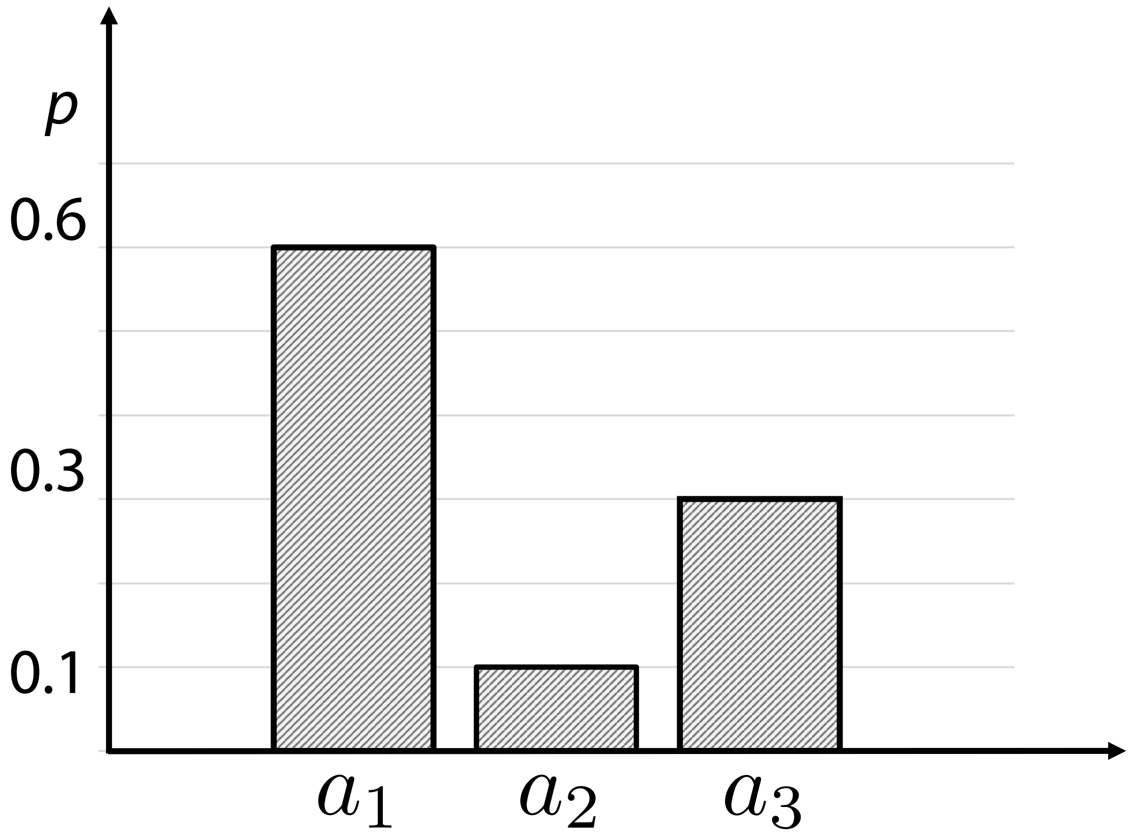
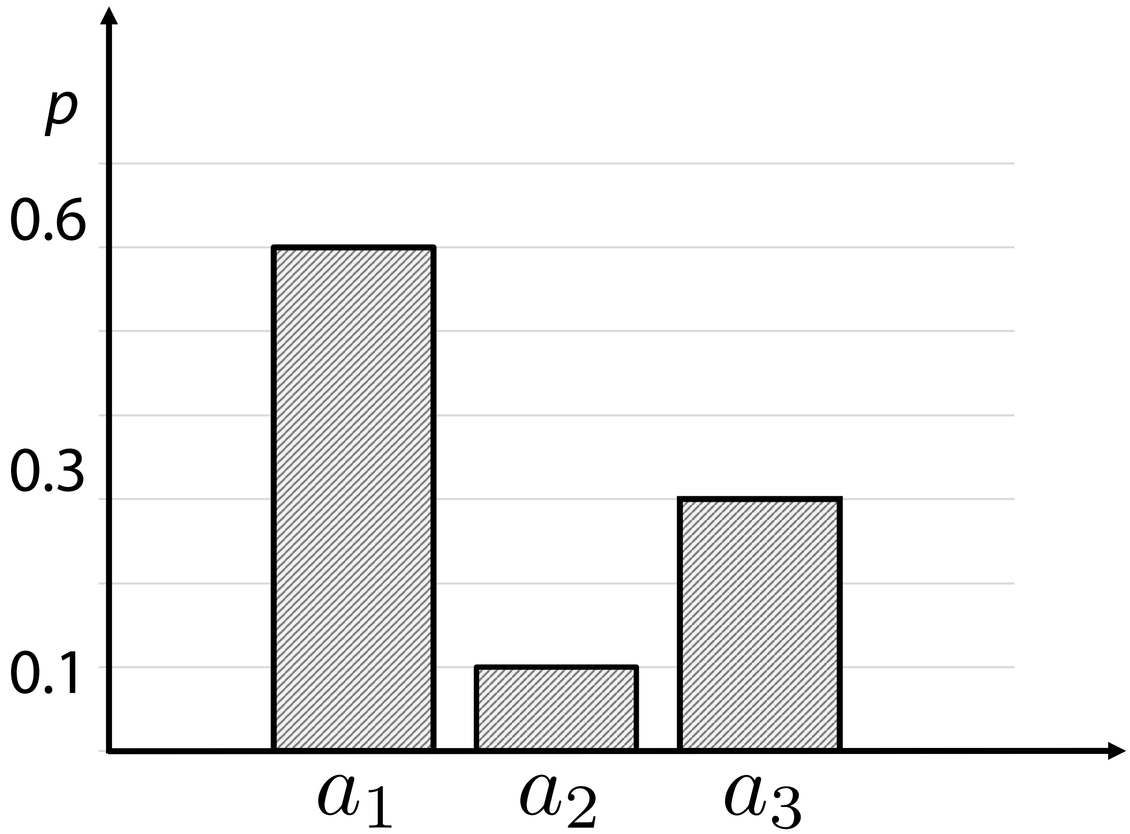
Sampling from 1d distributions

1d sampling (discrete)

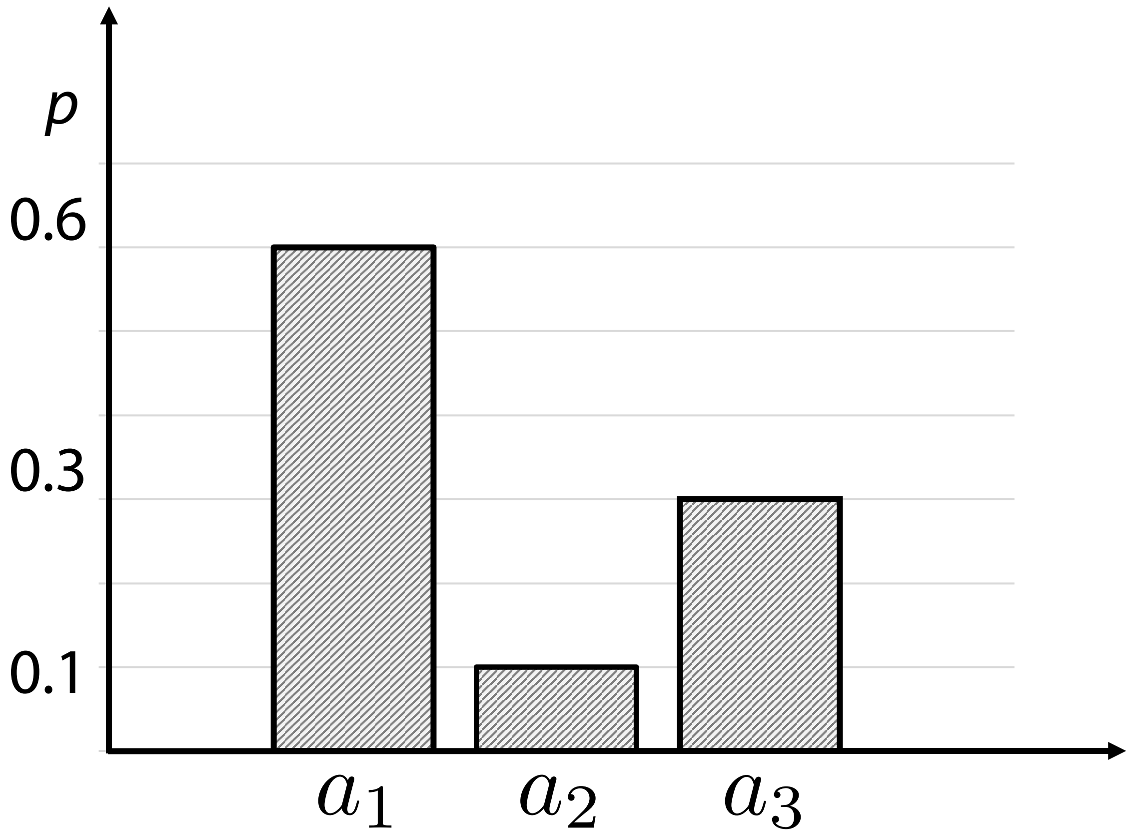


1d sampling (discrete)



