

**Regression metrics:  
(R)MSPE, MAPE, (R)MSLE**

# Plan for the video

## 1) Regression

- MSE, RMSE, R-squared
- MAE
- (R)MSPE, MAPE
- (R)MSLE

## 2) Classification:

- Accuracy, LogLoss, AUC
- Cohen's (Quadratic weighted) Kappa

# From MSE and MAE to MSPE and MAPE

- **Shop 1:** predicted 9, sold 10, MSE = 1
- **Shop 2:** predicted 999, sold 1000, MSE = 1

# From MSE and MAE to MSPE and MAPE

- **Shop 1:** predicted 9, sold 10, MSE = 1
- **Shop 2:** predicted 999, sold 1000, MSE = 1
  
- **Shop 1:** predicted 9, sold 10, MSE = 1
- **Shop 2:** predicted 900, sold 1000, MSE = 10000
  
- **Shop 1:** predicted 9, sold 10, relative\_metric = 1
- **Shop 2:** predicted 900, sold 1000, relative\_metric = 1

# From MSE and MAE to MSPE and MAPE

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$

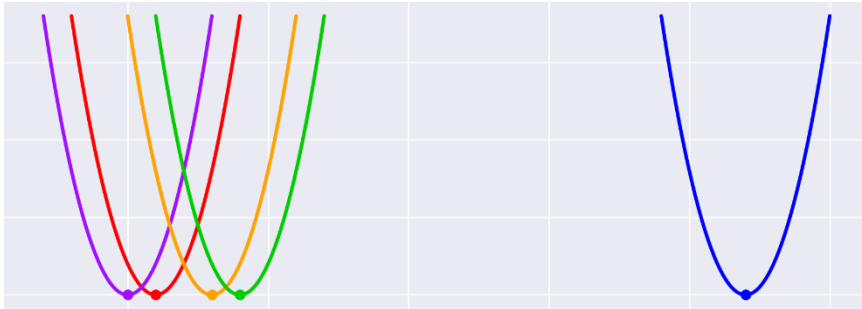


$$\text{MAE} = \frac{1}{N} \sum_{i=1}^N |y_i - \hat{y}_i|$$

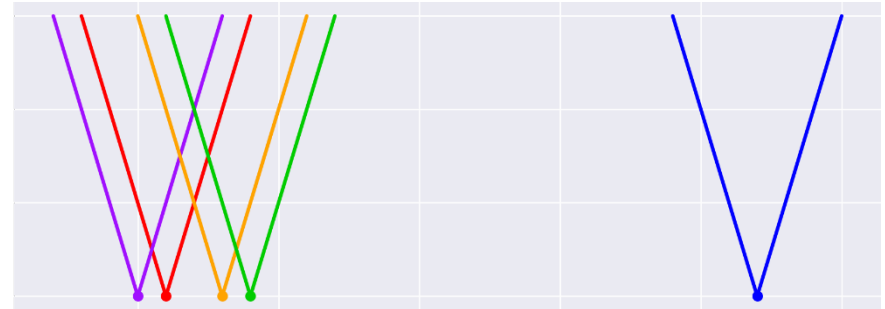


# From MSE and MAE to MSPE and MAPE

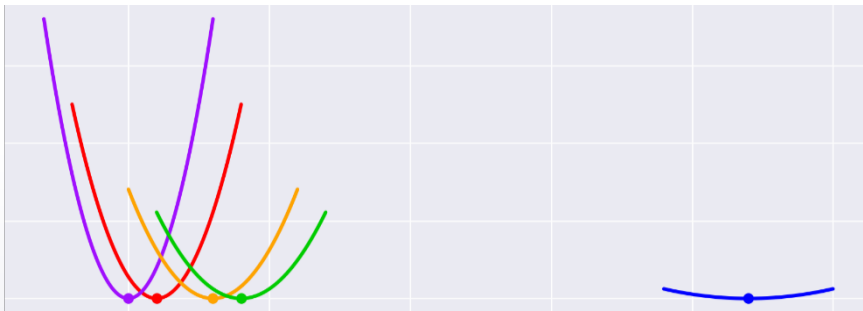
$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$



