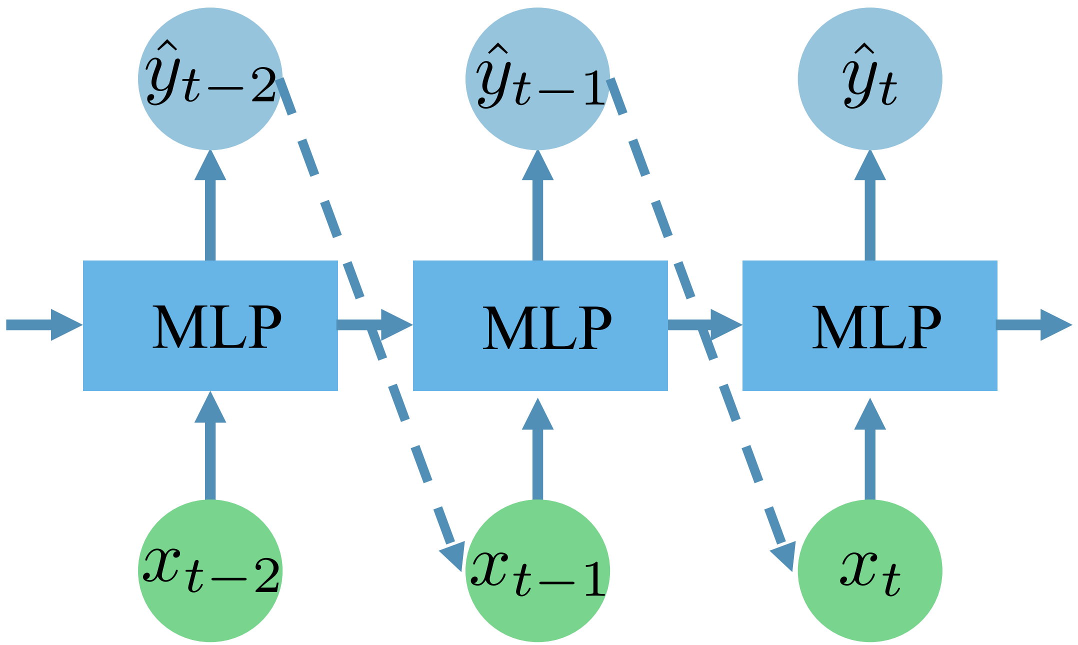
Previously on this week: Recurrent Architecture

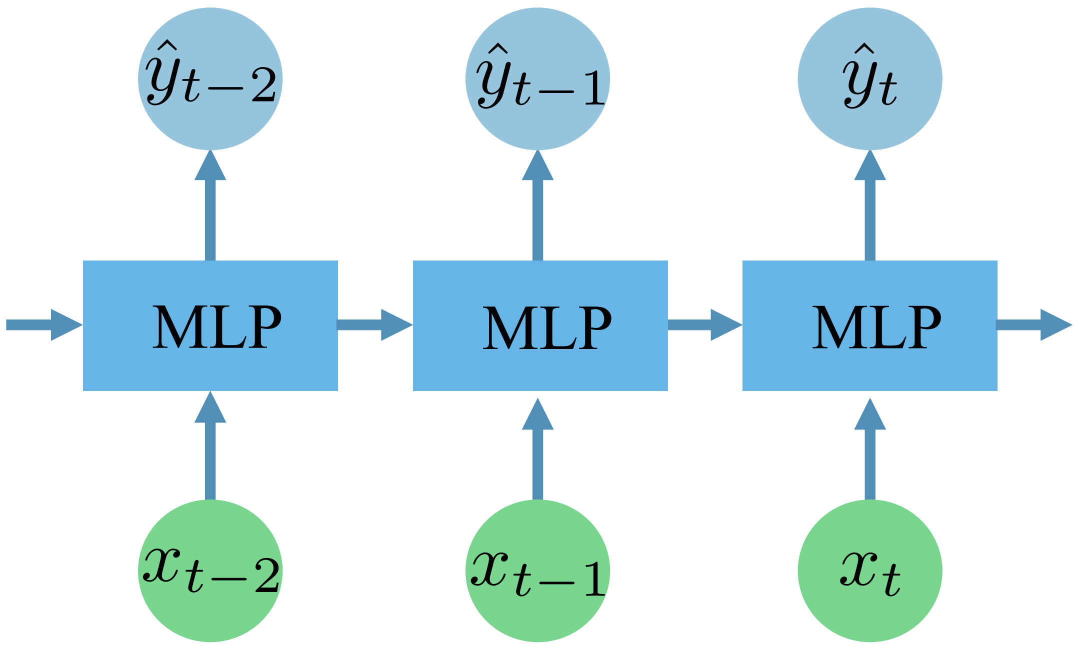


Language model:

x - word embedding

$\hat{y}$ - probability distribution for the next word

Previously on this week: Recurrent Architecture

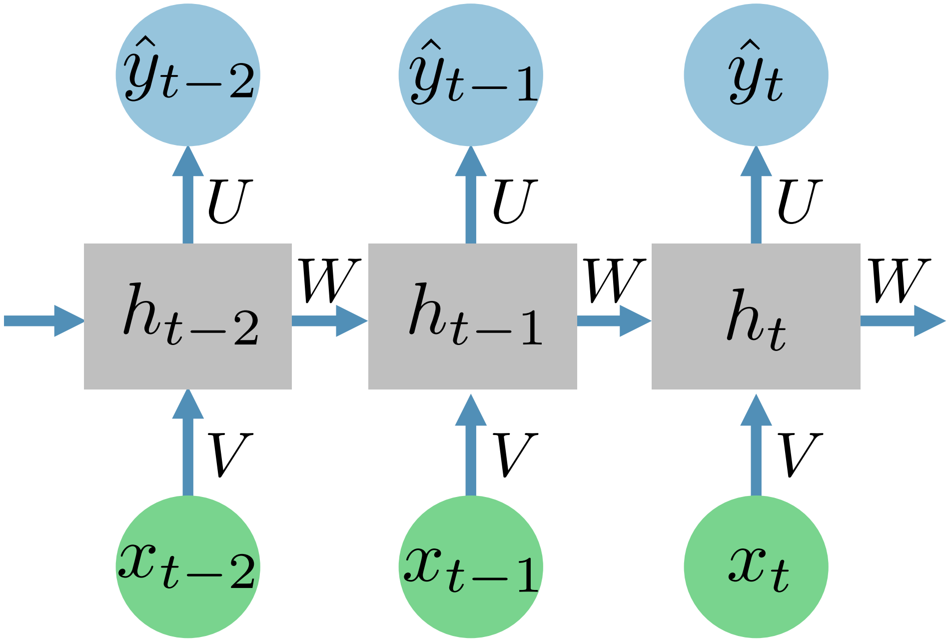


POS tagging:

x - word embedding

$\hat{y}$ - probability distribution for a POS tag of the current word

Recurrent Neural Network (RNN)

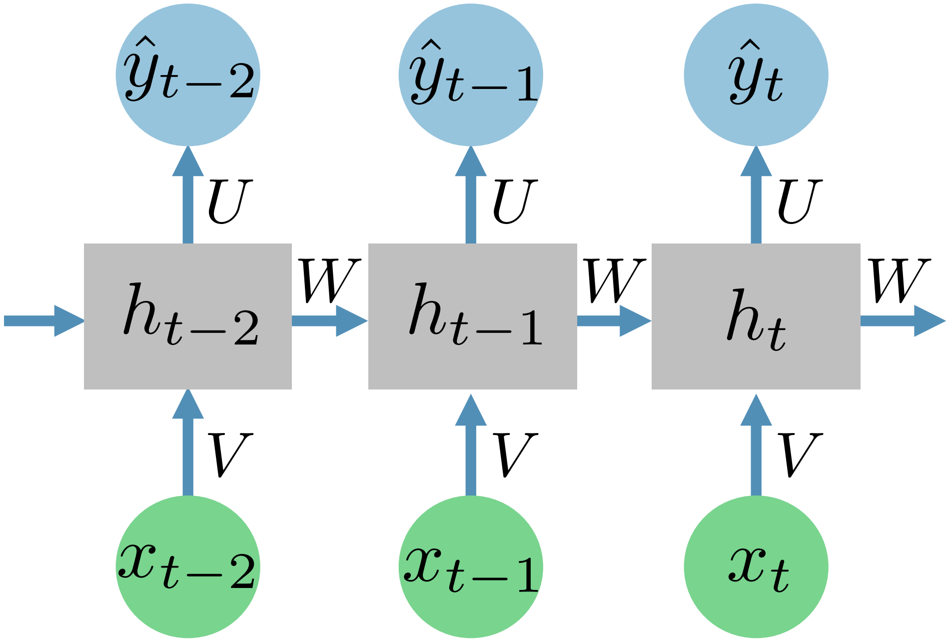


x - input

$\hat{y}$ - output (prediction)

h - hidden state

Recurrent Neural Network (RNN)



