \mid t_i, \theta) = \mathcal{N}(Wt_i + b, \Sigma)$$

$$\max_{\theta} p(X \mid \theta) = \prod_{i=1}^{N} p(x_i \mid \theta)$$

Principal Component Analysis



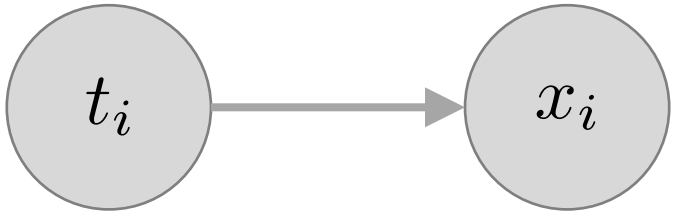
$$p(t_i) = \mathcal{N}(0, I)$$

$$p(x_i \mid t_i, \theta) = \mathcal{N}(Wt_i + b, \Sigma)$$

$$\max_{\theta} p(X \mid \theta) = \prod_{i=1}^{N} p(x_i \mid \theta)$$

$$= \prod_{i} \int p(x_i \mid t_i, \theta) p(t_i) dt_i$$

Principal Component Analysis



$$p(t_i) = \mathcal{N}(0, I)$$

$$p(x_i \mid t_i, \theta) = \mathcal{N}(Wt_i + b, \Sigma)$$

$$\max_{\theta} p(X \mid \theta) = \prod_{i=1}^{N} p(x_i \mid \theta)$$

$$= \prod_{i} \underbrace{\int p(x_i \mid t_i, \theta) p(t_i) dt_i}_{\text{conjugacy, } \mathcal{N}(\mu_i, \Sigma_i)}$$