# General form of Expectation Maximization

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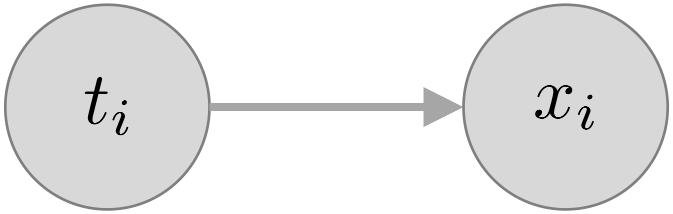

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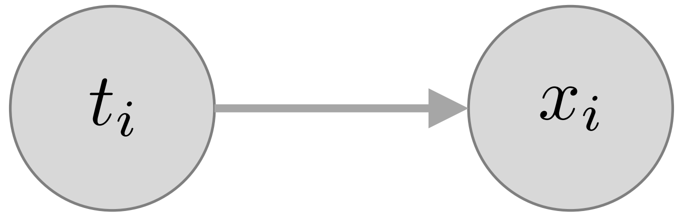


$$p(x_i \mid \theta) = \sum_{c=1}^{3} p(x_i \mid t_i = c, \theta)\, p(t_i = c \mid \theta)$$

# General form of Expectation Maximization

$$\max_{\theta}. \qquad p(X \mid \theta)$$

# General form of Expectation Maximization

$$\max_{\theta}.\ \log p(X \mid \theta)$$

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Jensen's inequality

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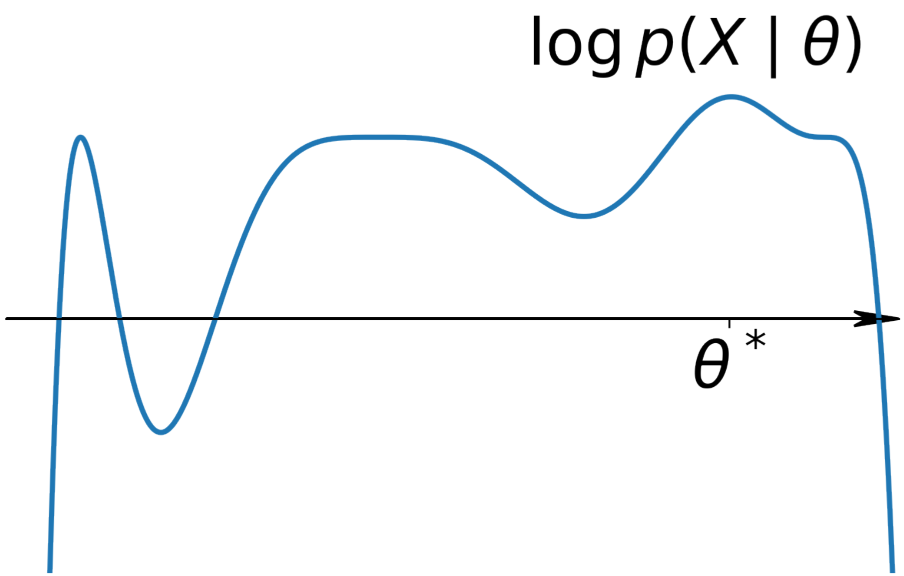