We can always sample from uniform $\mathcal{U}[0,1]$

1d sampling (discrete)



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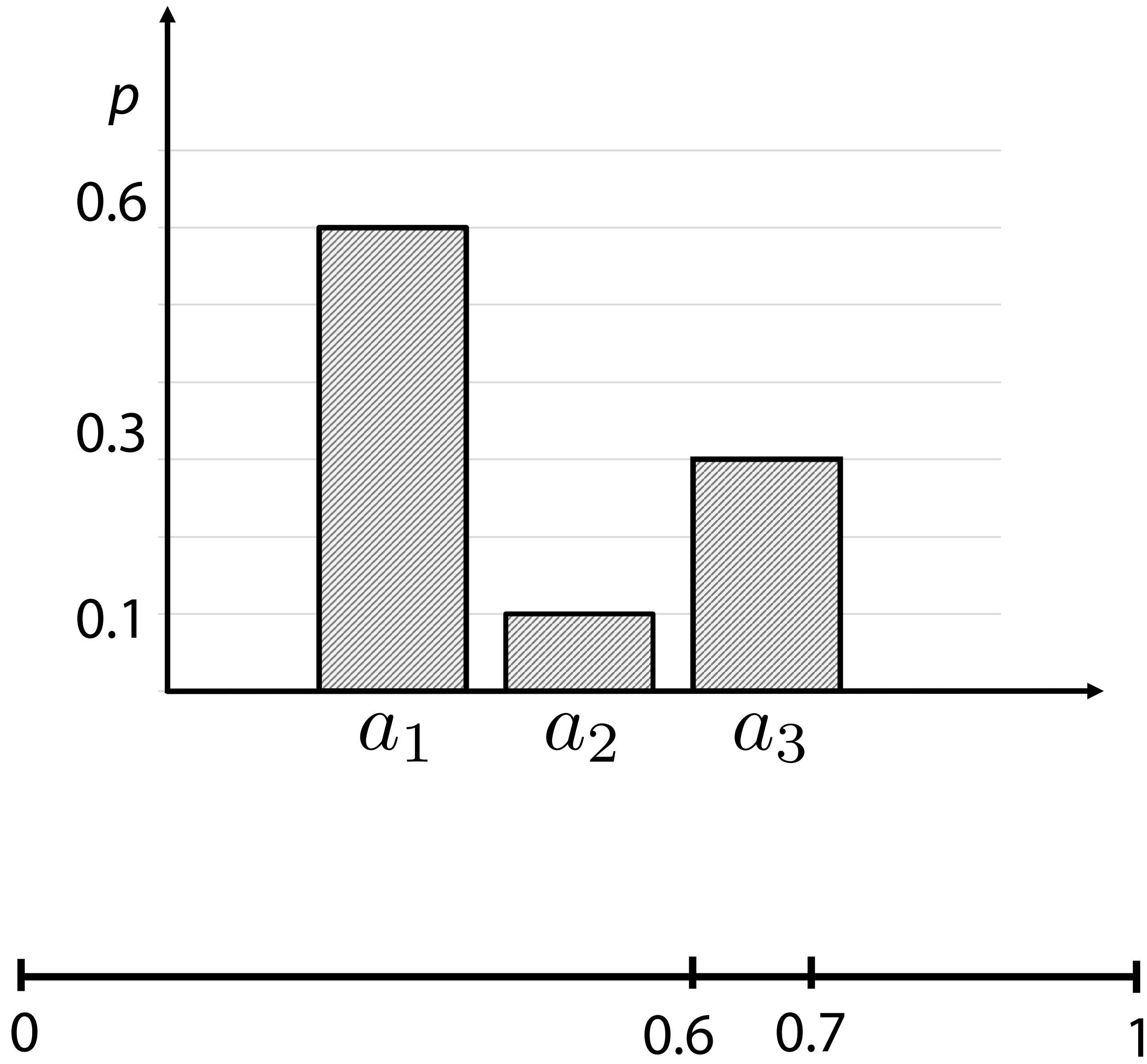

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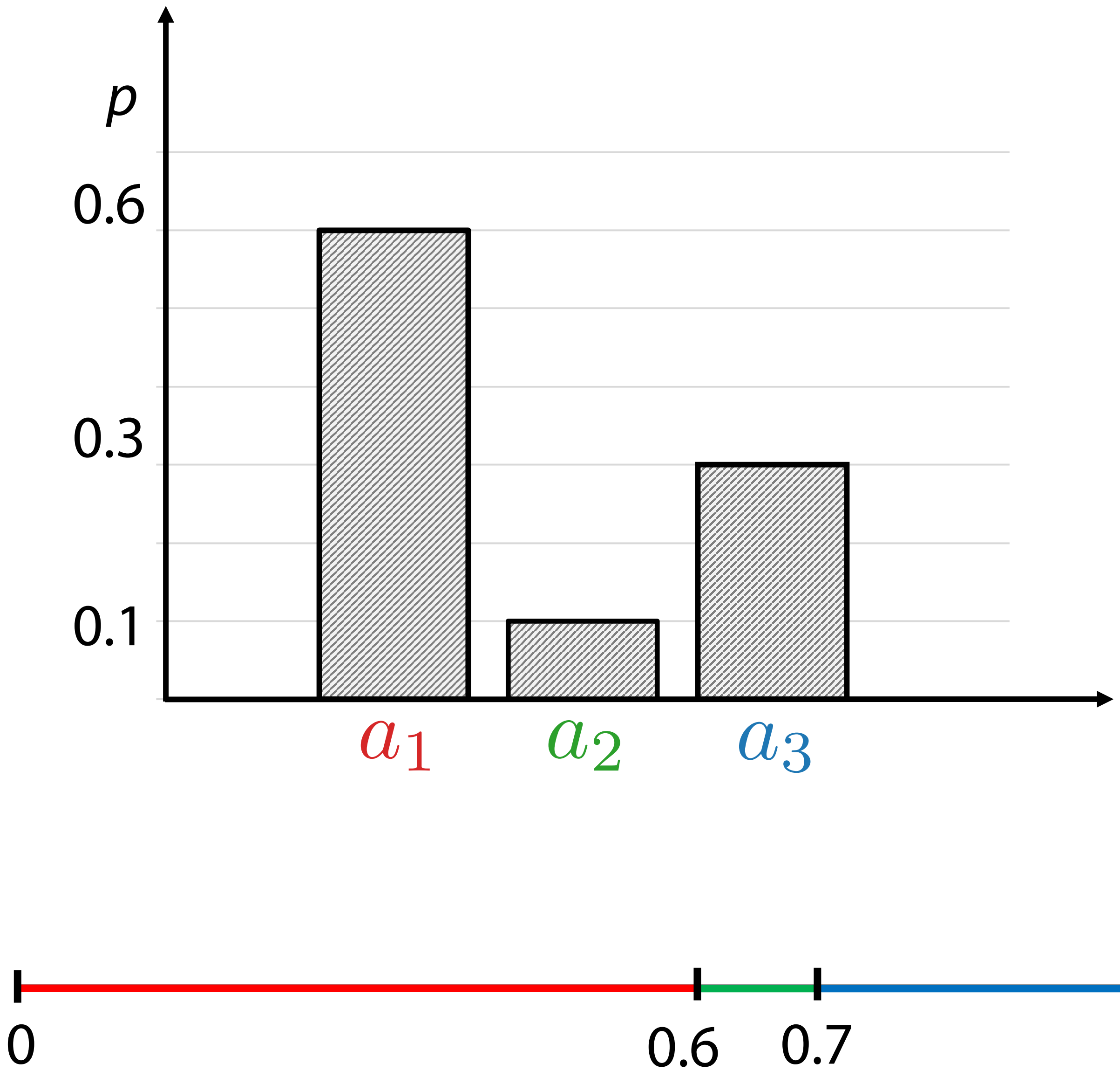