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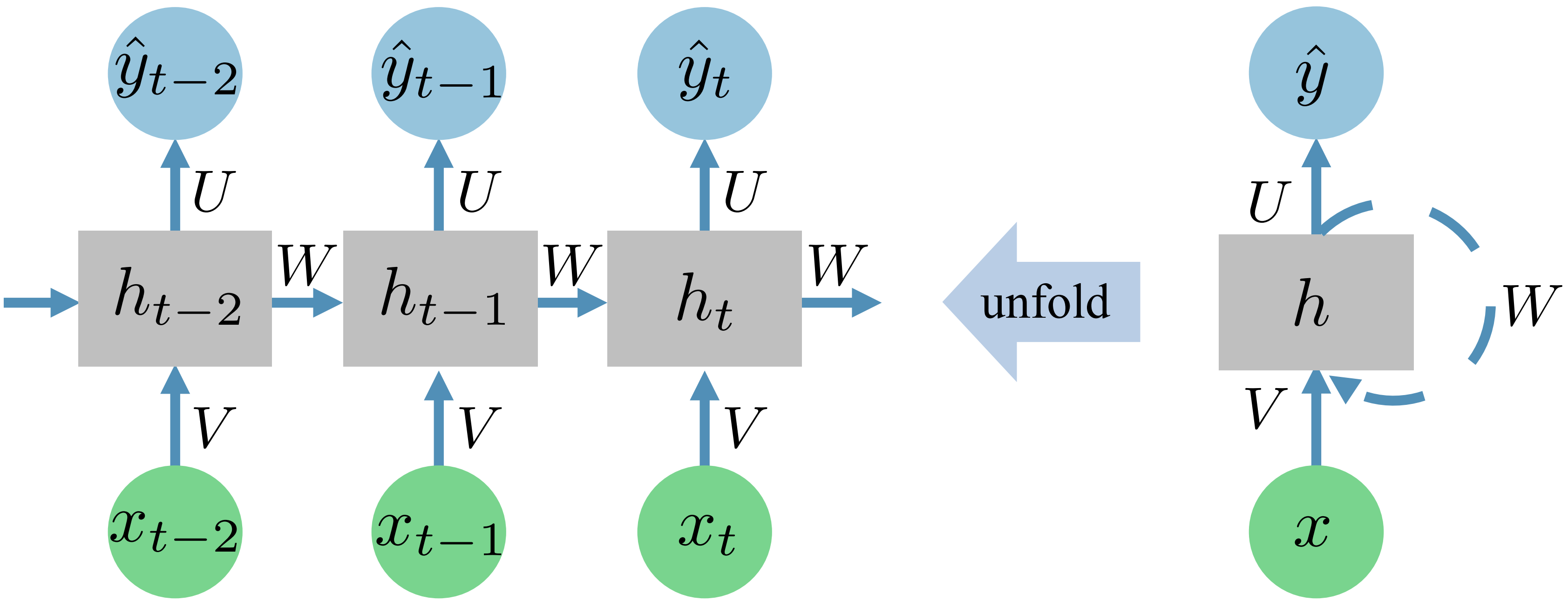
h - hidden state

$$h_t = f_h(Vx_t + Wh_{t-1} + b_h)$$

$$\hat{y}_t = f_y(Uh_t + b_y)$$

Recurrent Neural Network (RNN)



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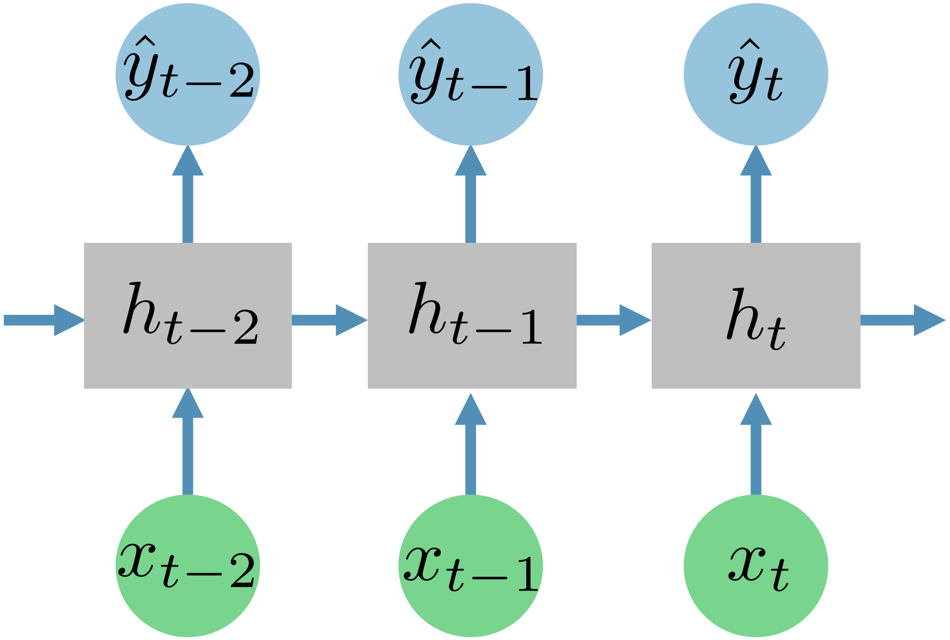
How to train RNN?

Let's consider an RNN
in the unfolded form.

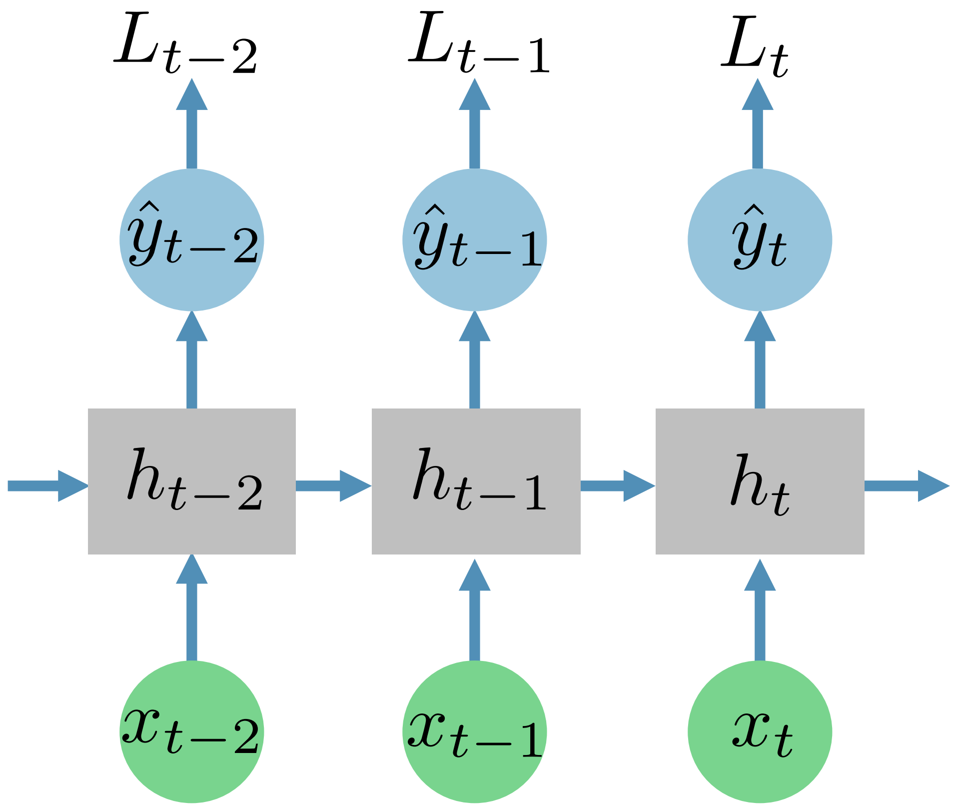


How to train RNN?

Let's consider an RNN
in the unfolded form.

At each time step:

- y_t - true label
- $\hat{y}_t$ - prediction
- $L_t(y_t, \hat{y}_t)$ - some loss function



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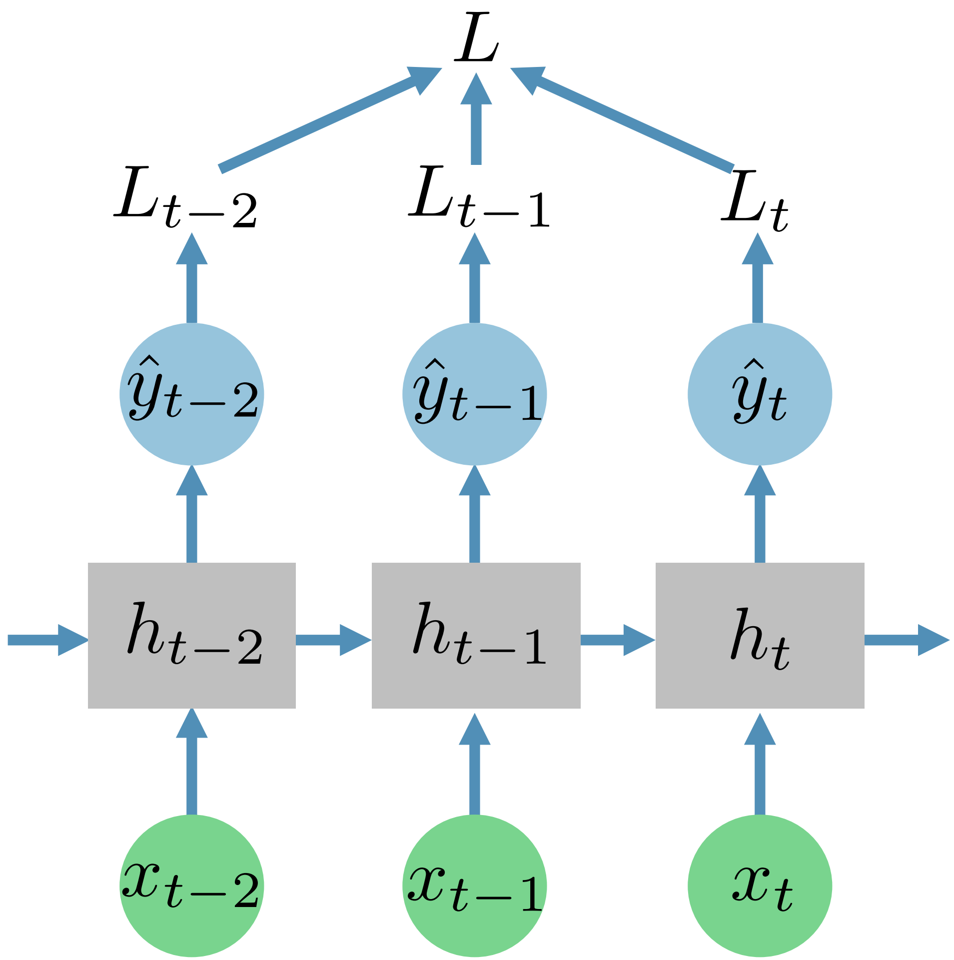
Loss:

$$L = \sum_i L_i(y_i, \hat{y}_i)$$



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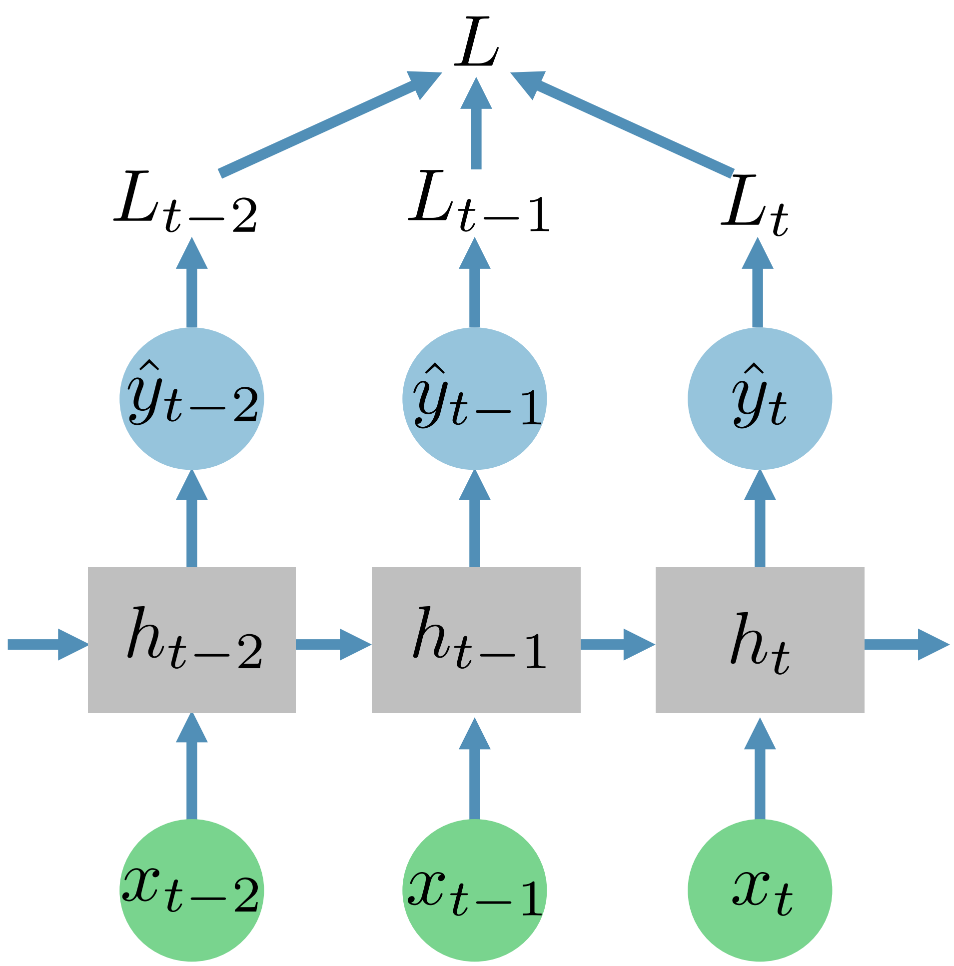
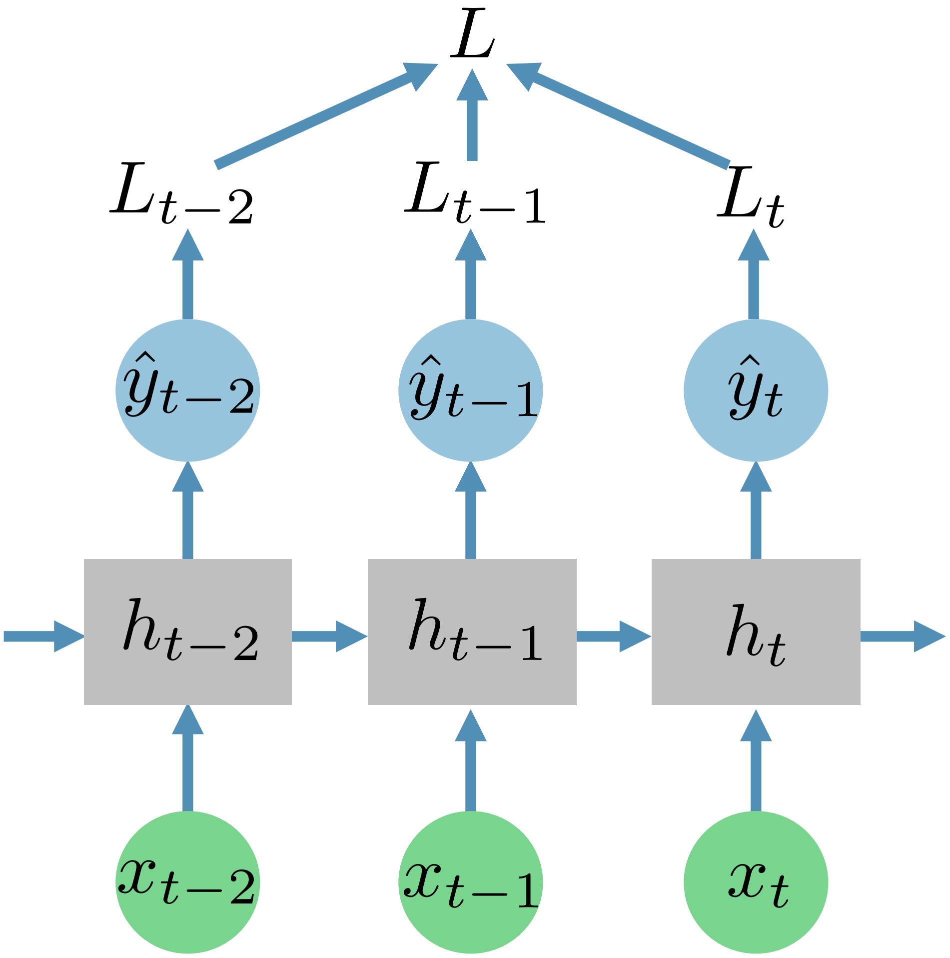


We can use Backpropagation to train the RNN!

Backpropagation Through Time (BPTT)

As usual we do forward and backward passes.

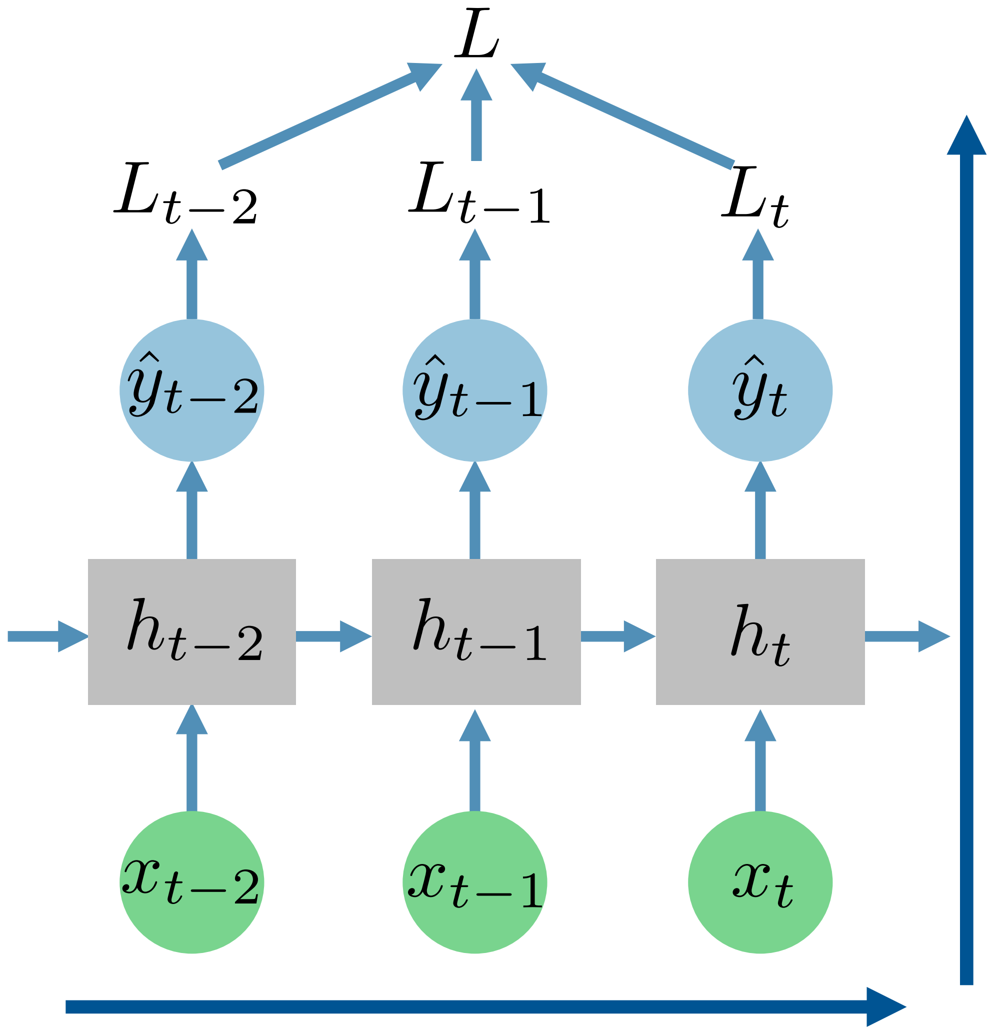


Backpropagation Through Time (BPTT)