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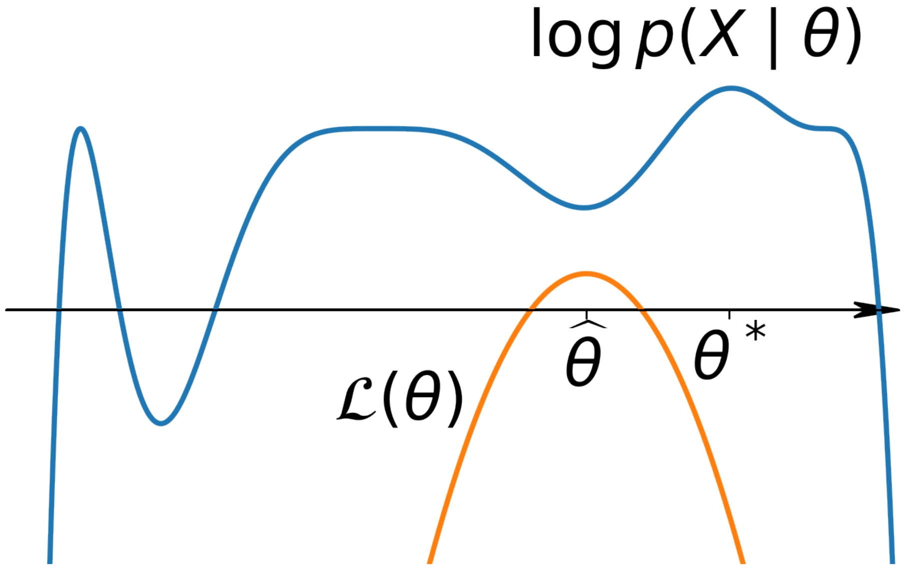

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$$\boxed{\log\left(\sum_c \alpha_c v_c\right) \geq \sum_c \alpha_c \log(v_c)}$$

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# General form of Expectation Maximization

$$
\begin{aligned}
\log p(X \mid \theta) &= \sum_{i=1}^{N} \log p(x_i \mid \theta) \\
&= \sum_{i=1}^{N} \log \sum_{c=1}^{3} \frac{q(t_i = c)}{q(t_i = c)} p(x_i, t_i = c \mid \theta) \\
&\geq \sum_{i=1}^{N} \sum_{c=1}^{3} q(t_i = c) \log \frac{p(x_i, t_i = c \mid \theta)}{q(t_i = c)} \\
&= \mathcal{L}(\theta, q)
\end{aligned}
$$

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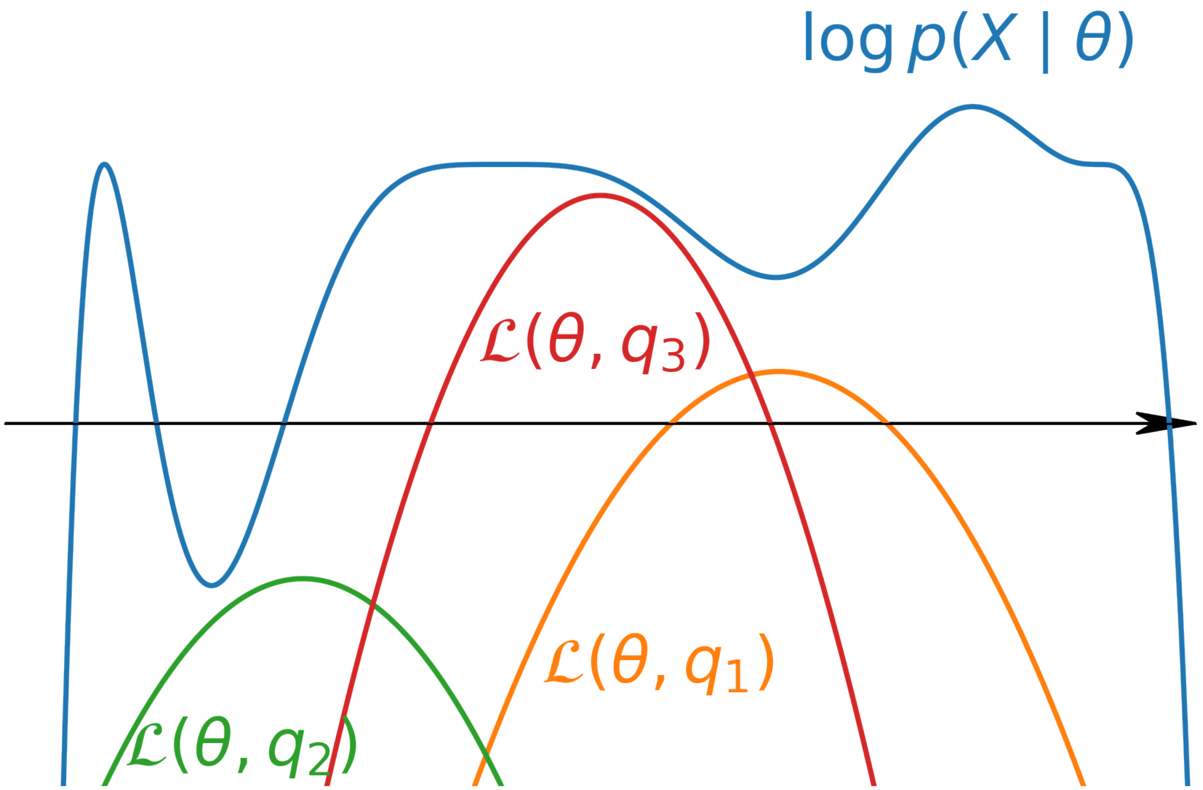
$$\log p(X \mid \theta) \geq \mathcal{L}(\theta, q) \text{ for any q}$$