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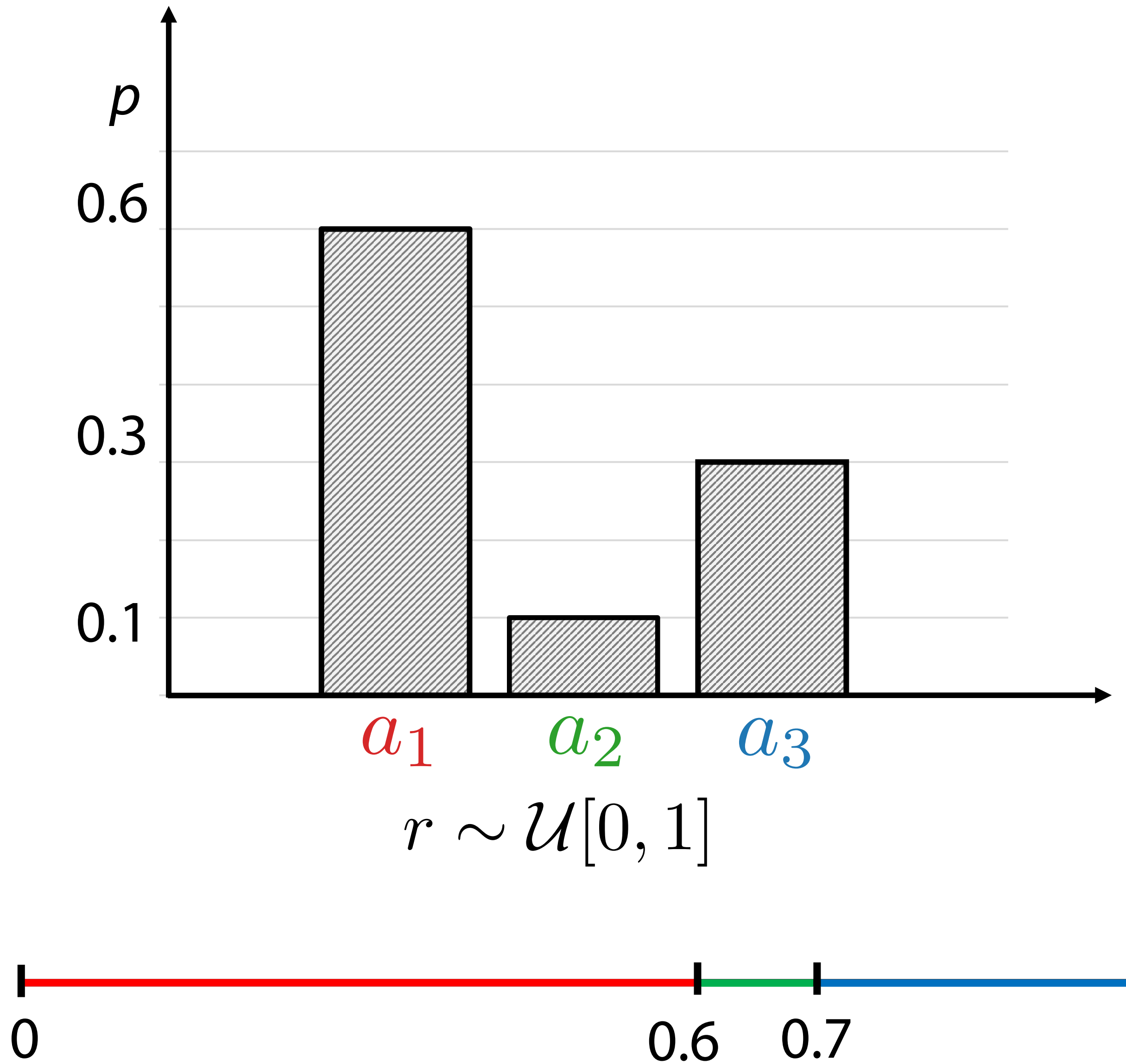

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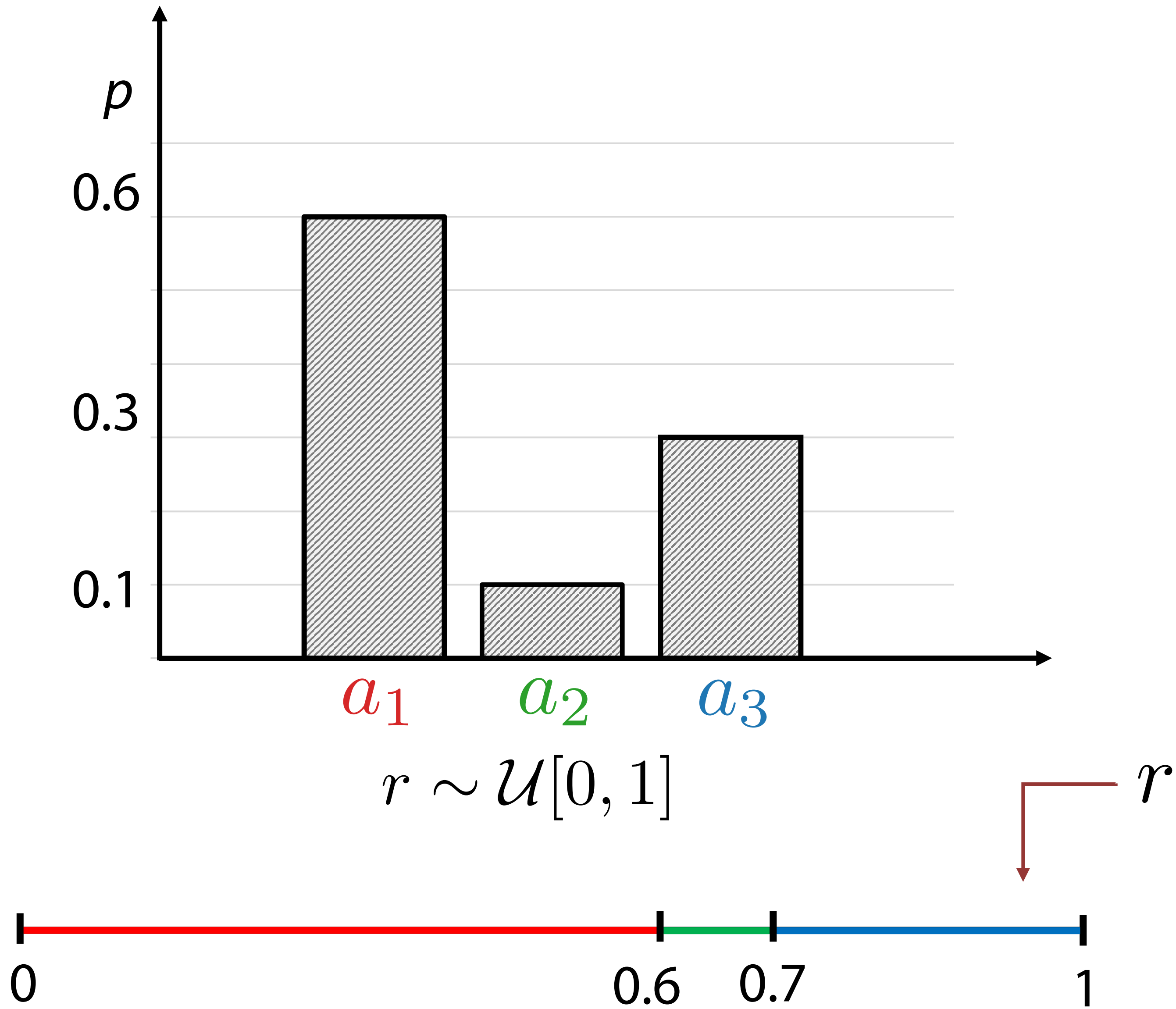

Summary

1d discrete distributions with finite number of values are easy

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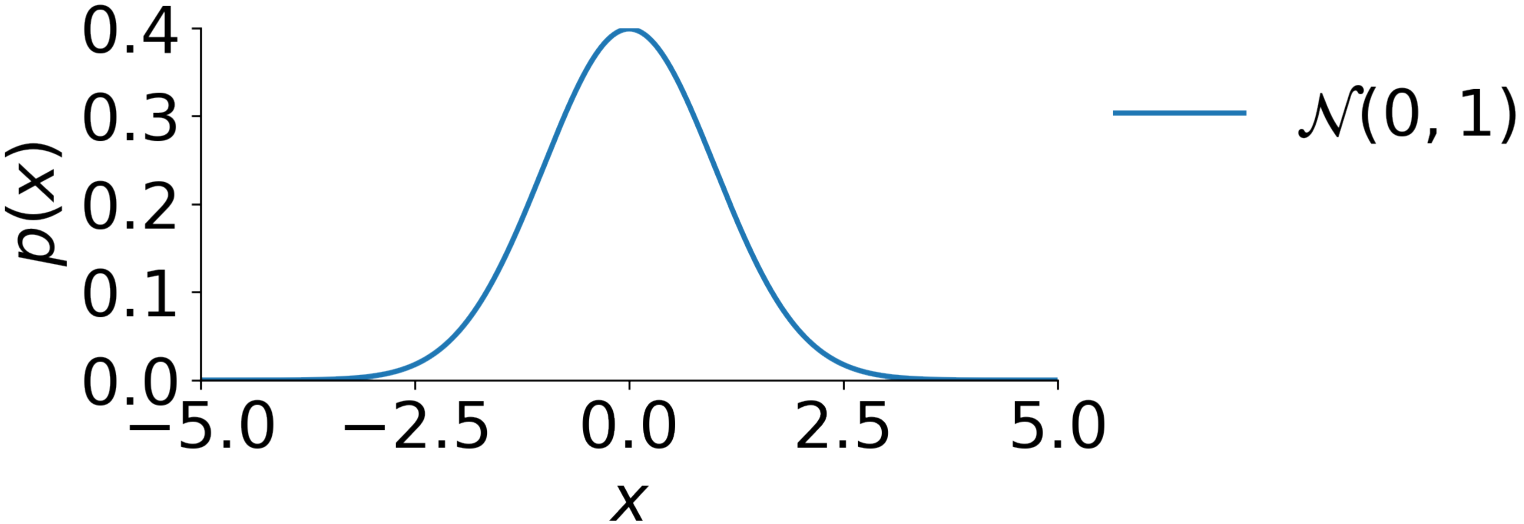
At least then number of values is $<$ 100 000

Continuous sampling

1d sampling (continuous)

