M-step details

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(Usually) concave function w.r.t. θ, easy to optimize

Expectation Maximization algorithm

For k = 1, …

E-step

$$q^{k+1} = \arg\min_q \mathcal{KL}\left[q(T) \parallel p(T \mid X, \theta^k)\right]$$

$$\Leftrightarrow$$

$$q^{k+1}(t_i) = p(t_i \mid x_i, \theta^k)$$

M-step

$$\theta^{k+1} = \arg\max_\theta \mathbb{E}_{q^{k+1}} \log p(X, T \mid \theta)$$

Convergence guaranties

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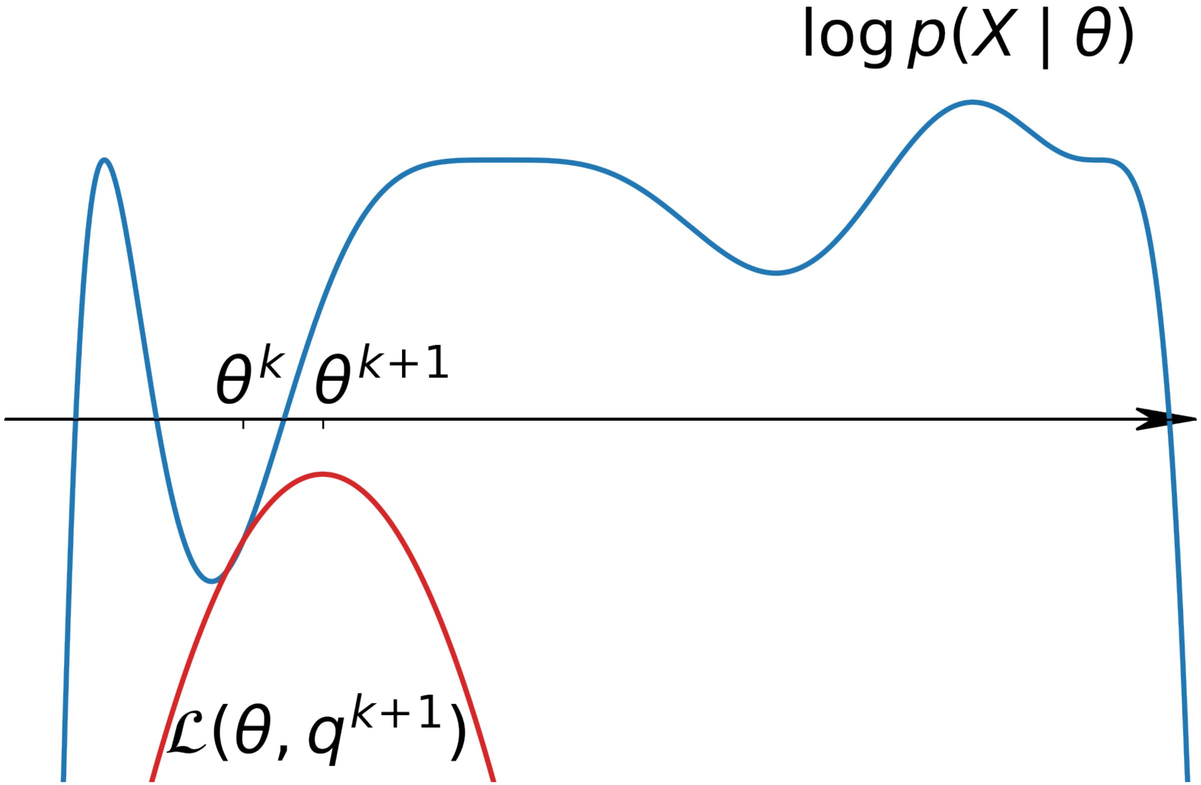
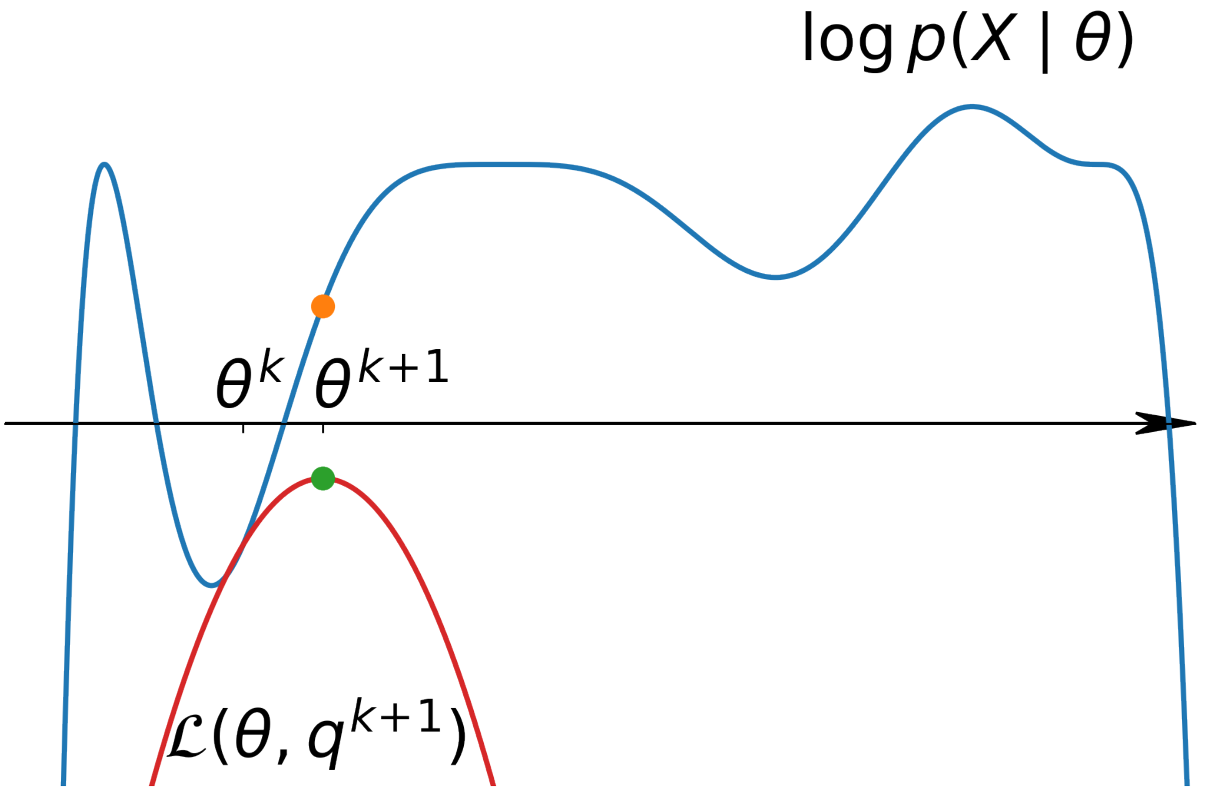