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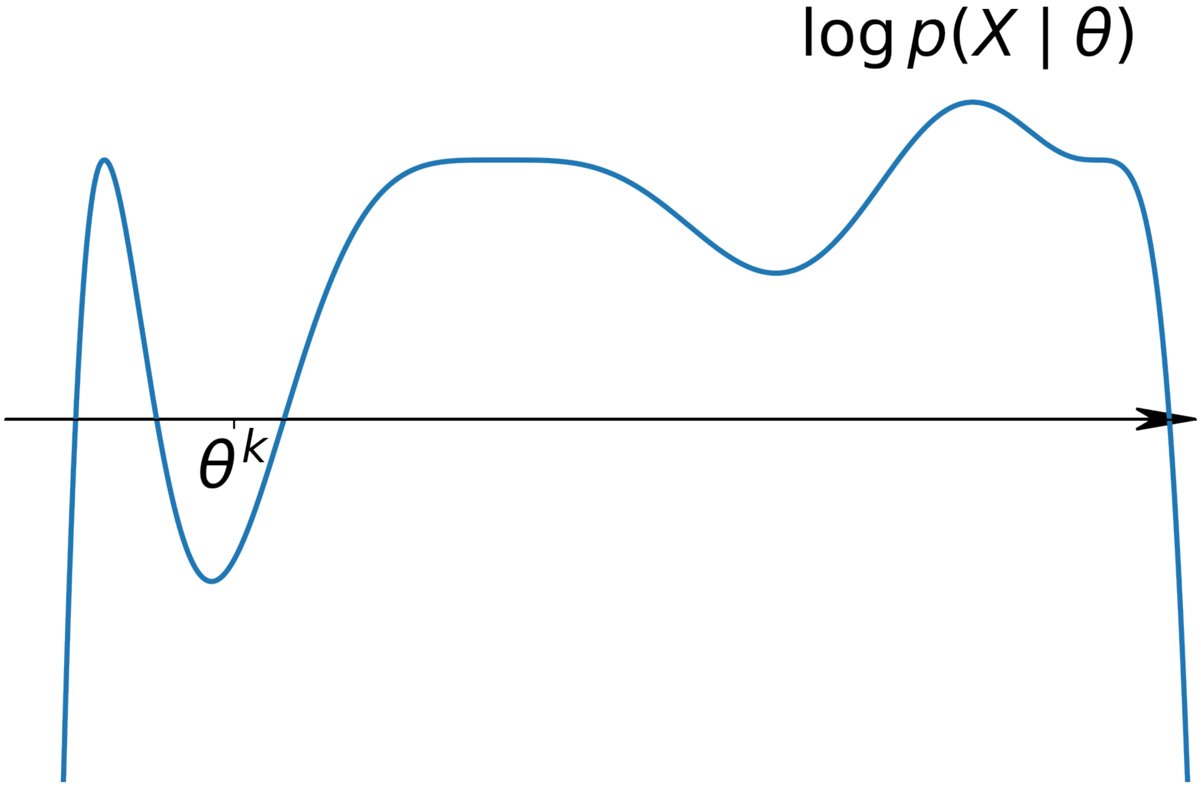