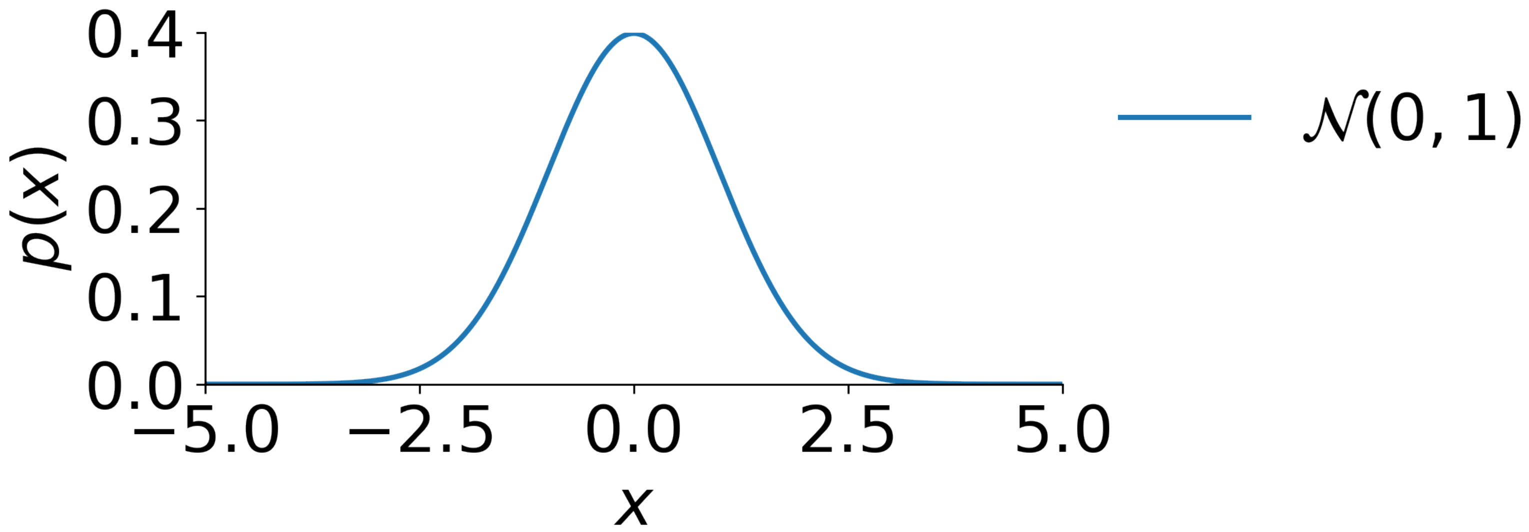
Sampling from Gaussian distribution



1d sampling (continuous)

Sampling from Gaussian distribution

$$z = \sum_{i=1}^{12} x_i - 6, \quad x_i \sim \mathcal{U}[0,1]$$

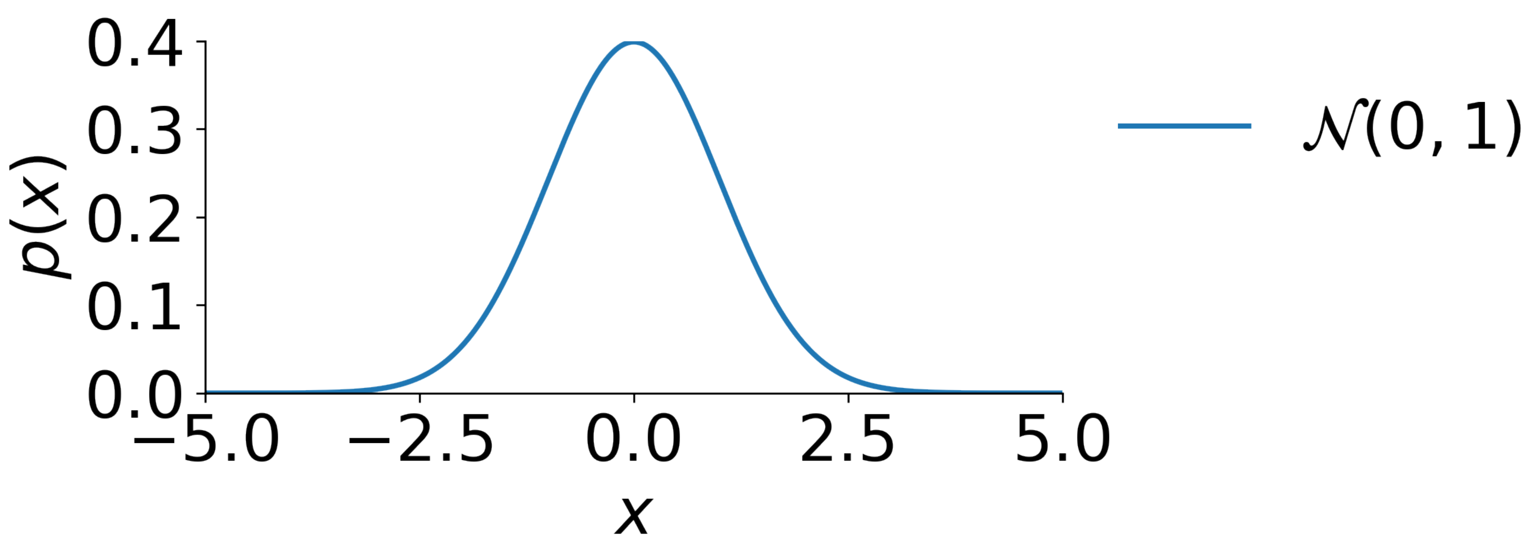


1d sampling (continuous)

Sampling from Gaussian distribution

$$z = \sum_{i=1}^{12} x_i - 6, \quad x_i \sim \mathcal{U}[0,1]$$

$$p(z) \approx \mathcal{N}(0,1)$$

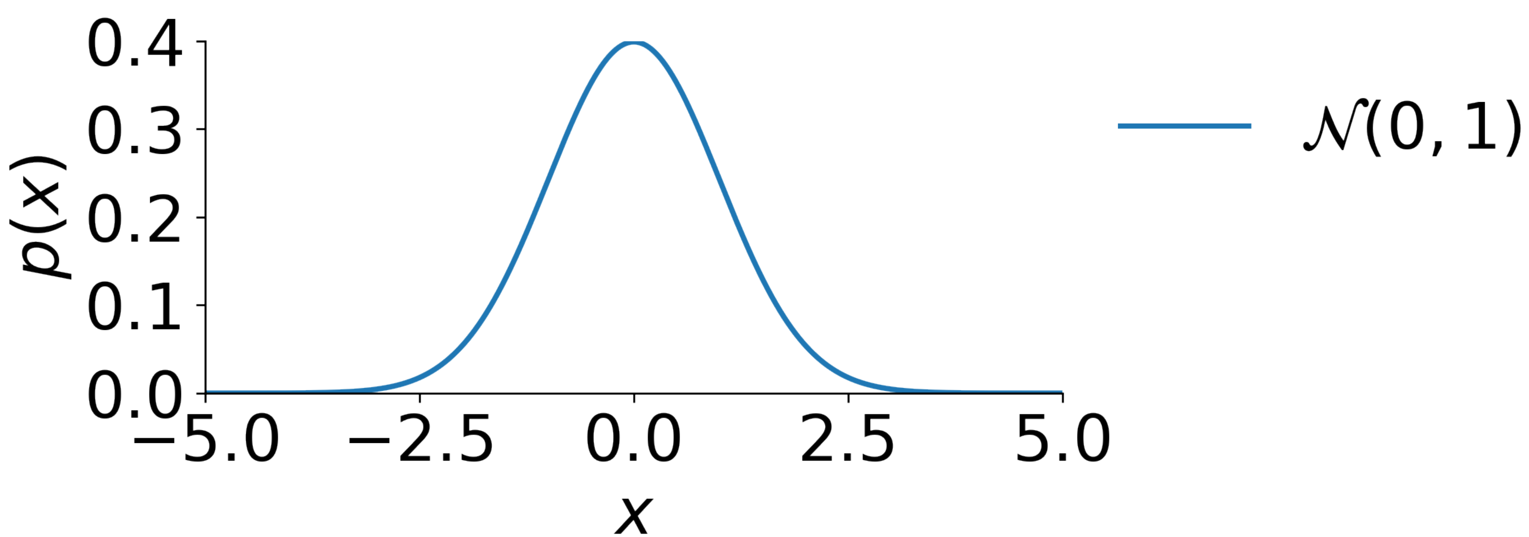


1d sampling (continuous)