As usual we do forward and backward passes.

Forward pass:

$$h_t,\ \hat{y}_t,\ L_t,\ L$$



Backpropagation Through Time (BPTT)

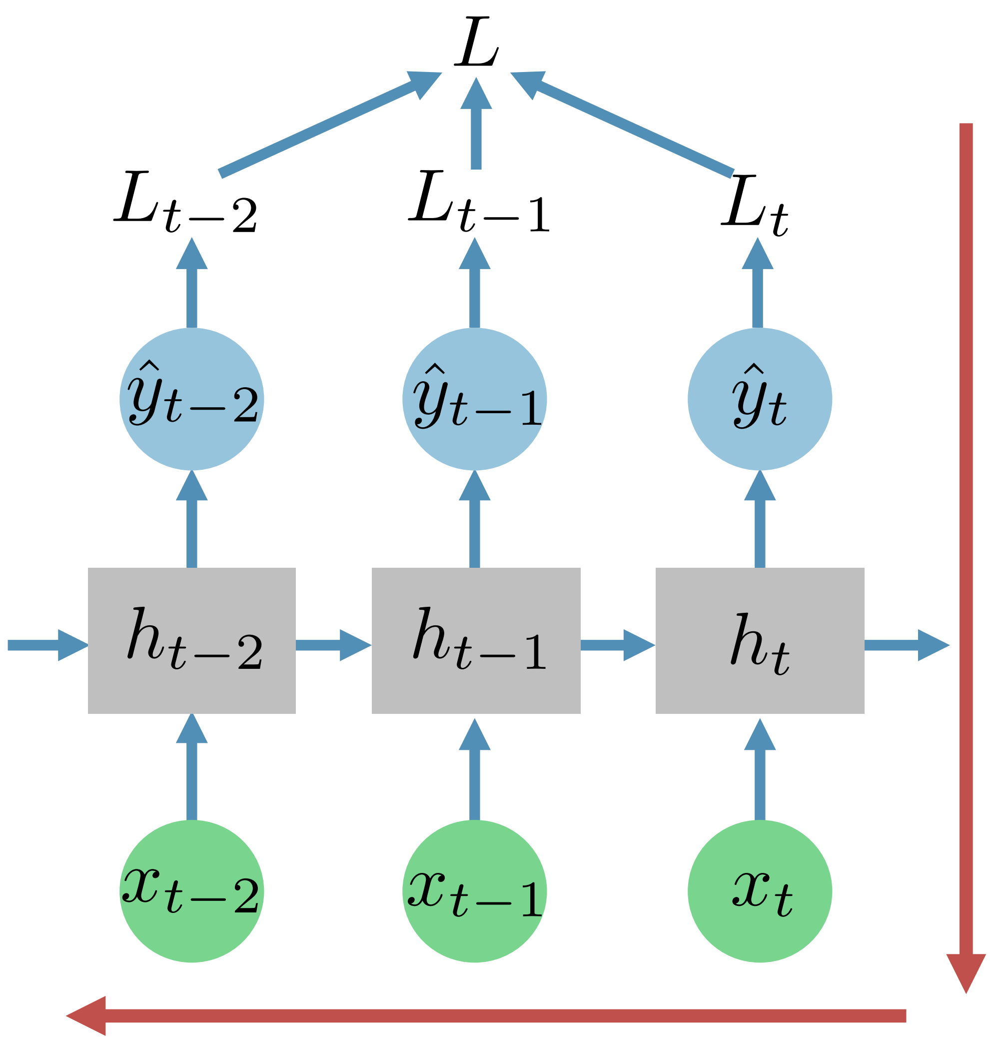
As usual we do forward and backward passes.

Forward pass:

$$h_t,\ \hat{y}_t,\ L_t,\ L$$

Backward pass:

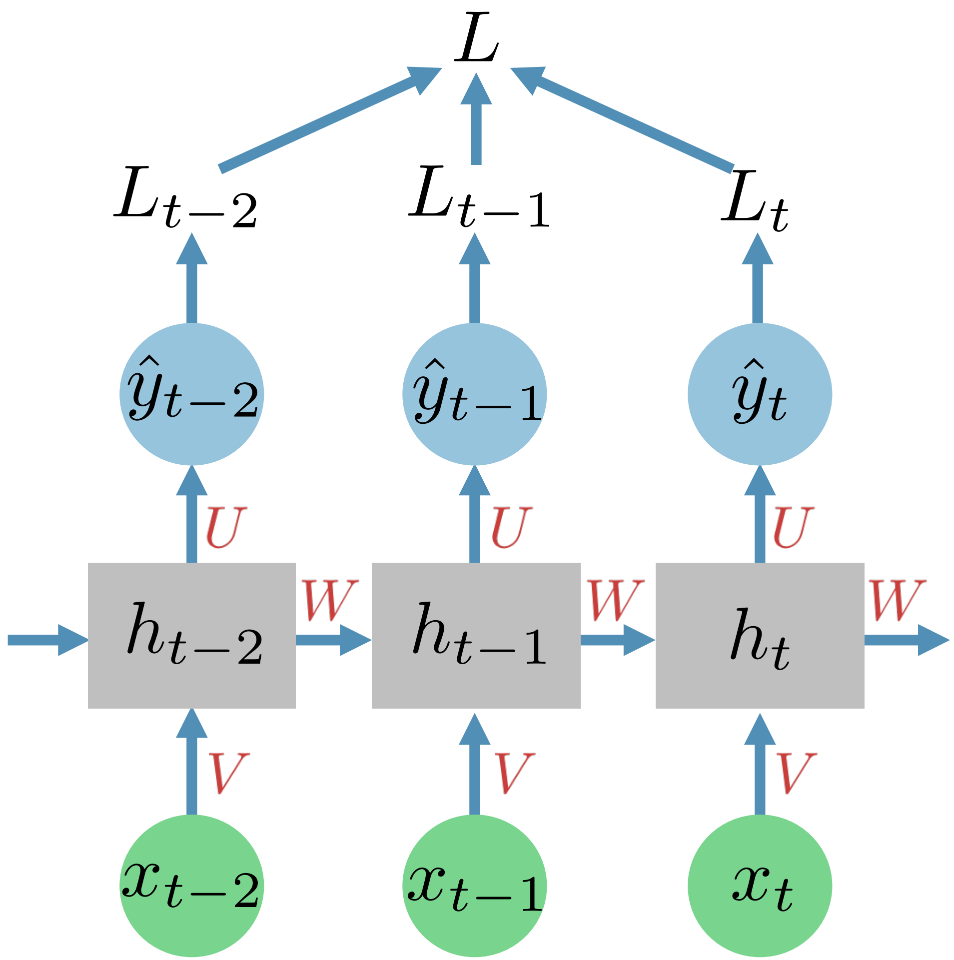
$$\frac{\partial L}{\partial U},\ \frac{\partial L}{\partial V},\ \frac{\partial L}{\partial W},$$
$$\frac{\partial L}{\partial b_x},\ \frac{\partial L}{\partial b_h}$$



We backpropagate through layers and time.

Backpropagation Through Time (BPTT)

All weights are shared across time steps!