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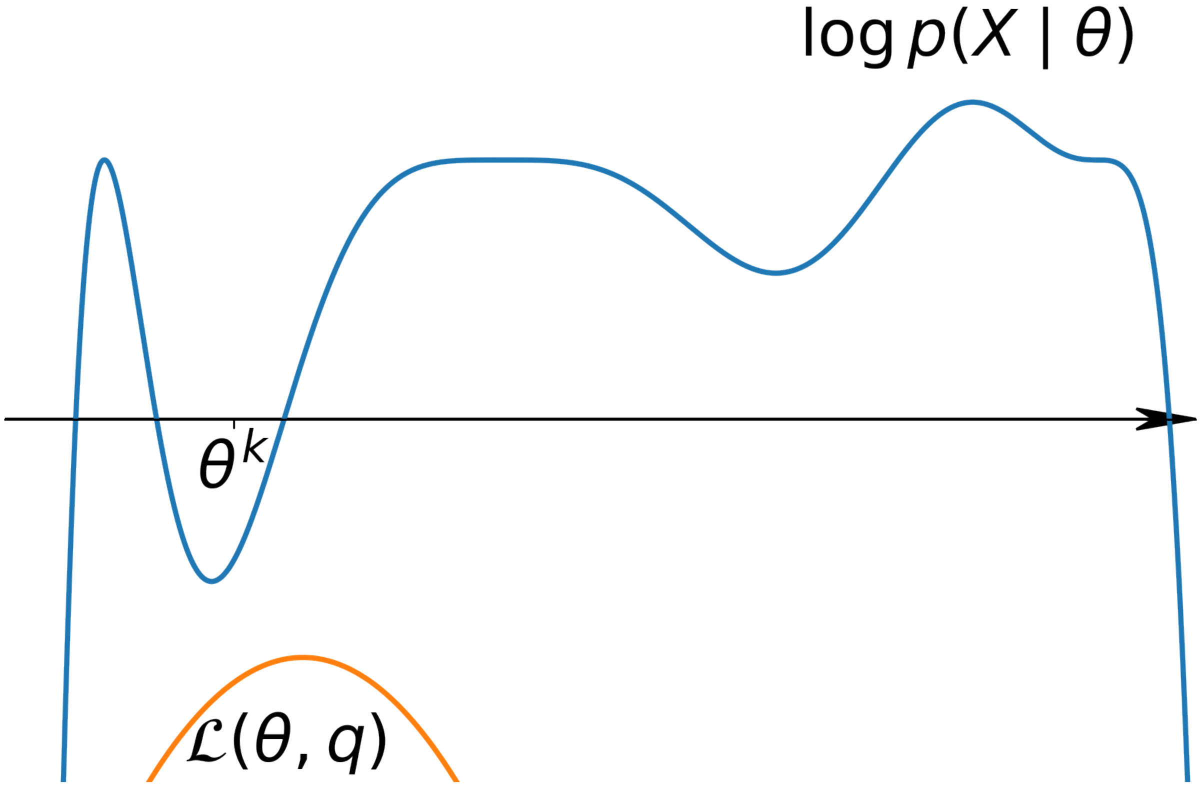
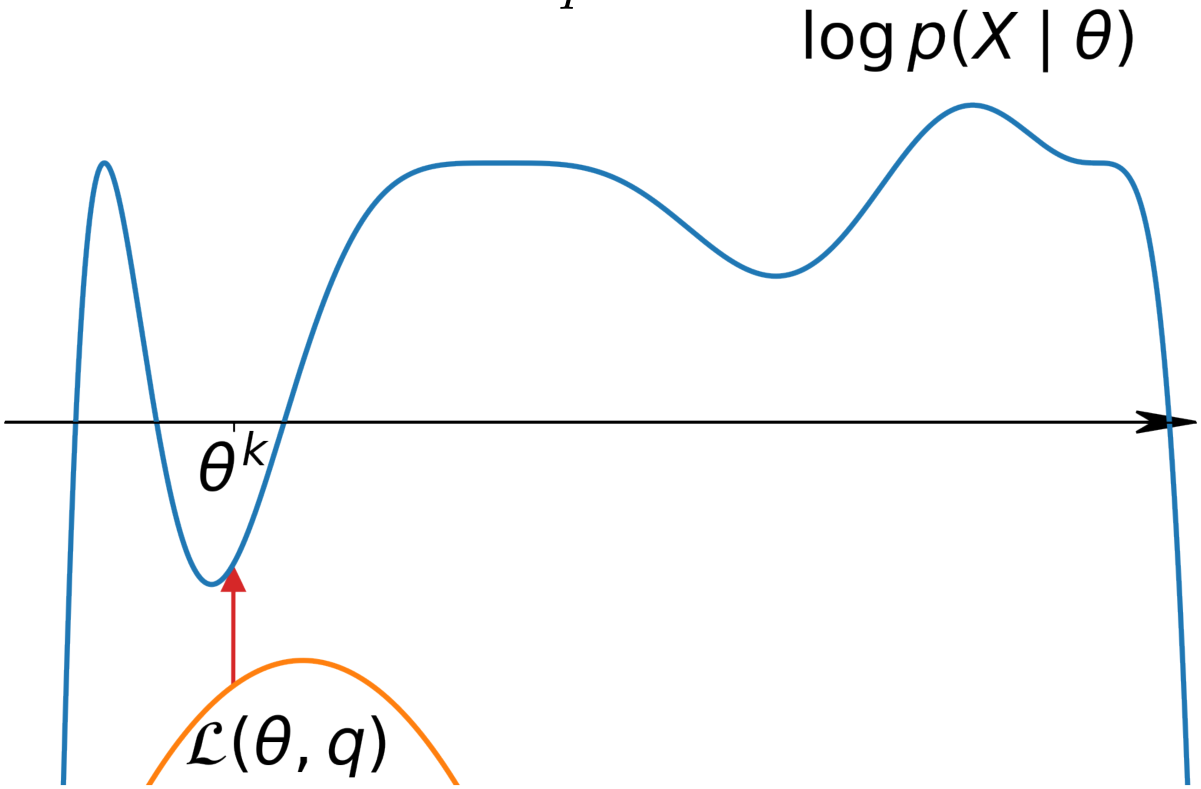