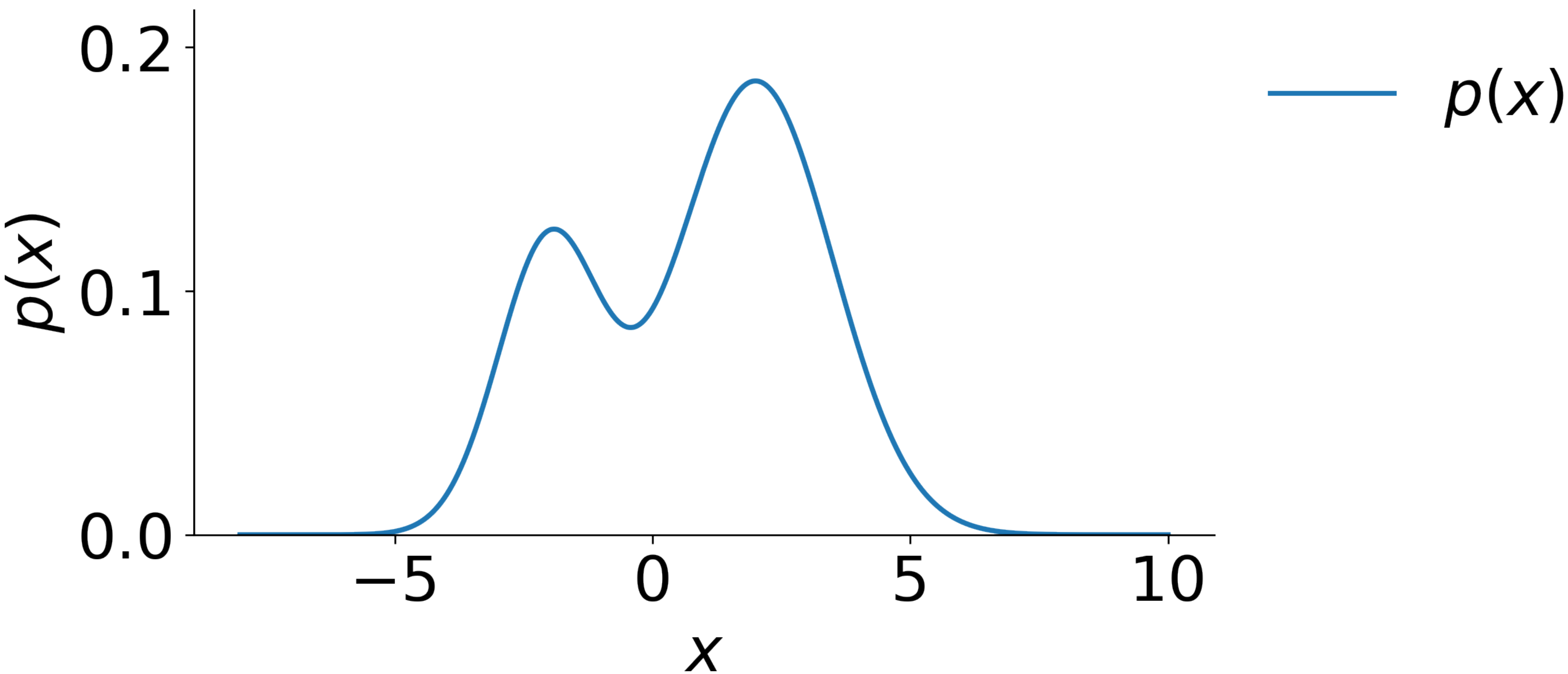
Sampling from Gaussian distribution

Or call library function ☺

z = numpy.random.randn()



1d sampling (continuous)



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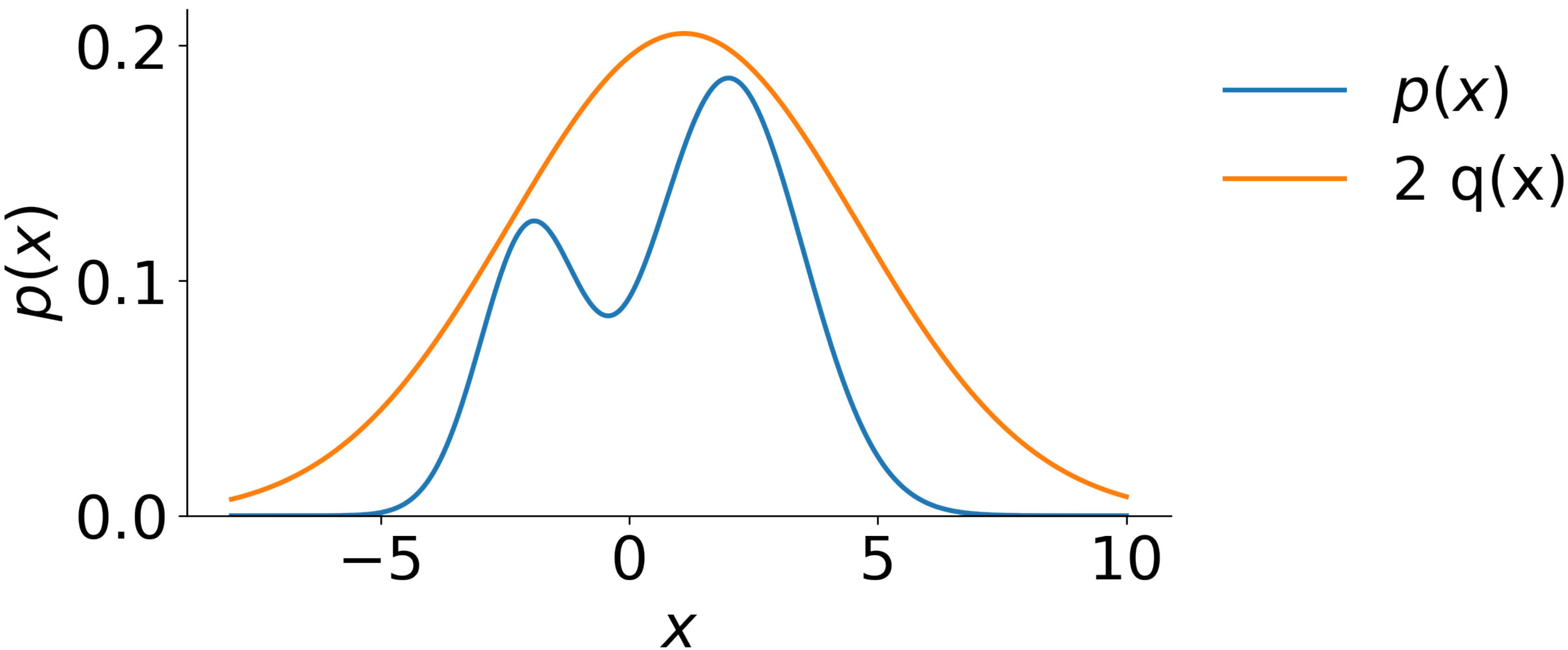
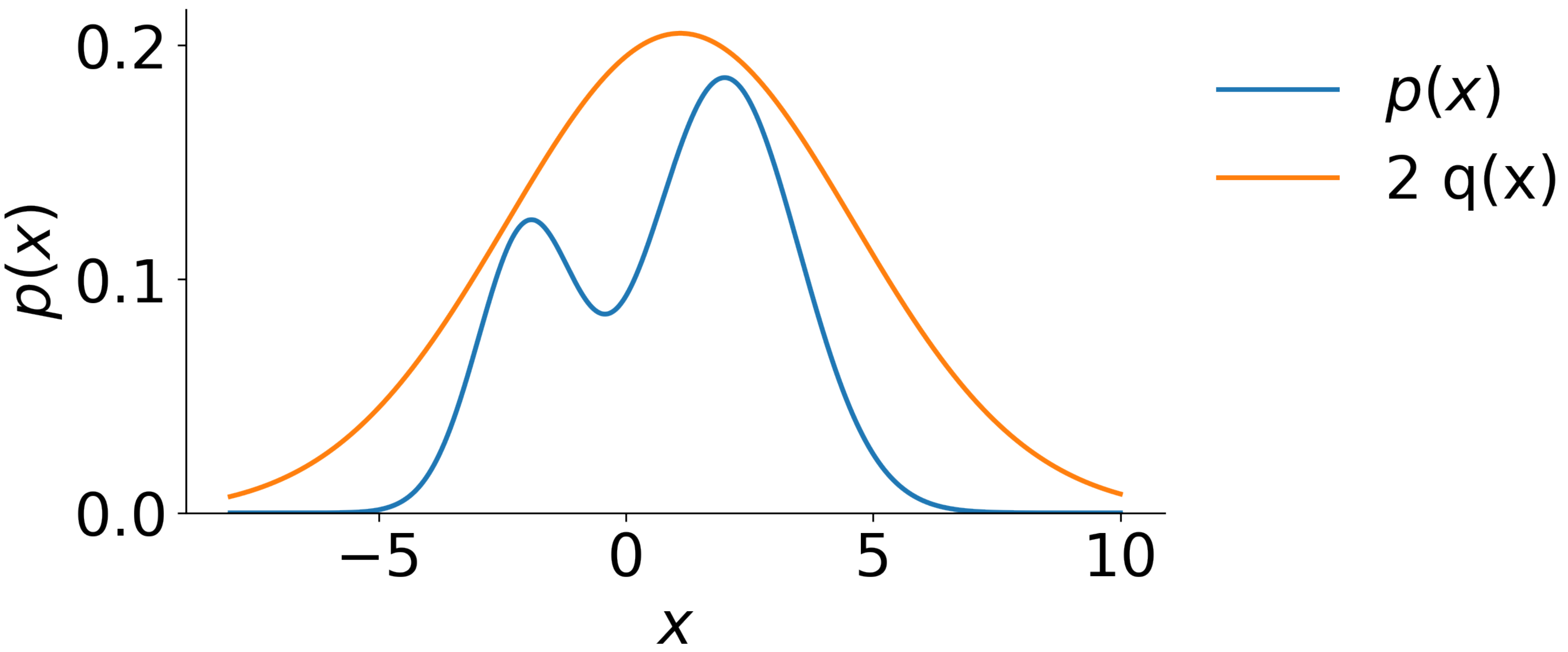


