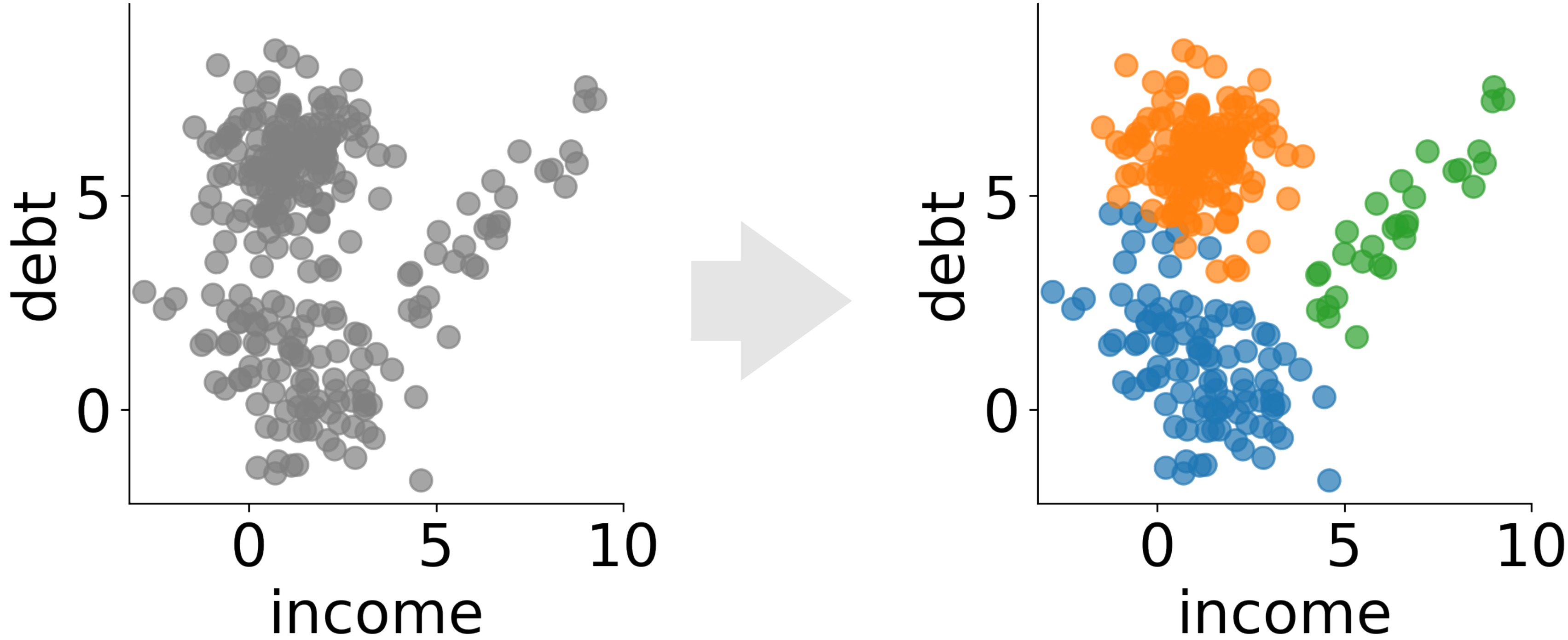
K-Means connection



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   b) Update centroids

$$\mu_c = \frac{\sum_{i:c_i=c} x_i}{\#\{i : c_i = c\}}$$

K-Means from GMM perspective

From GMM to K-means:

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$$p(x_i \mid t_i = c, \theta) = \frac{1}{Z} \exp\left(-0.5\|x_i - \mu_c\|^2\right)$$

K-Means from EM perspective

E-step

$$q^{k+1} = \arg\min_q \mathcal{KL}\left[q(T) \parallel p(T \mid X, \theta^k)\right]$$