# General form of Expectation Maximization

$$\log p(X \mid \theta) \geq \mathcal{L}(\theta, q) \text{ for any q}$$

$$q^{k+1} = \arg\max_q \mathcal{L}(\theta^k, q)$$



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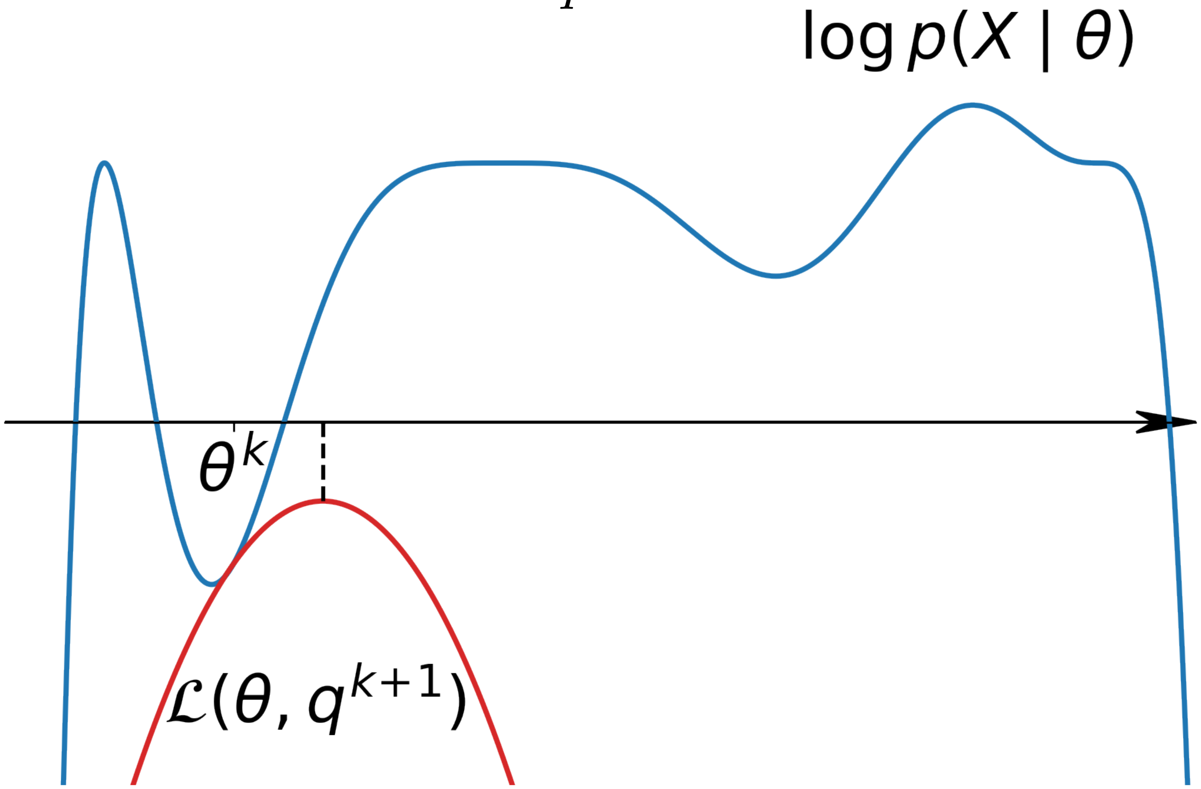