$$q(x) = \mathcal{N}(1, 3^2)$$

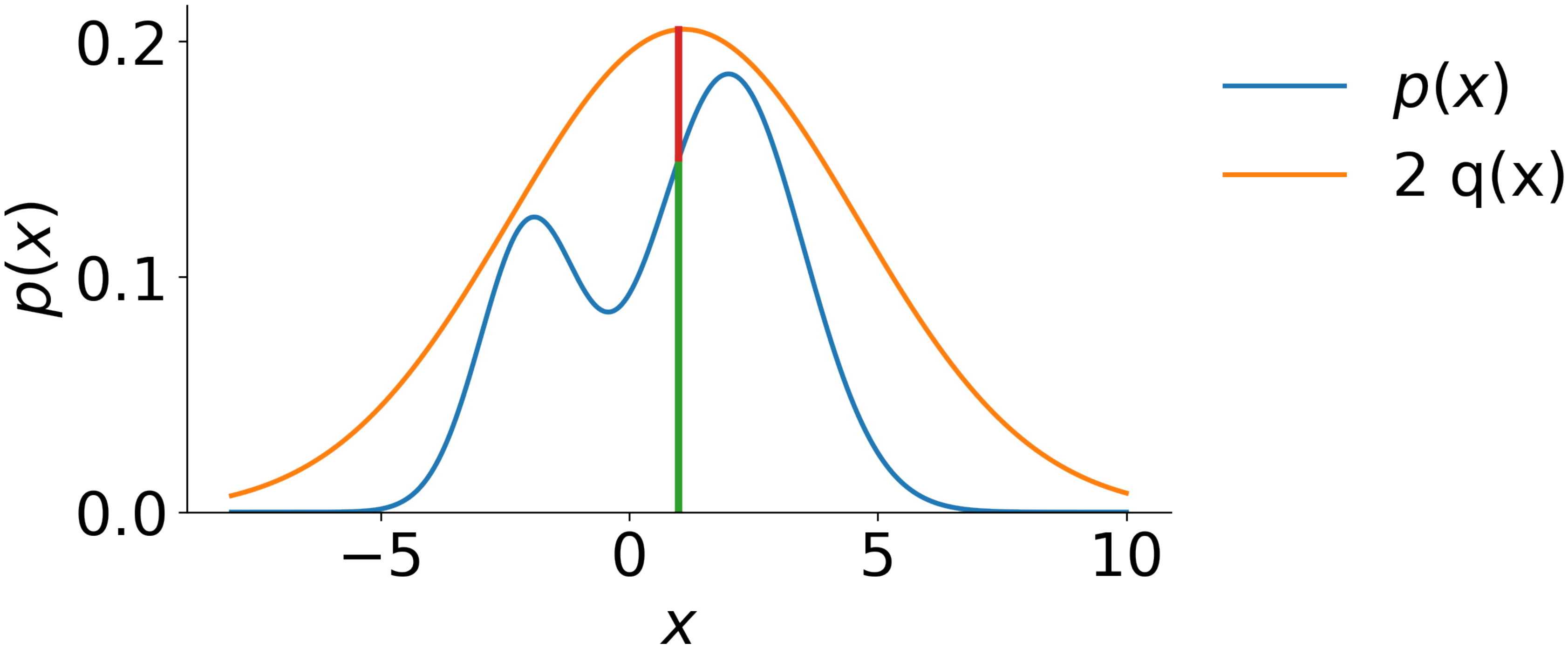
$$p(x) \leq 2q(x)$$

1d sampling (continuous)



$$\widetilde{x} \sim q(x)$$

1d sampling (continuous)



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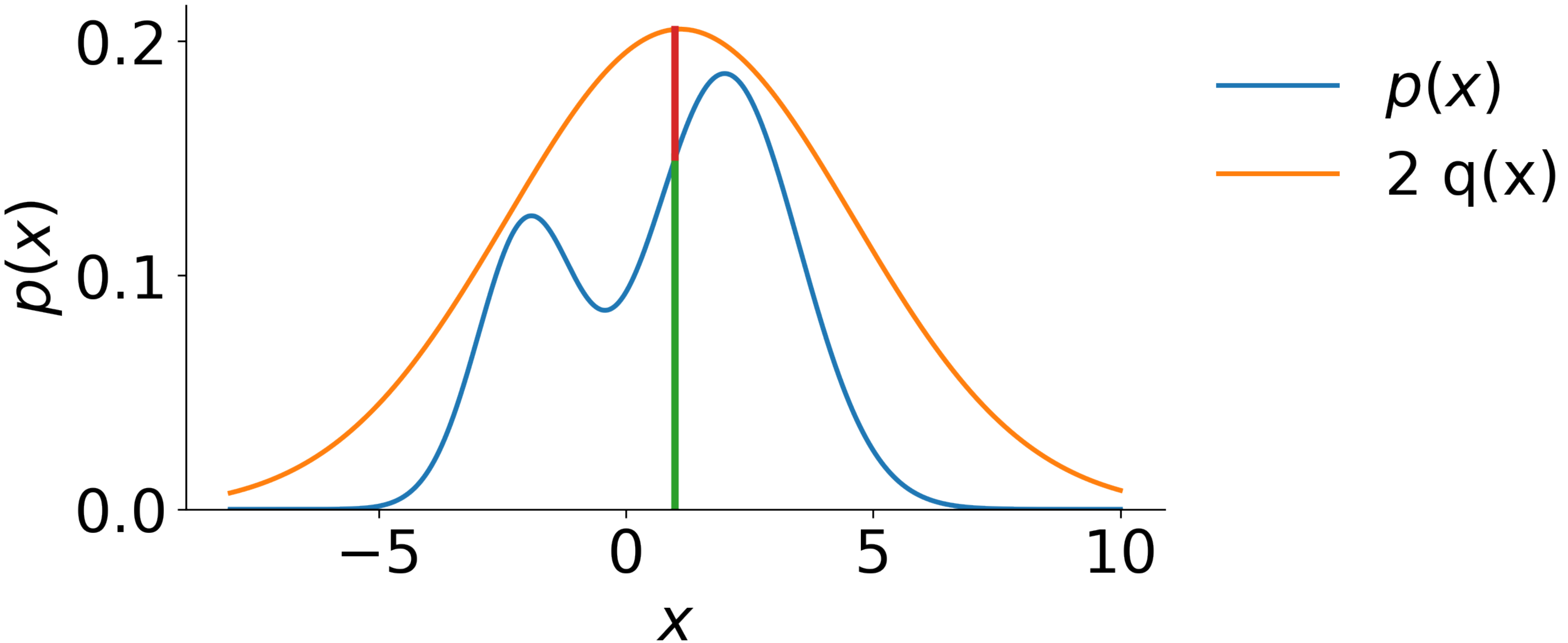
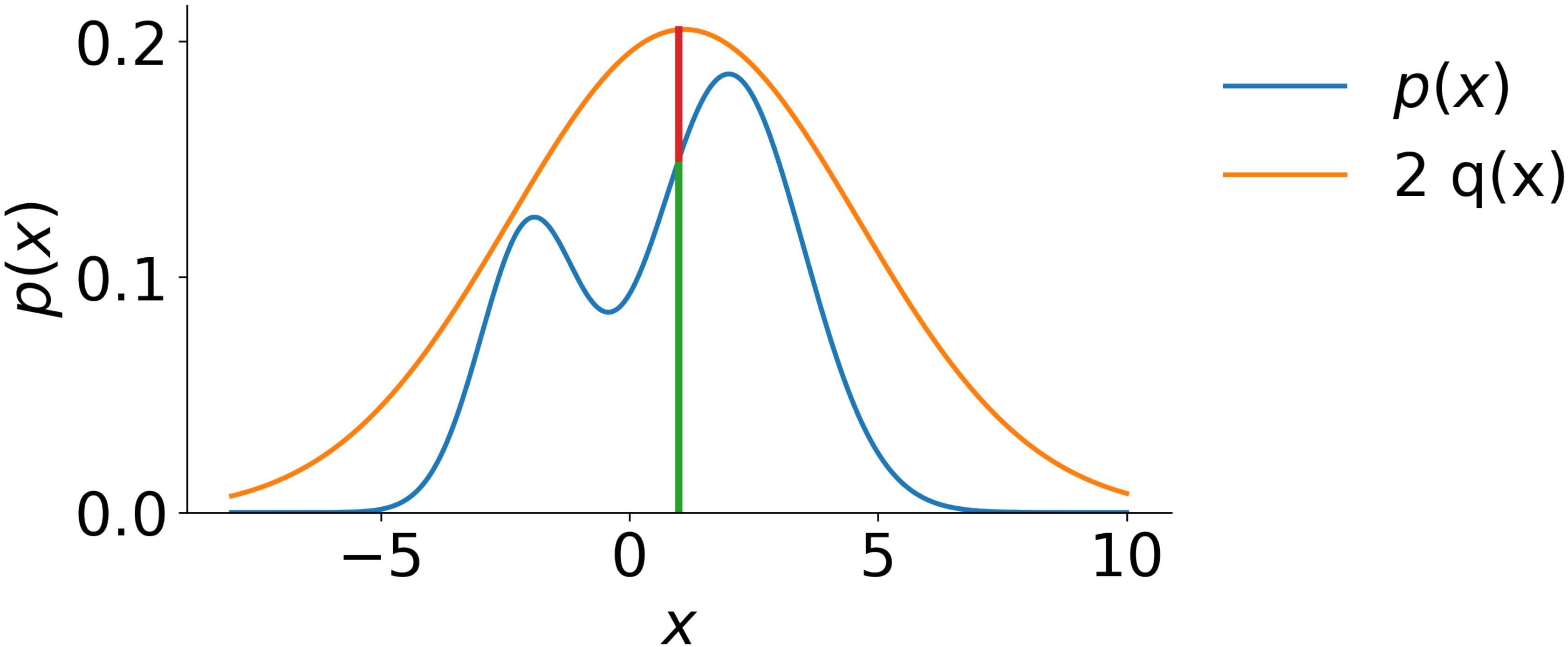


$$\widetilde{x} \sim q(x) \qquad\qquad y \sim \mathcal{U}[0, 2q(\widetilde{x})]$$