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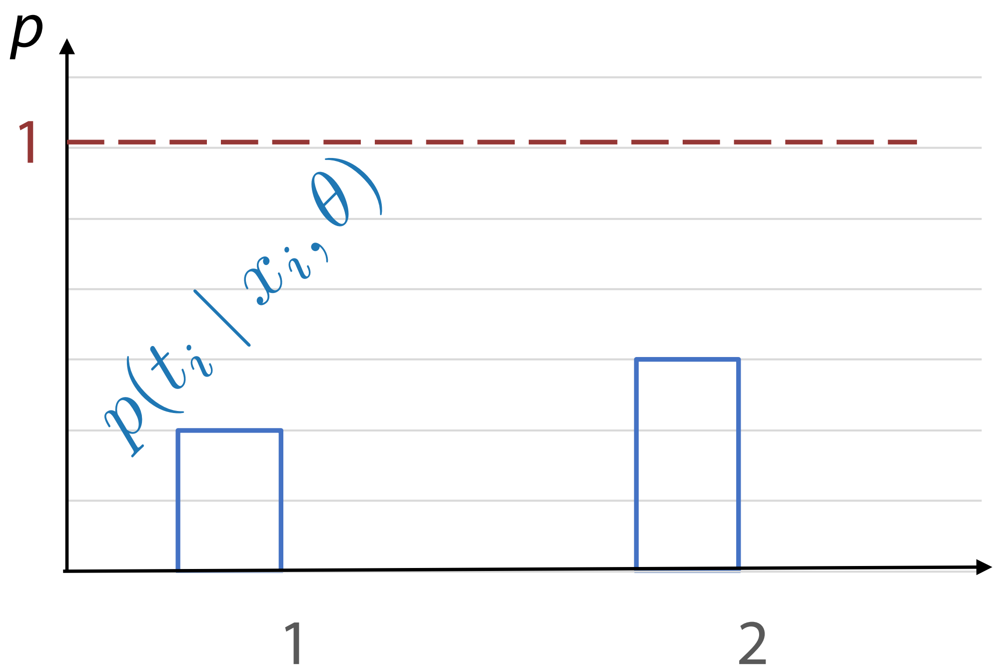

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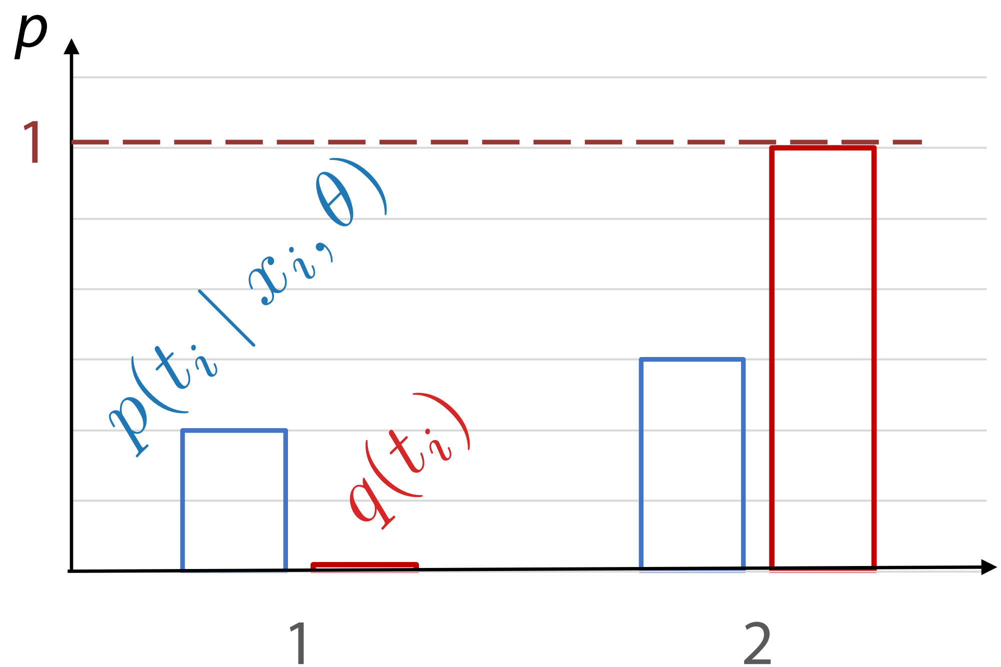

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Exactly like in K-Means!