Convergence guaranties

$$\log p(X \mid \theta^{k+1}) \geq \mathcal{L}(\theta^{k+1}, q^{k+1})$$



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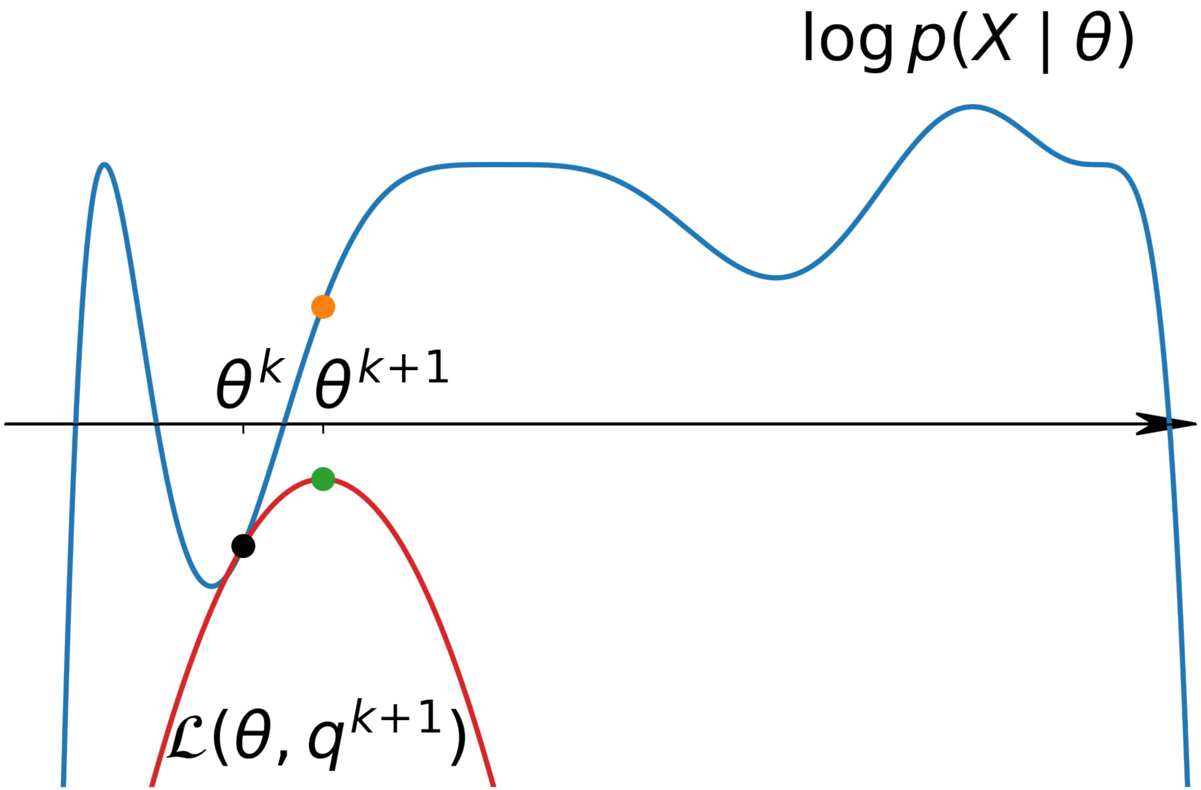
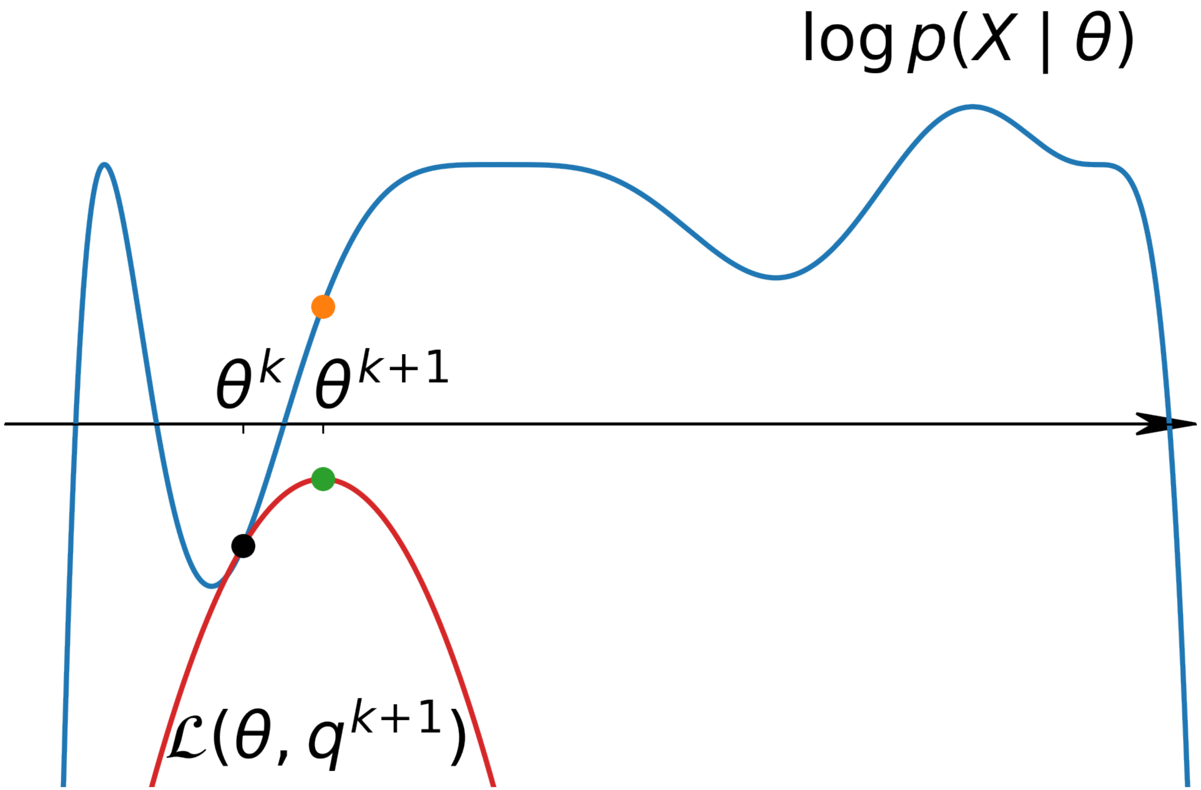