Accept $\widetilde{x}$ with probability $\dfrac{p(x)}{2q(x)}$

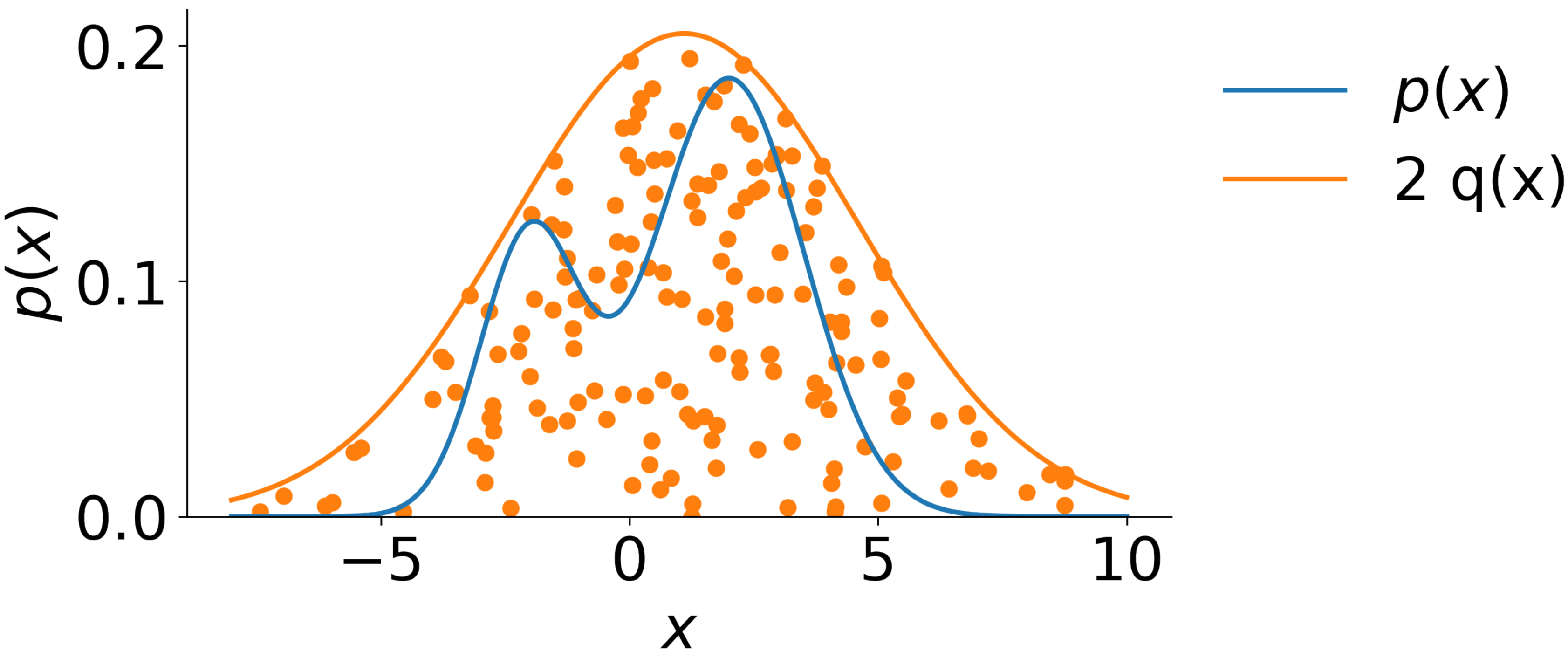
1d sampling (continuous)



$$\widetilde{x} \sim q(x) \qquad\qquad y \sim \mathcal{U}[0, 2q(\widetilde{x})]$$

Accept $\widetilde{x}$ with probability $\dfrac{p(x)}{2q(x)}$: if $y \leq p(x)$

1d sampling (continuous)



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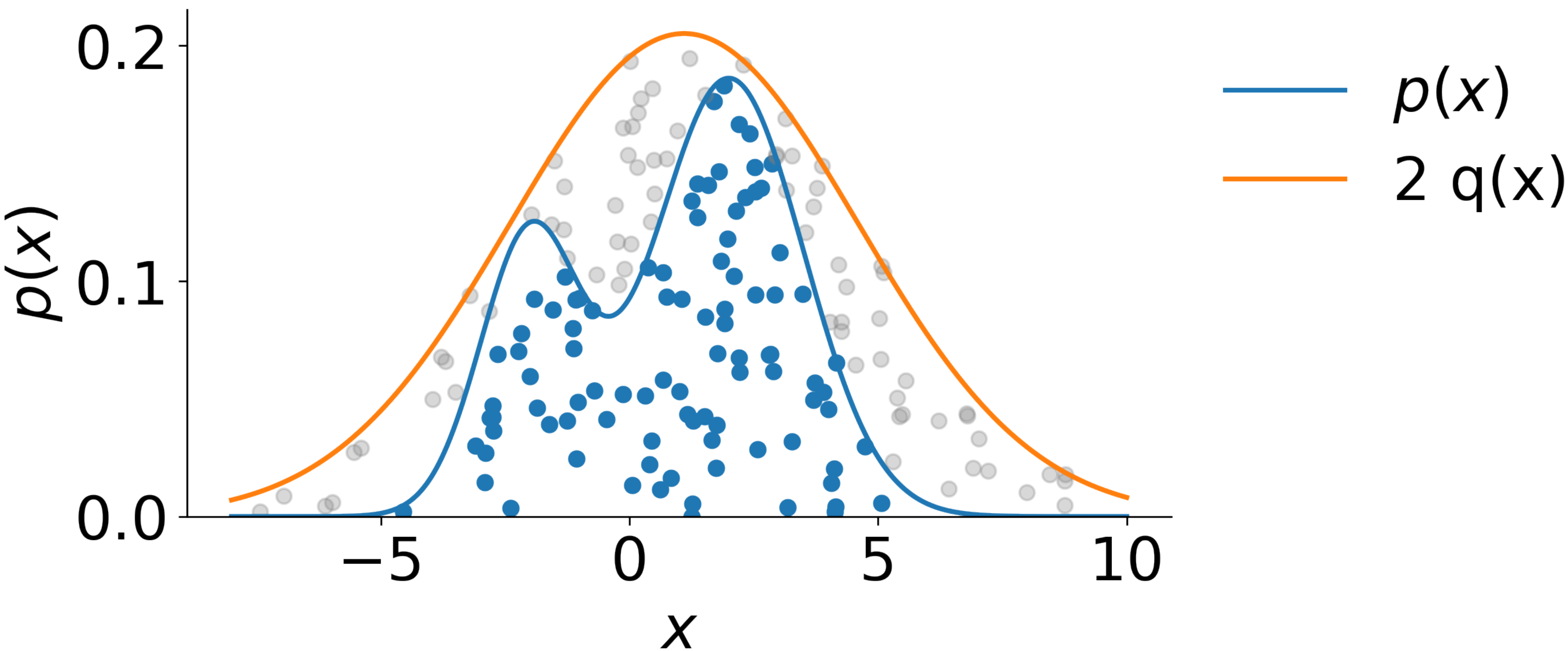