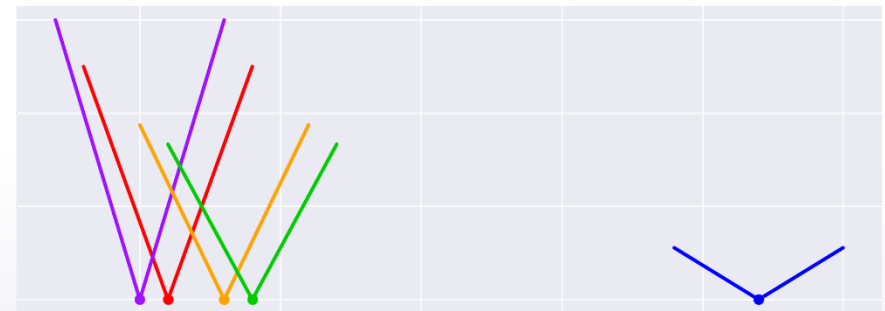
$$\text{MAE} = \frac{1}{N} \sum_{i=1}^N |y_i - \hat{y}_i|$$



$$\text{MSPE} = \frac{100\%}{N} \sum_{i=1}^N \left( \frac{y_i - \hat{y}_i}{y_i} \right)^2$$



$$\text{MAPE} = \frac{100\%}{N} \sum_{i=1}^N \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$