Backpropagation Through Time (BPTT)

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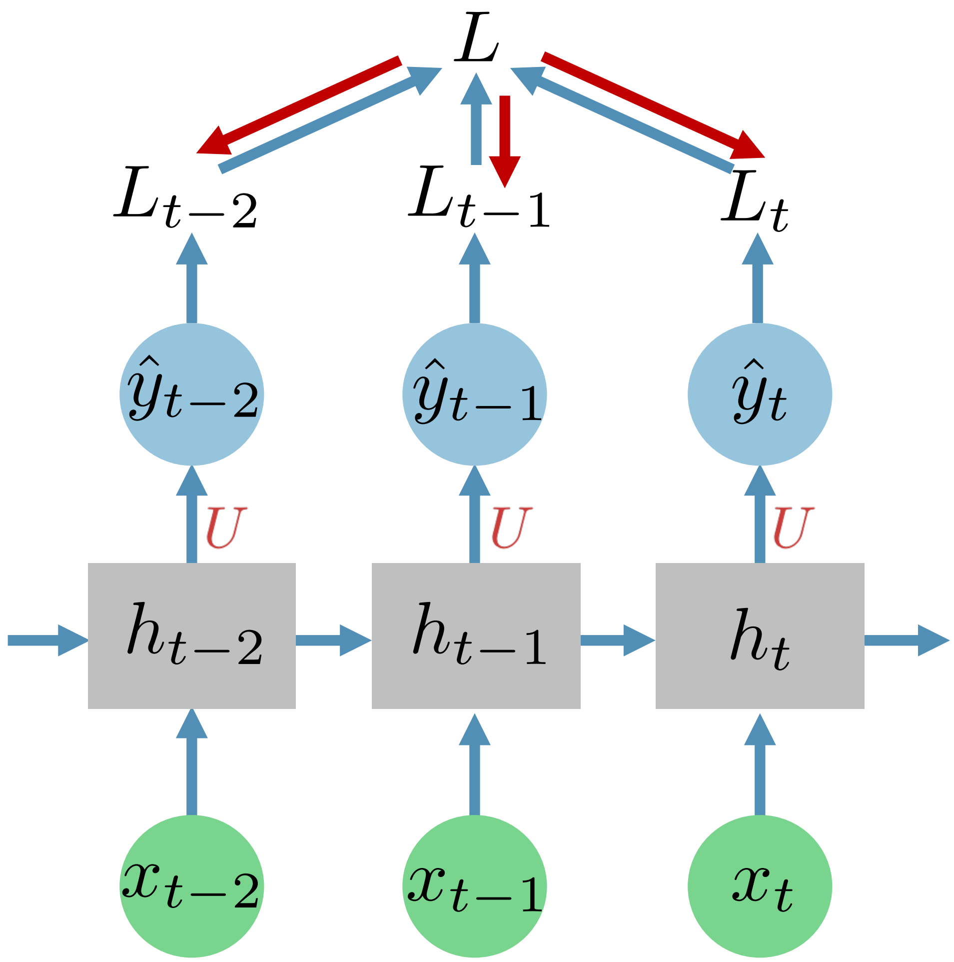
$$\frac{\partial L}{\partial U} = \sum_{i=0}^{T} \frac{\partial L_i}{\partial U}$$



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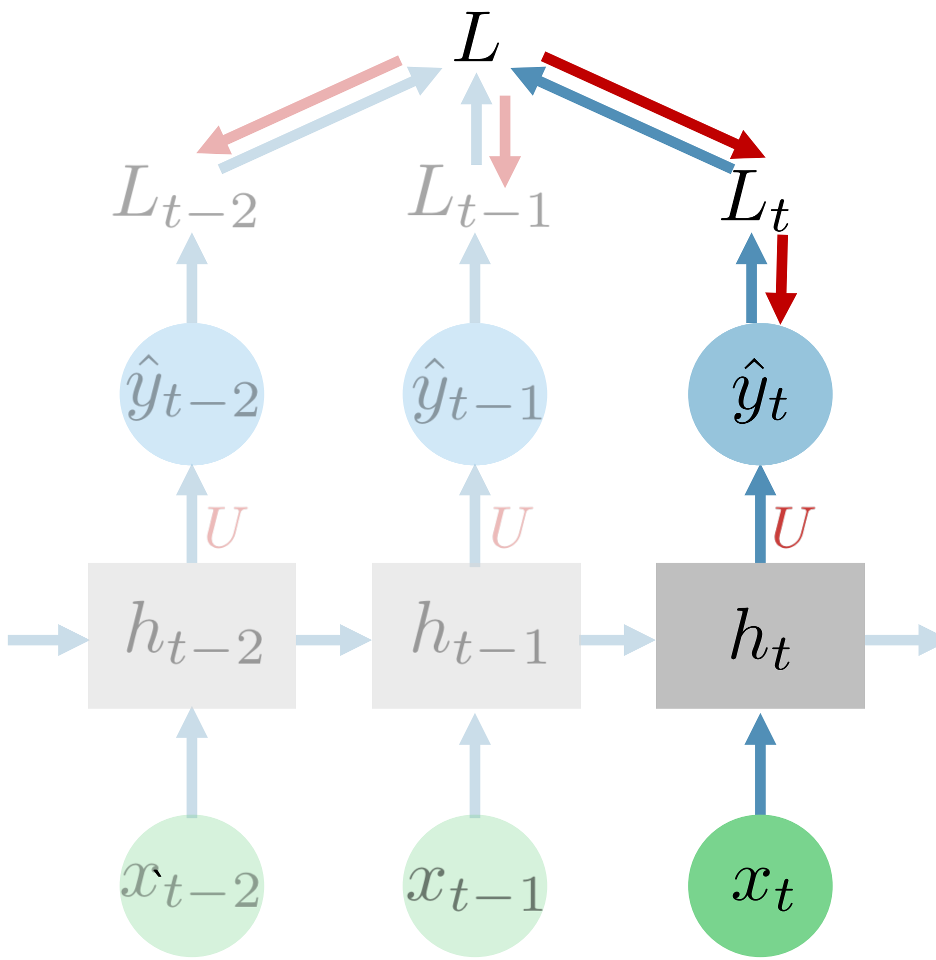

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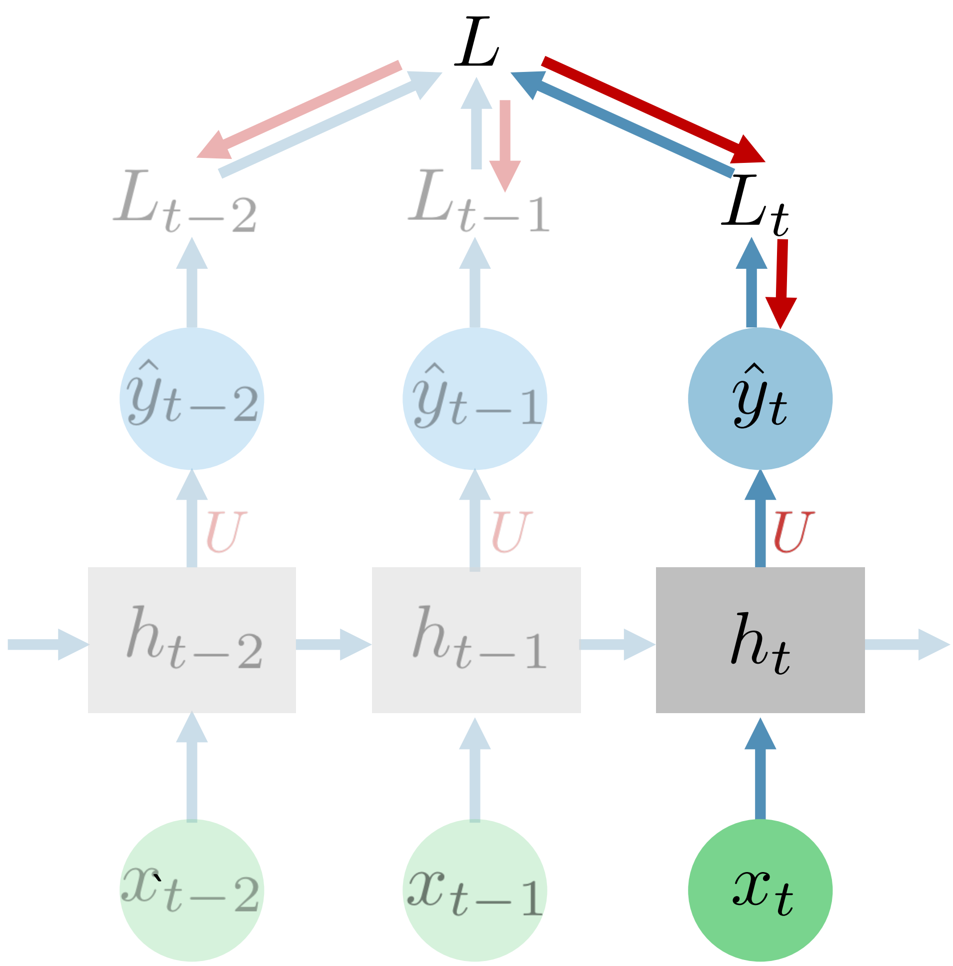
$$\hat{y}_t = f_y(\boxed{U} h_t + b_y)$$