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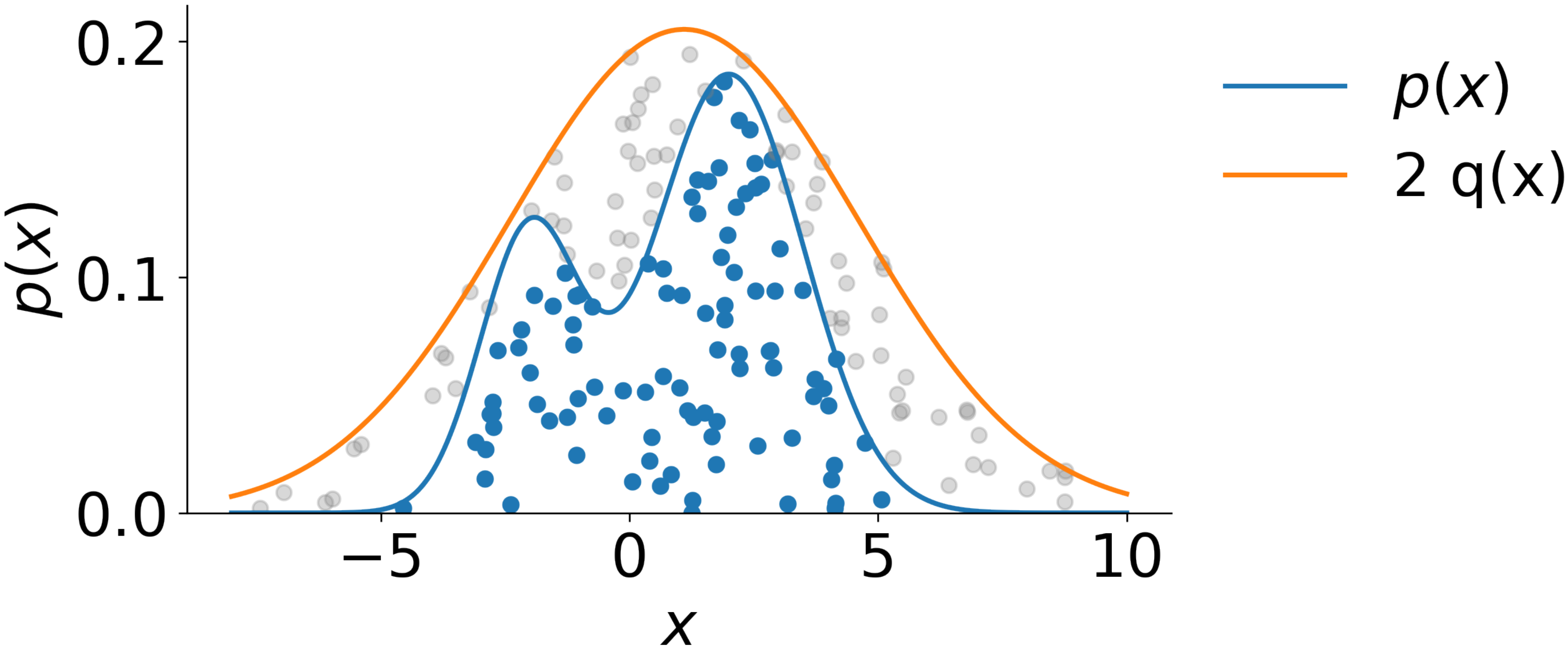
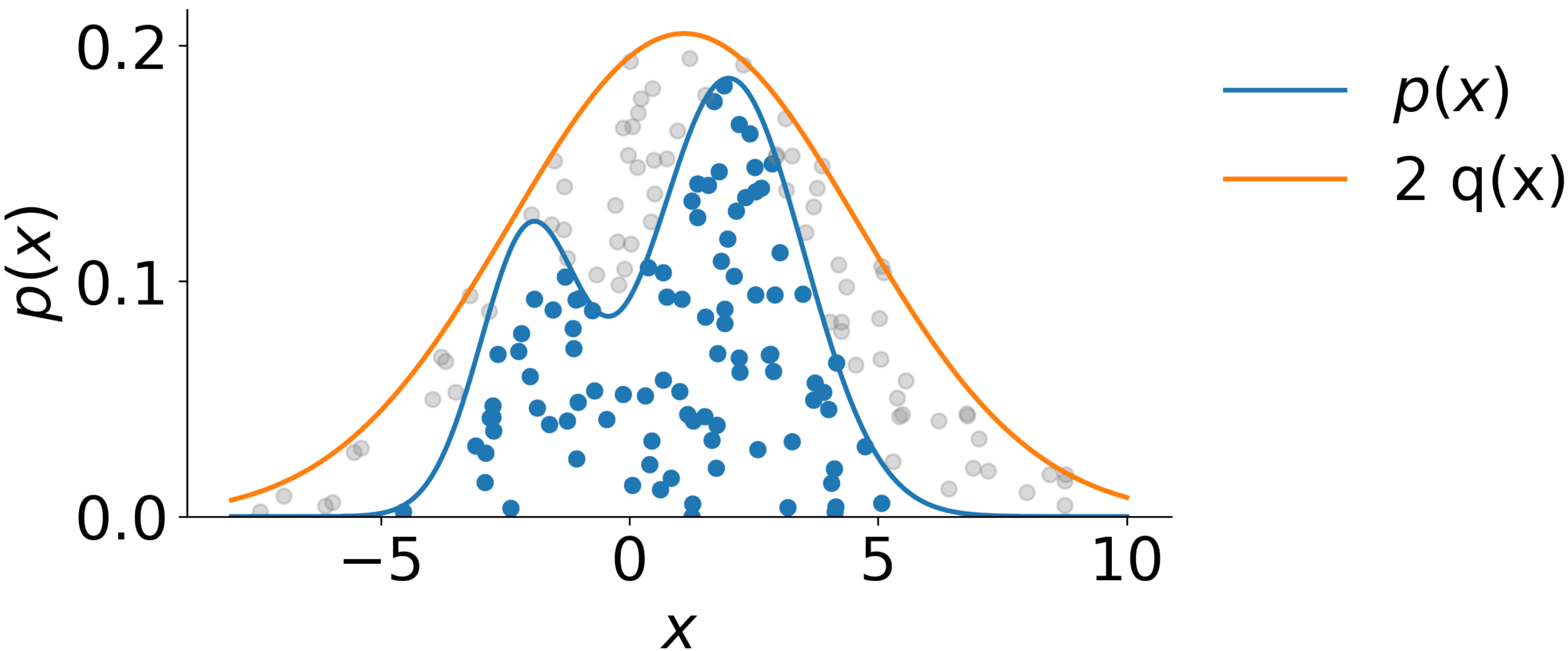


$$p(x) \leq Mq(x)$$

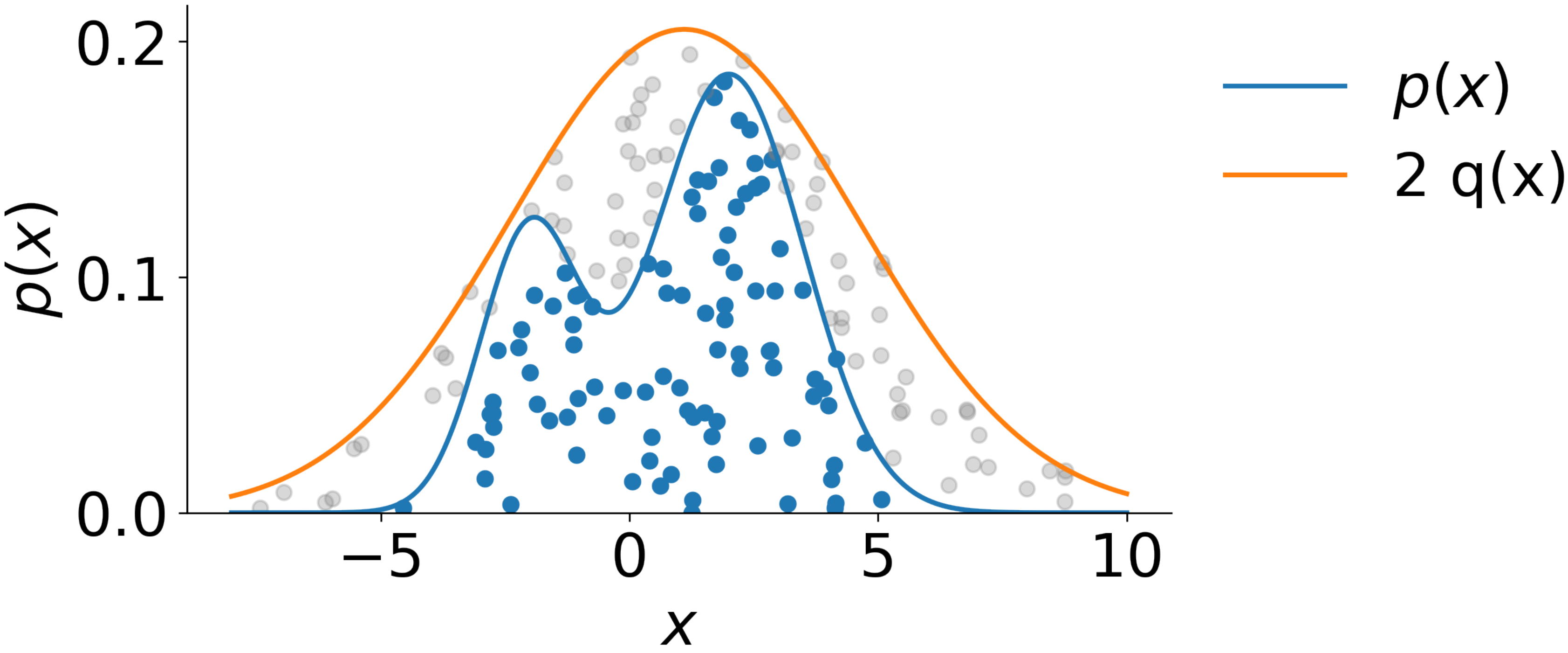
Accepts $\frac{1}{M}$ points on average

1d sampling (continuous)



$$\frac{\widehat{p}(x)}{Z} \leq Mq(x)$$

1d sampling (continuous)



$$\widehat{p}(x) \leq \underbrace{ZM}_{\widetilde{M}} q(x)$$

Summary

Pros:

- Works for most distributions (even unnormalized)

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- Works for most distributions (even unnormalized)

Cons:

- If q and p are too different (M is large), rejects most of the points
- M is large for d-dimensional distributions