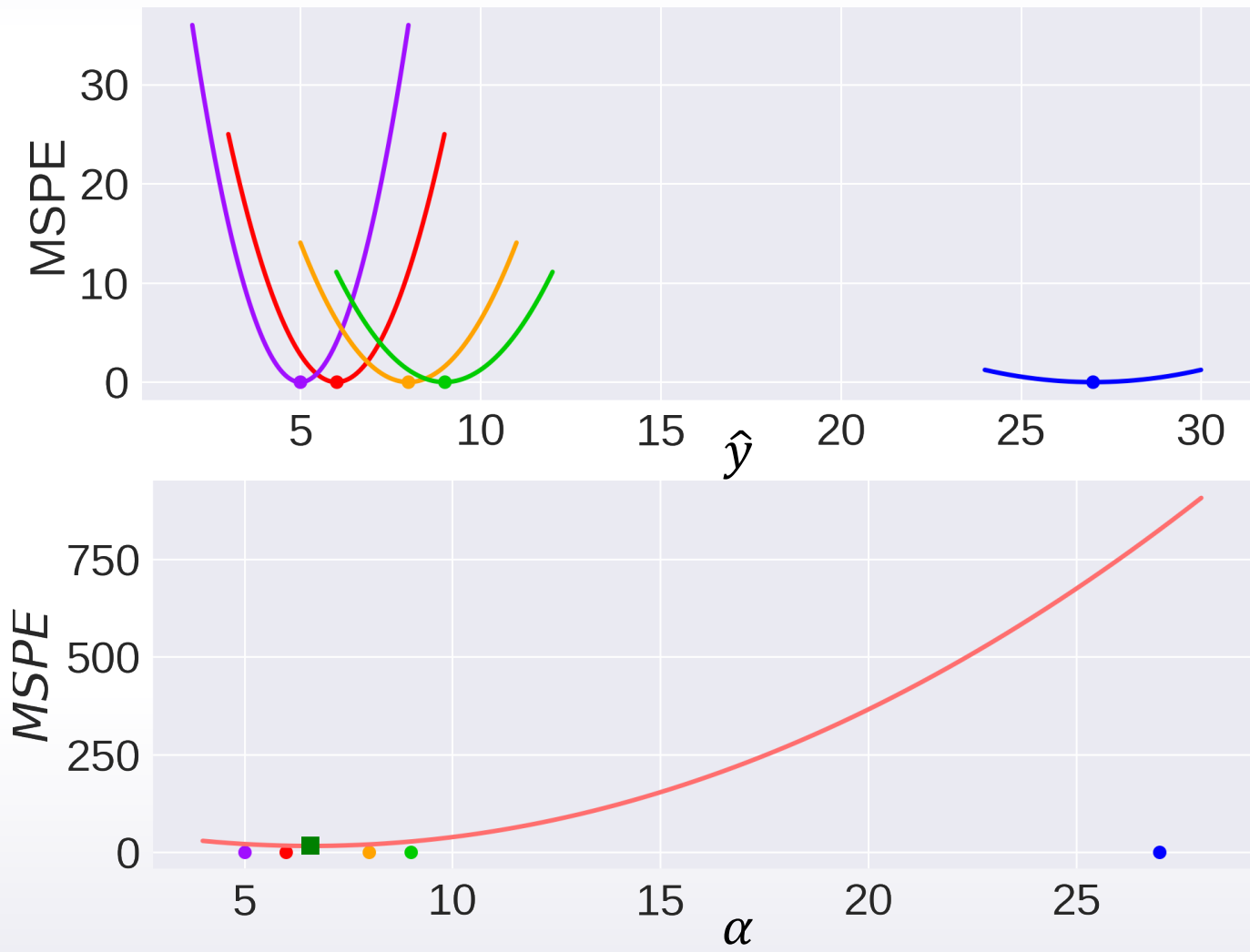
# MSPE: constant

$$\text{MSPE} = \frac{100\%}{N} \sum_{i=1}^N \left( \frac{y_i - \alpha}{y_i} \right)^2$$

Best constant:  
weighted target mean

Data:

X	Y
-1	4
1	3
-2	6
3	7
3	25



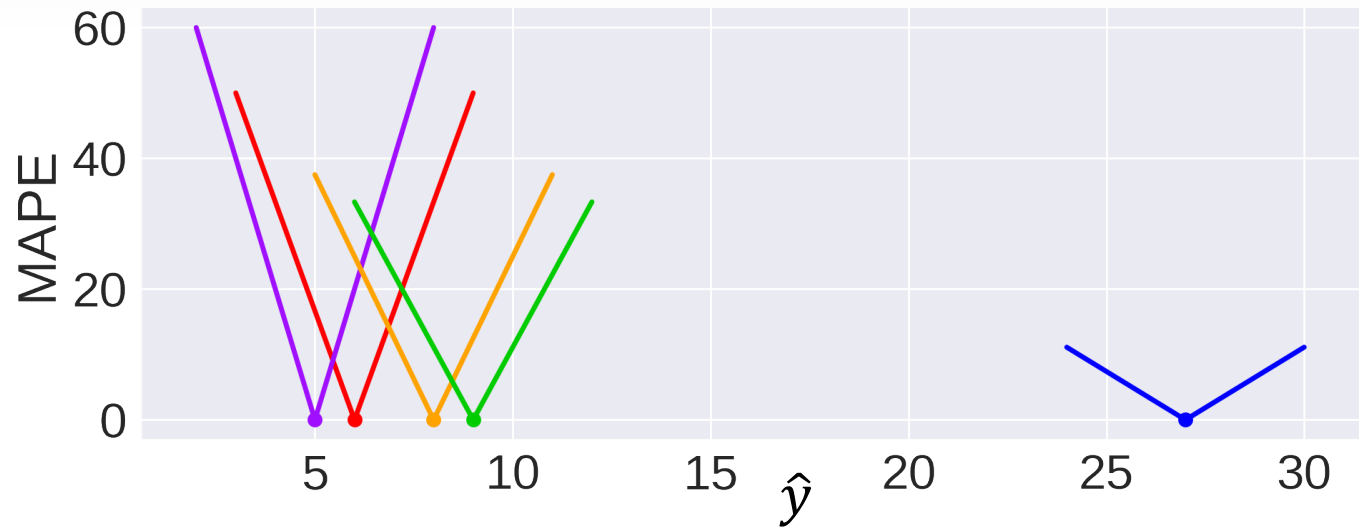
# MAPE: constant

$$\text{MAPE} = \frac{100\%}{N} \sum_{i=1}^N \left| \frac{y_i - \alpha}{y_i} \right|$$

Best constant:  
?