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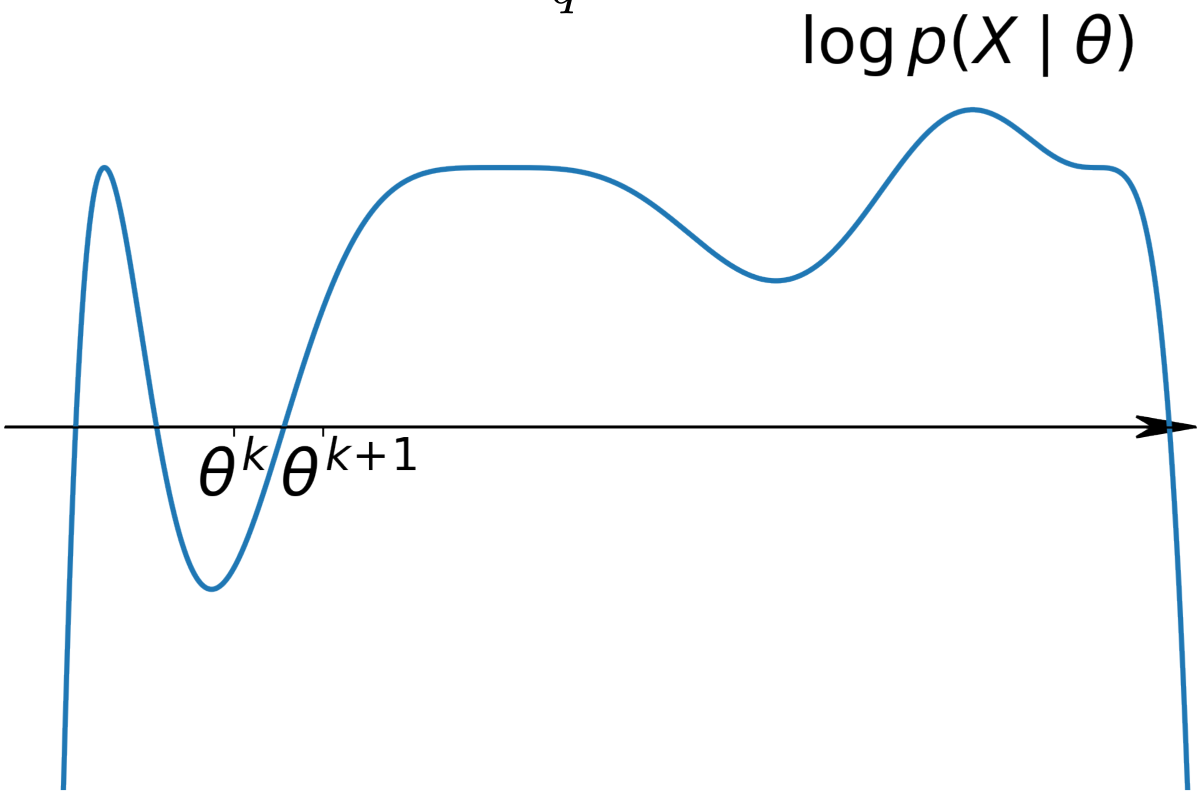