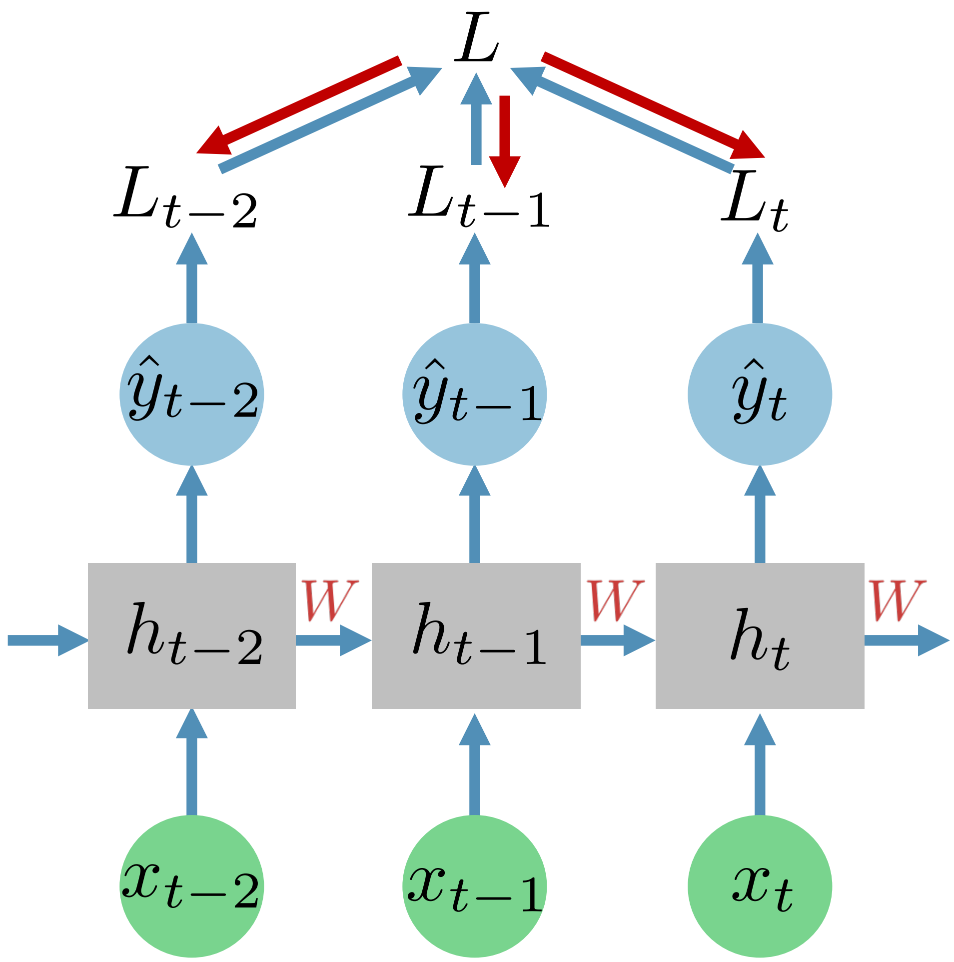
this is the only dependence



Backpropagation Through Time (BPTT)

All weights are shared across time steps!

$$\frac{\partial L}{\partial W} = \sum_{i=0}^{T} \frac{\partial L_i}{\partial W}$$



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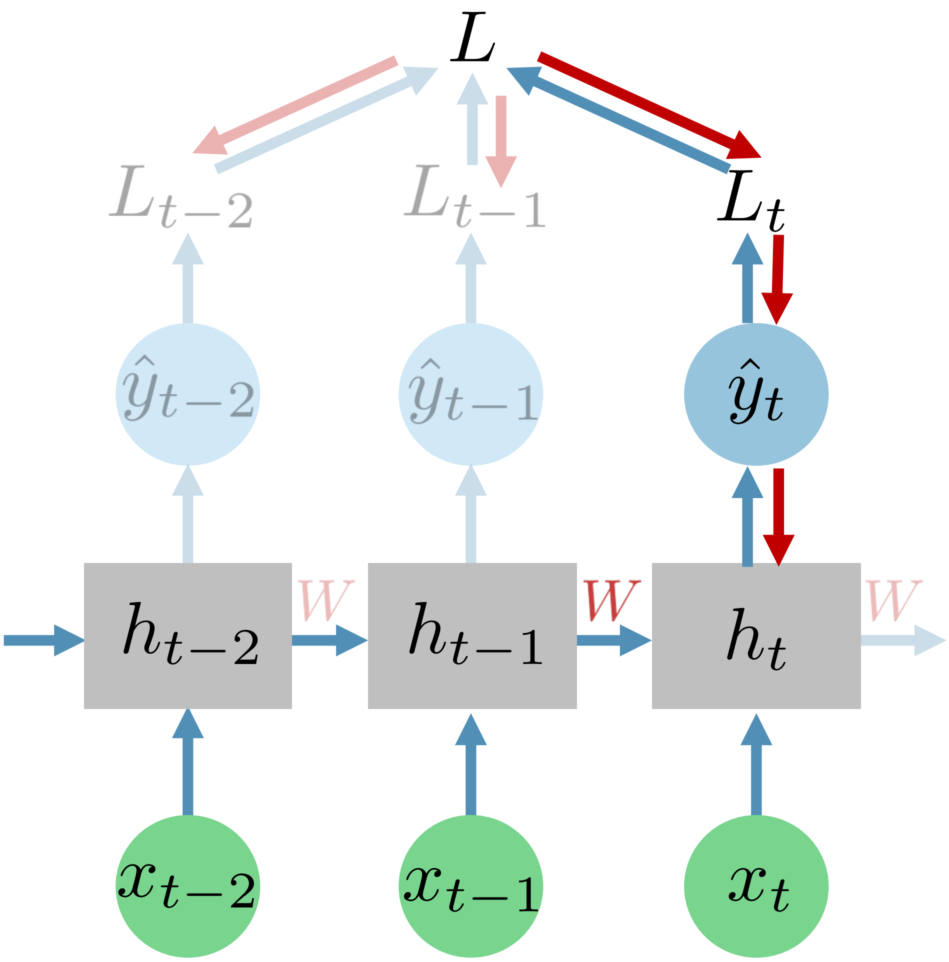

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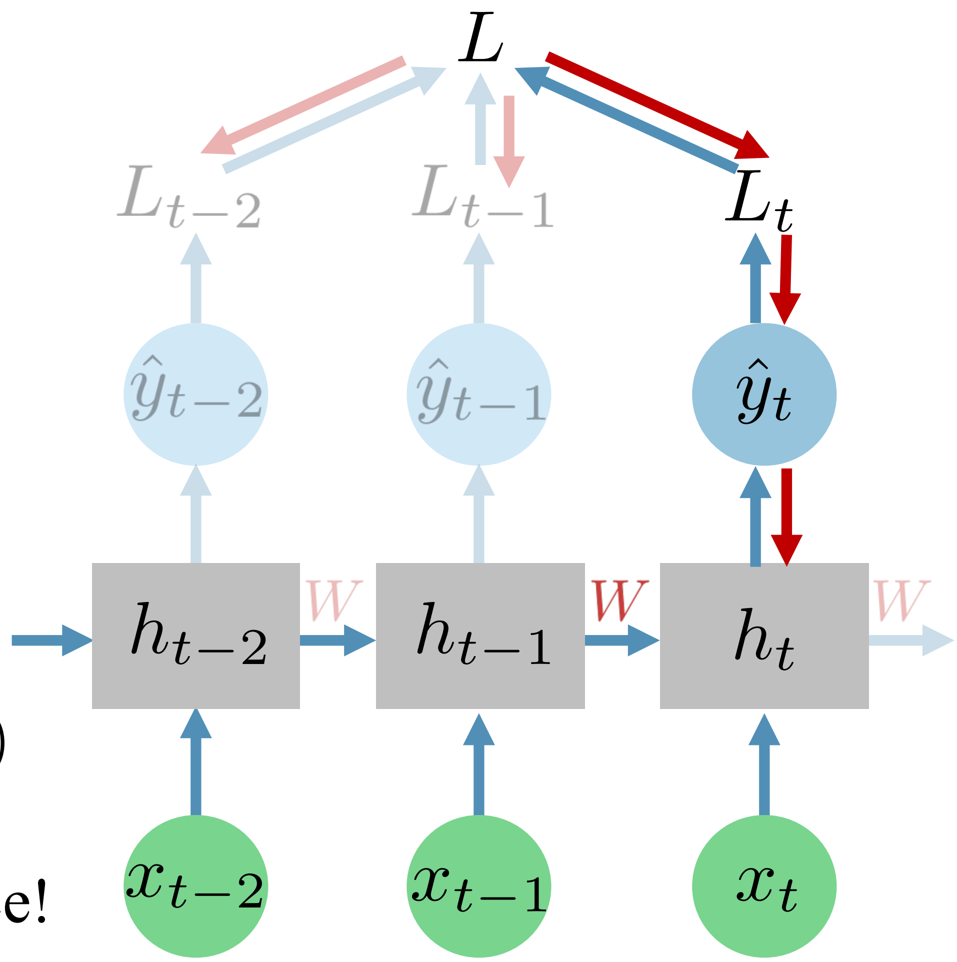
$$h_t = f_h(V x_t + \boxed{W} h_{t-1} + b_h)$$

This is NOT the only dependence!