Data:

X	Y
-1	4
1	3
-2	6
3	7
3	25





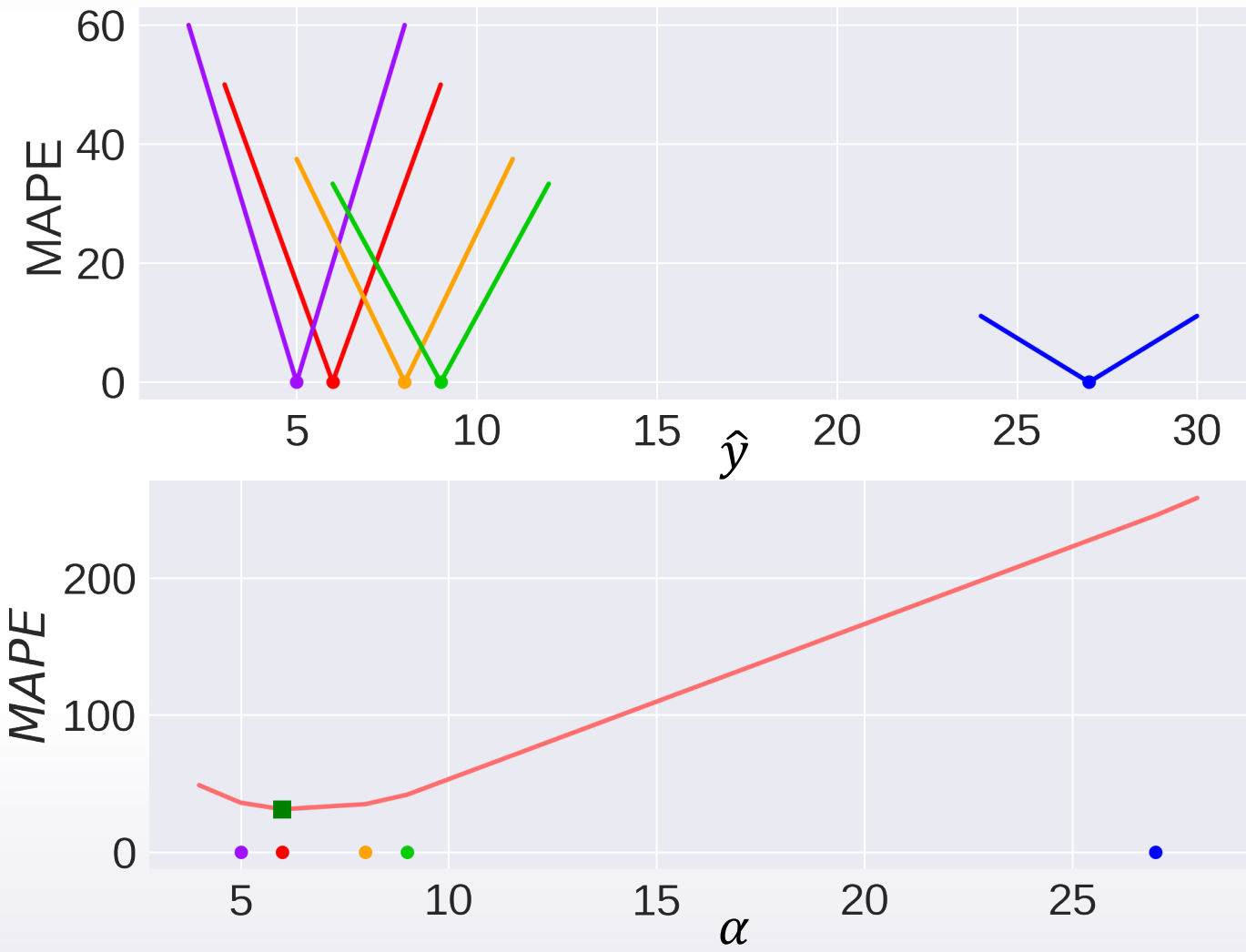
# MAPE: constant

$$\text{MAPE} = \frac{100\%}{N} \sum_{i=1}^N \left| \frac{y_i - \alpha}{y_i} \right|$$

Best constant:  
weighted target median

Data:

X	Y
-1	4
1	3
-2	6
3	7
3	25



# (R)MSLE: Root Mean Square Logarithmic Error

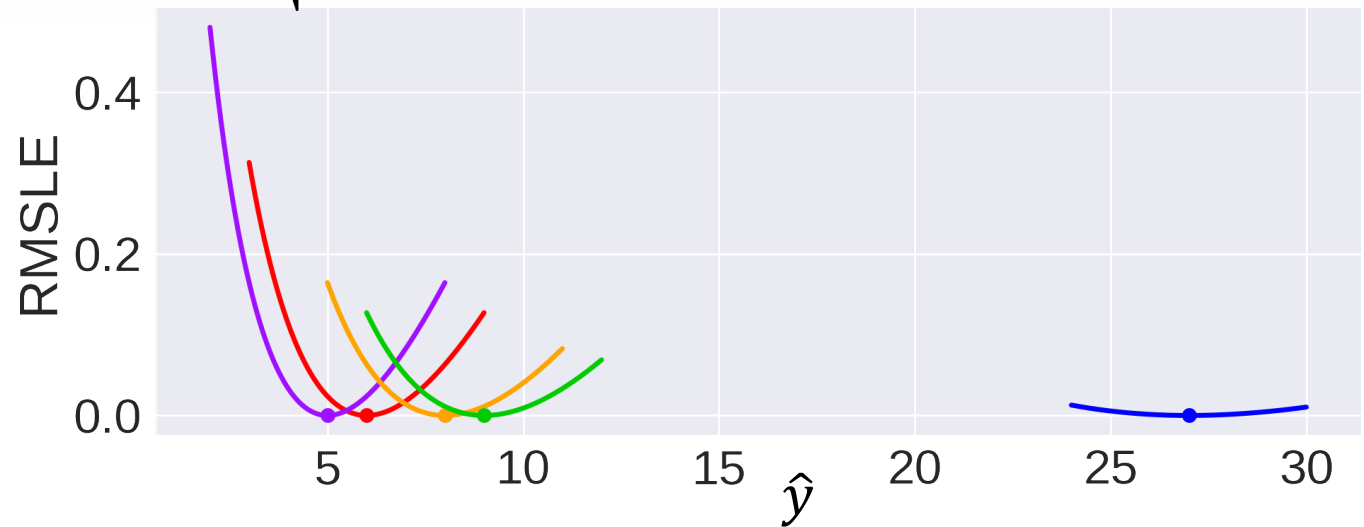
$$\begin{aligned}\text{RMSLE} &= \sqrt{\frac{1}{N} \sum_{i=1}^N (\log(y_i + 1) - \log(\hat{y}_i + 1))^2} = \\ &= \text{RMSE}(\log(y_i + 1), \log(\hat{y}_i + 1)) = \\ &= \sqrt{\text{MSE}(\log(y_i + 1), \log(\hat{y}_i + 1))}\end{aligned}$$

# (R)MSLE: Root Mean Square Logarithmic Error

$$\text{RMSLE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\log(y_i + 1) - \log(\hat{y}_i + 1))^2}$$

Data:

X	Y
-1	4
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## (R)MSLE: constant

$$\begin{aligned}\text{RMSLE} &= \sqrt{\frac{1}{N} \sum_{i=1}^N (\log(y_i + 1) - \log(\alpha + 1))^2} = \\ &= \text{RMSE}(\log(y_i + 1), \log(\alpha + 1)) = \\ &= \sqrt{\text{MSE}(\log(y_i + 1), \log(\alpha + 1))}\end{aligned}$$

## (R)MSLE: constant

$$\begin{aligned}\text{RMSLE} &= \sqrt{\frac{1}{N} \sum_{i=1}^N (\log(y_i + 1) - \log(\alpha + 1))^2} = \\ &= \text{RMSE}(\log(y_i + 1), \log(\alpha + 1)) = \\ &= \sqrt{\text{MSE}(\log(y_i + 1), \log(\alpha + 1))}\end{aligned}$$