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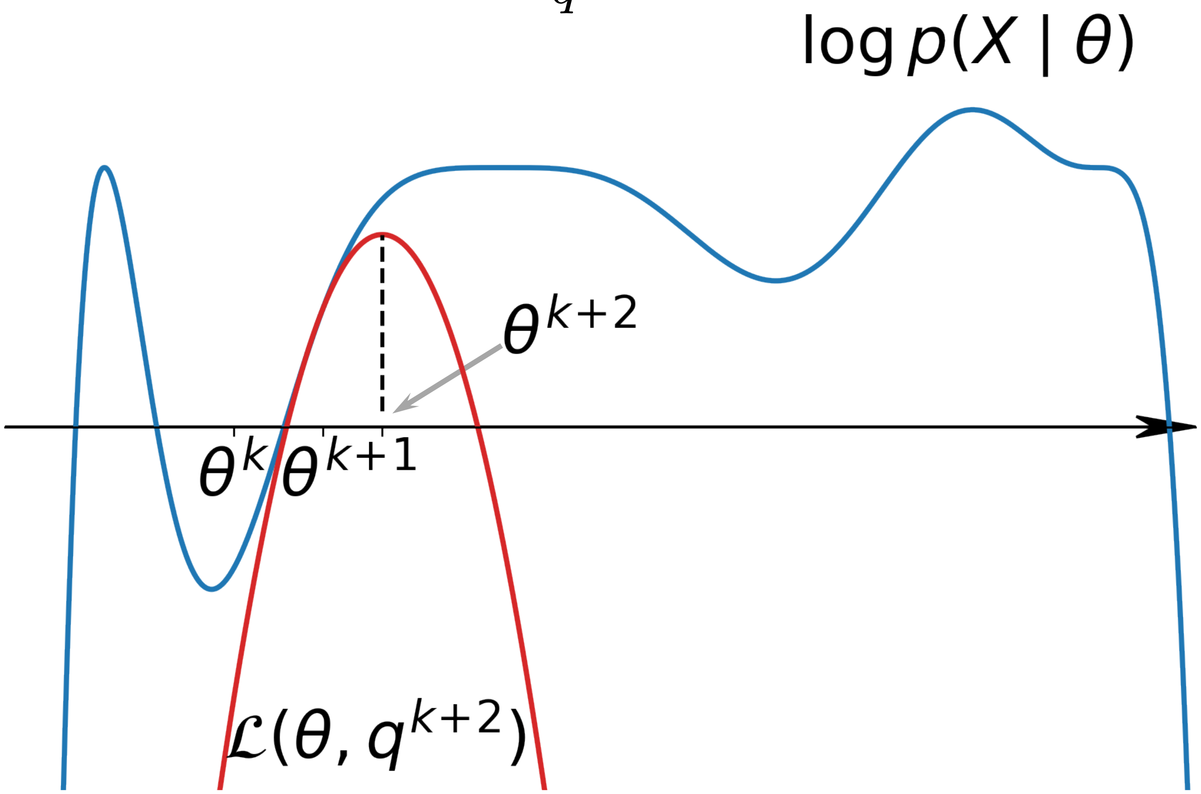

# Summary of Expectation Maximization

$$\log p(X \mid \theta) \geq \mathcal{L}(\theta, q) \text{ for any q}$$

Variational lower bound

E-step

$$q^{k+1} = \arg\max_q \mathcal{L}(\theta^k, q)$$

M-step

$$\theta^{k+1} = \arg\max_\theta \mathcal{L}(\theta, q^{k+1})$$