Convergence guaranties

$$\log p(X \mid \theta^{k+1}) \geq \mathcal{L}(\theta^{k+1}, q^{k+1}) \geq \mathcal{L}(\theta^{k}, q^{k+1}) = \log p(X \mid \theta^{k})$$



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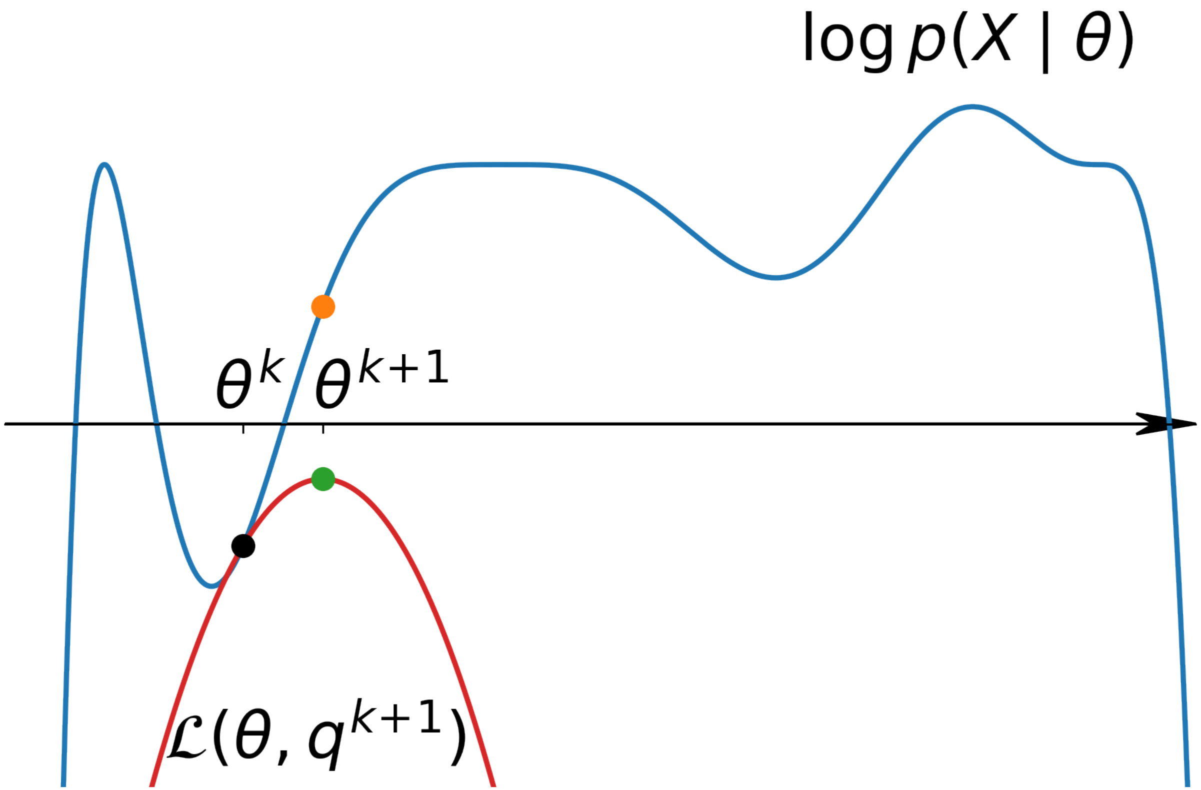

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- On each iteration EM doesn't decrease the objective (good for debugging!)
- Guarantied to converge to a local maximum (or saddle point)