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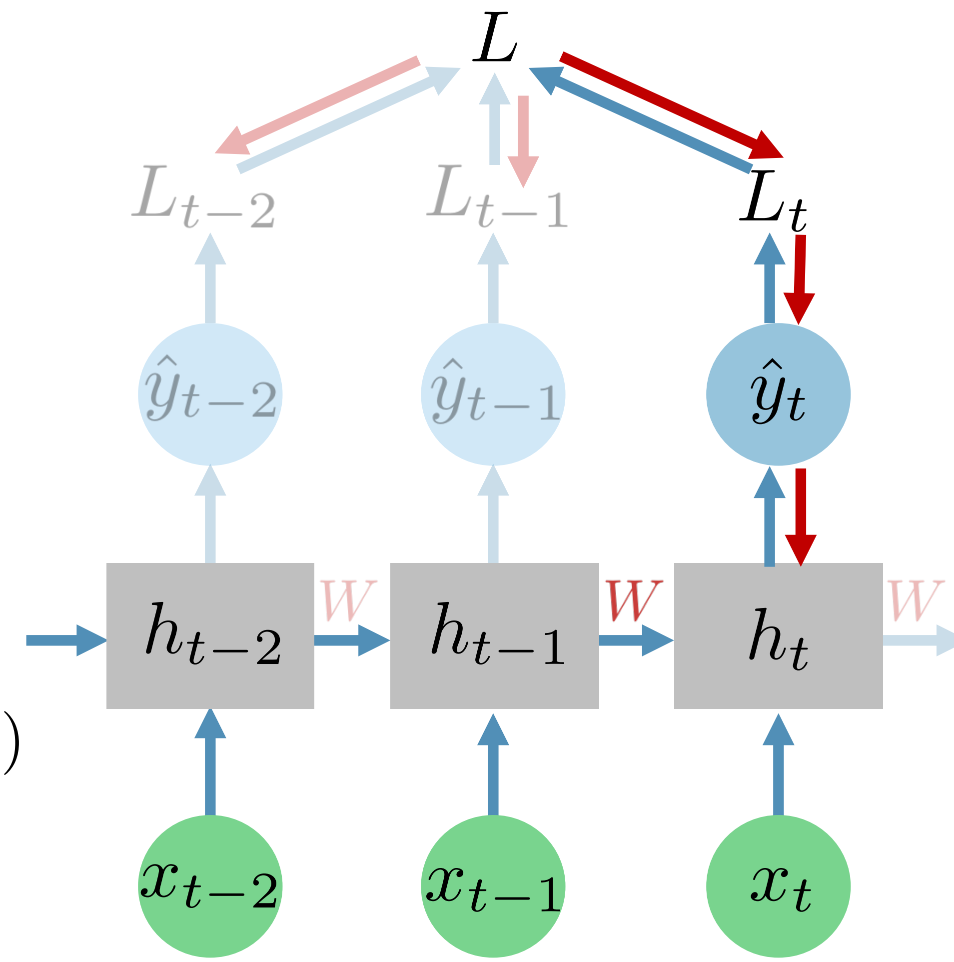
$$h_t = f_h(Vx_t + W\boxed{h_{t-1}} + b_h)$$

Depends on W too!



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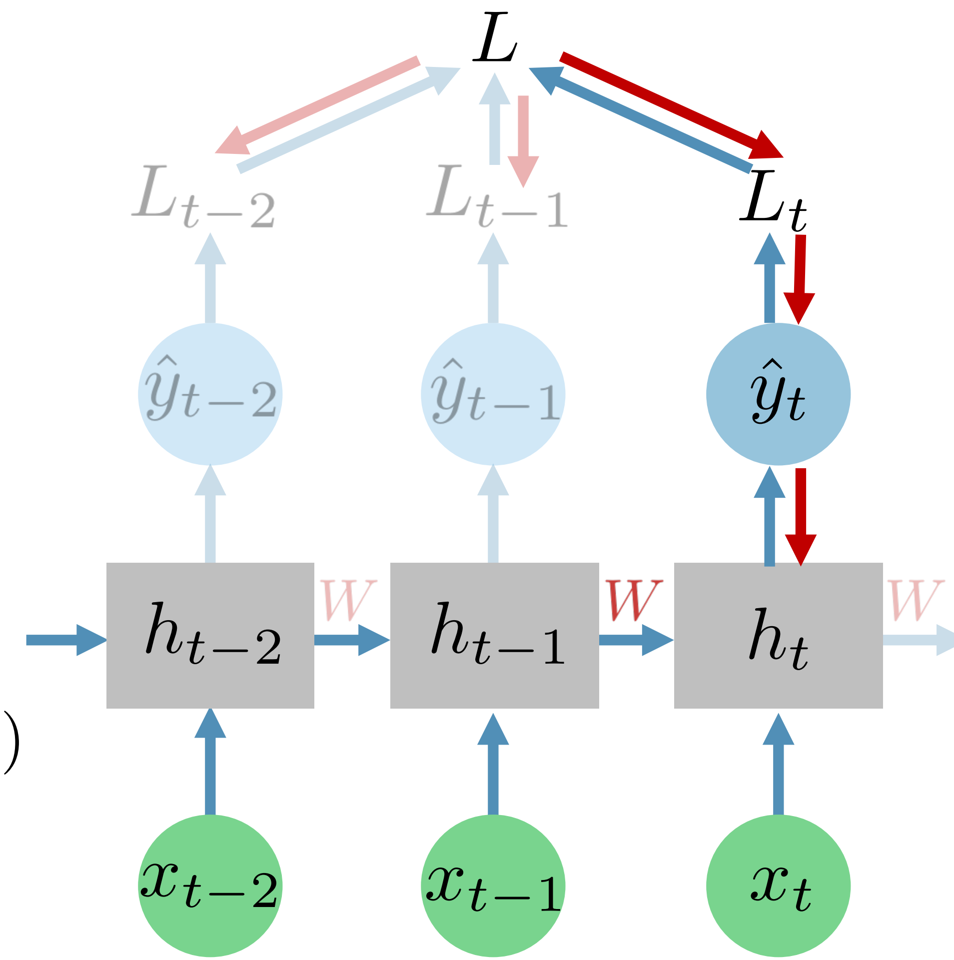

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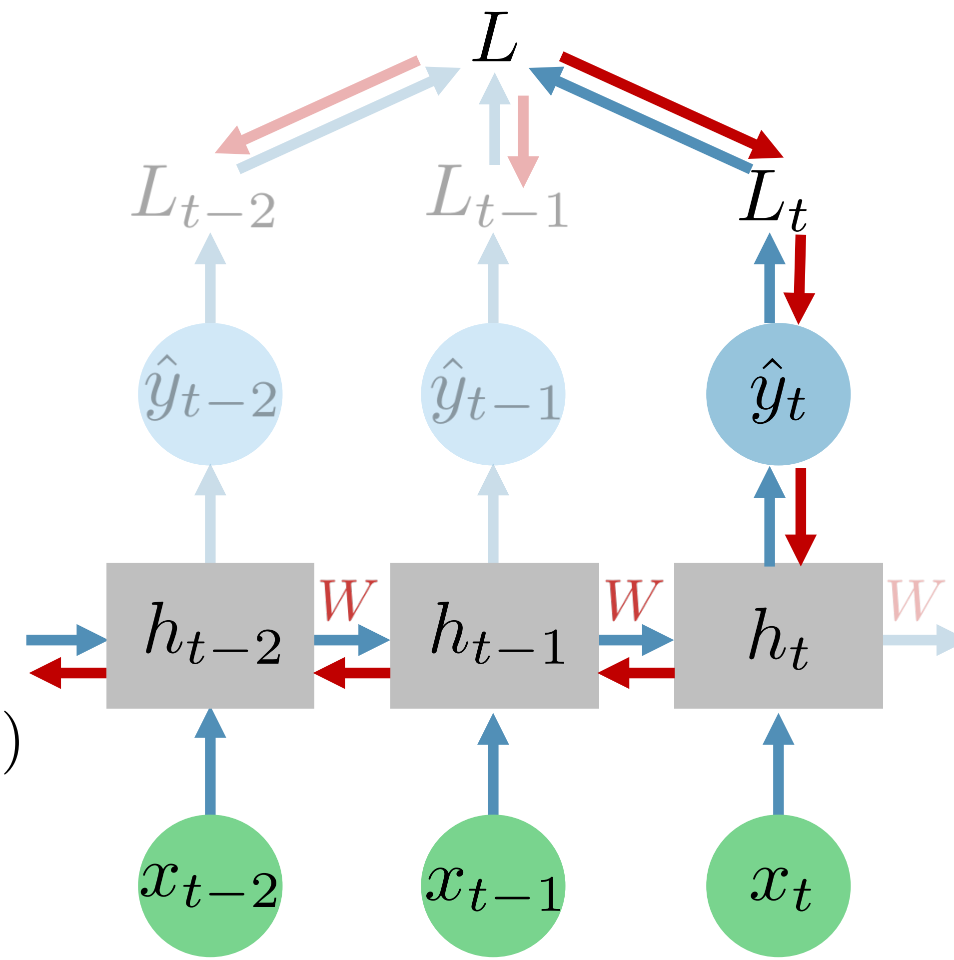


$$\frac{\partial L_t}{\partial W} = \frac{\partial L_t}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \boxed{\left(\frac{\partial h_t}{\partial W} + \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial W} + \dots \right)}$$

Backpropagation Through Time (BPTT)