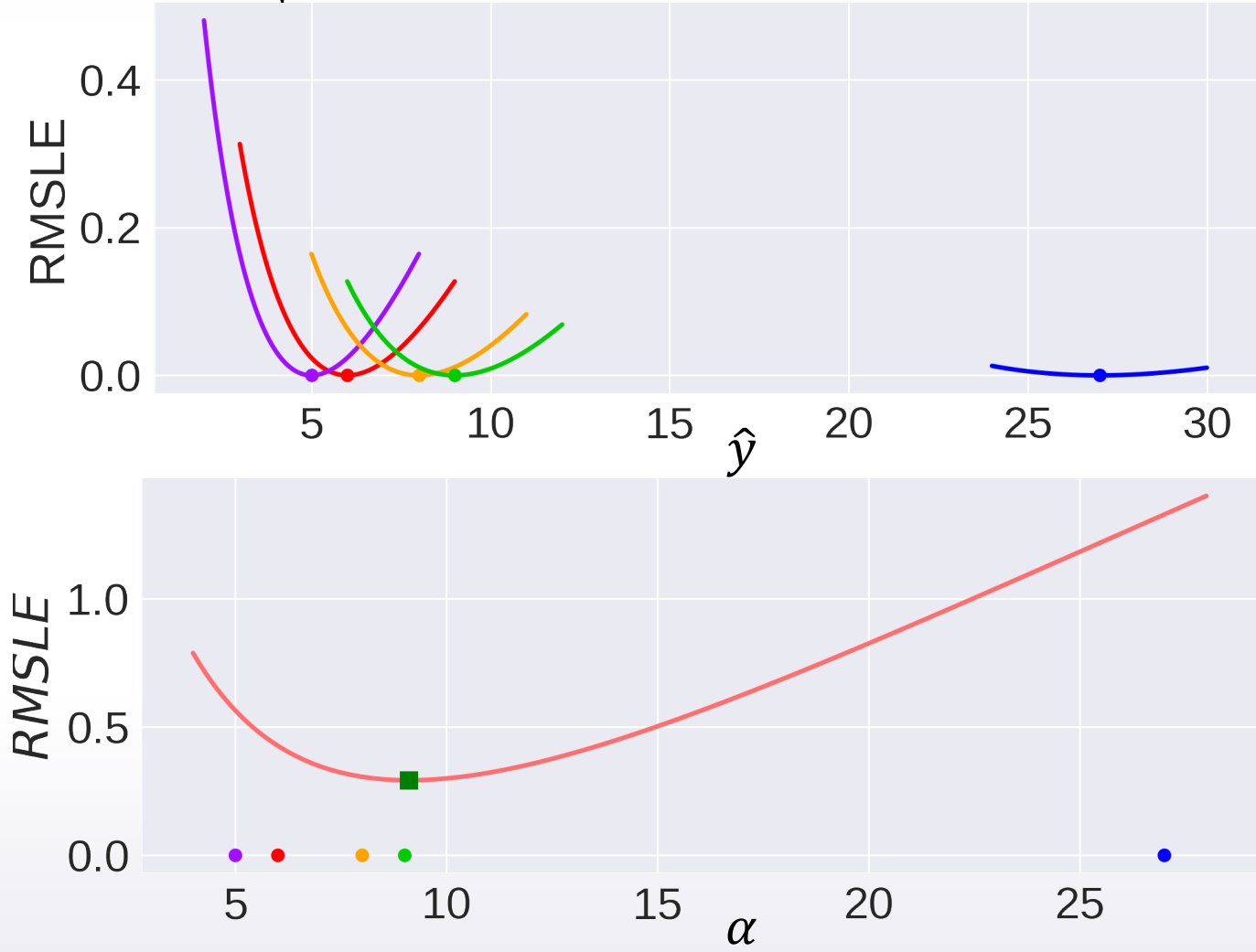
- Best constant in log space is a mean target value
- We need to exponentiate it to get an answer

# (R)MSLE: constant

$$\text{RMSLE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\log(y_i + 1) - \log(\alpha + 1))^2}$$

Data:

X	Y
-1	4
1	3
-2	6
3	7
3	25



# Compare the constants

Metric	Constant
MSE	11
RMSLE	9.11
MAE	8
MSPE	6.6
MAPE	6

# Conclusion

Discussed the metrics, sensitive to relative errors:

- **(R)MSPE**
  - Weighted version of MSE
- **MAPE**
  - Weighted version of MAE
- **(R)MSLE**
  - MSE in log space