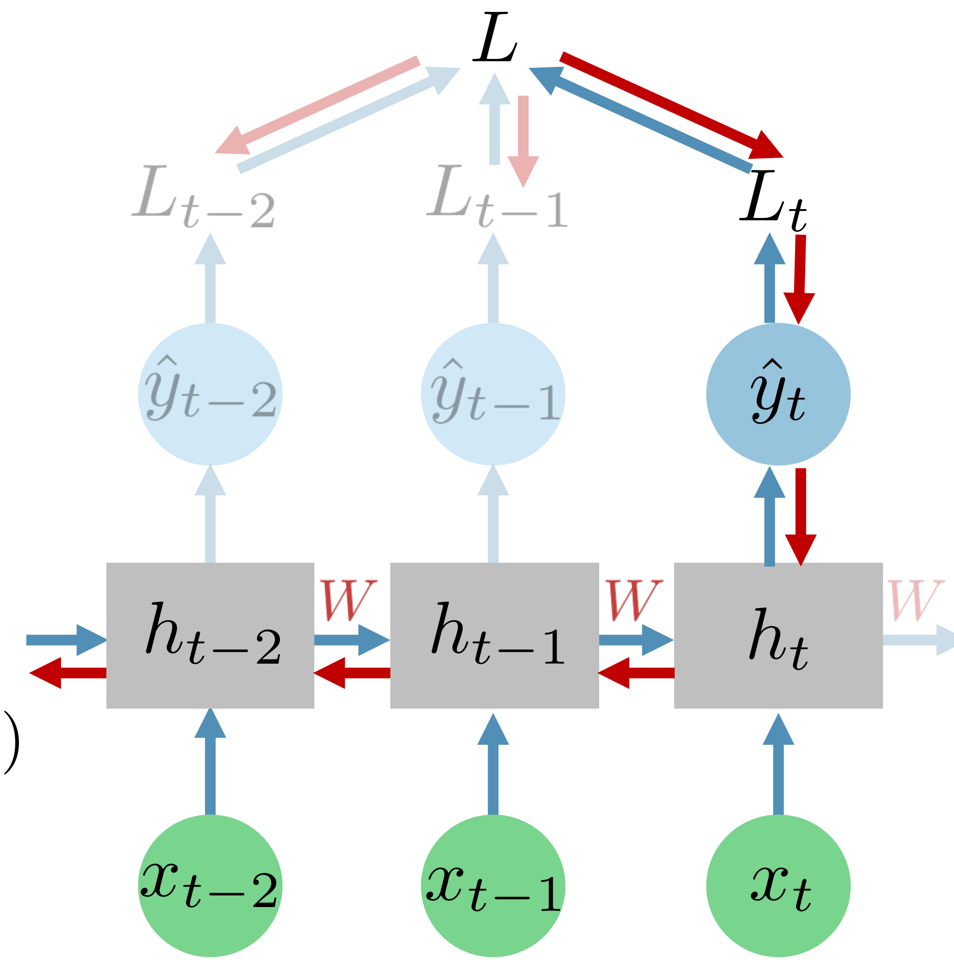
All weights are shared across time steps!

$$\frac{\partial L}{\partial W} = \sum_{i=0}^{T} \frac{\partial L_i}{\partial W}$$

$$\cancel{\frac{\partial L_t}{\partial W} = \frac{\partial L_t}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W}}$$

$$h_t = f_h(V x_t + W \boxed{h_{t-1}} + b_h)$$

Depends on W too!



$$\frac{\partial L_t}{\partial W} = \frac{\partial L_t}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \boxed{\sum_{k=0}^{t} \frac{\partial h_t}{\partial h_{t-1}} \cdots \frac{\partial h_{k+1}}{\partial h_k} \frac{\partial h_k}{\partial W}}$$

Backpropagation Through Time (BPTT)

All weights are shared across time steps!

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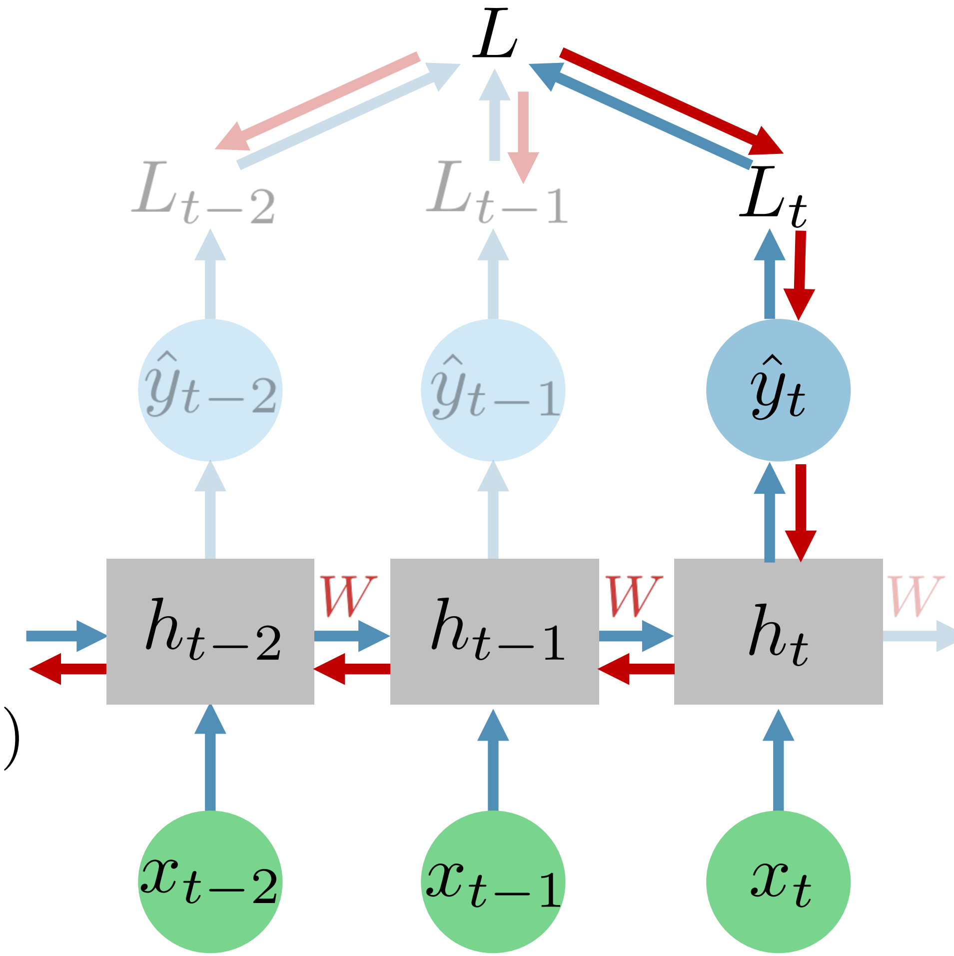
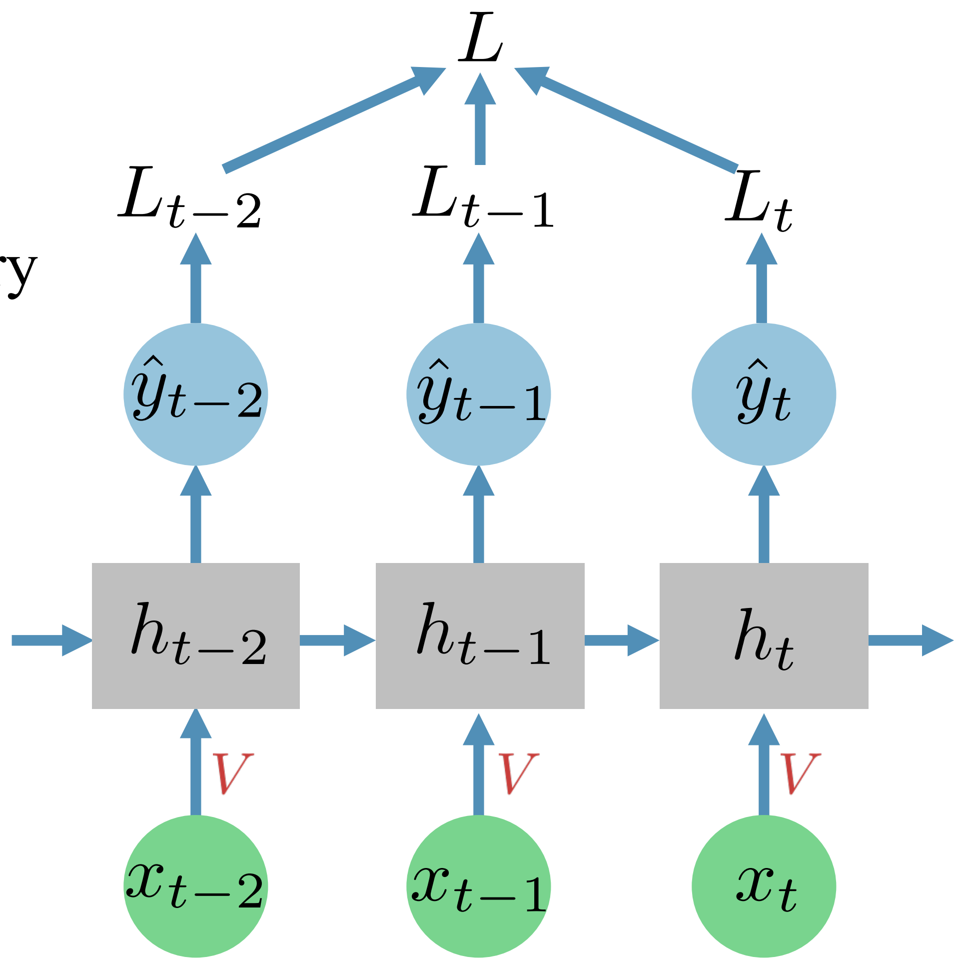


$$\frac{\partial L_t}{\partial W} = \frac{\partial L_t}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \boxed{\sum_{k=0}^{t} \left(\prod_{i=k+1}^{t} \frac{\partial h_i}{\partial h_{i-1}} \right) \frac{\partial h_k}{\partial W}}$$

Backpropagation Through Time (BPTT)

And what about the last weight matrix V? Is it necessary to go backwards in time to calculate $\dfrac{\partial L}{\partial V}$?



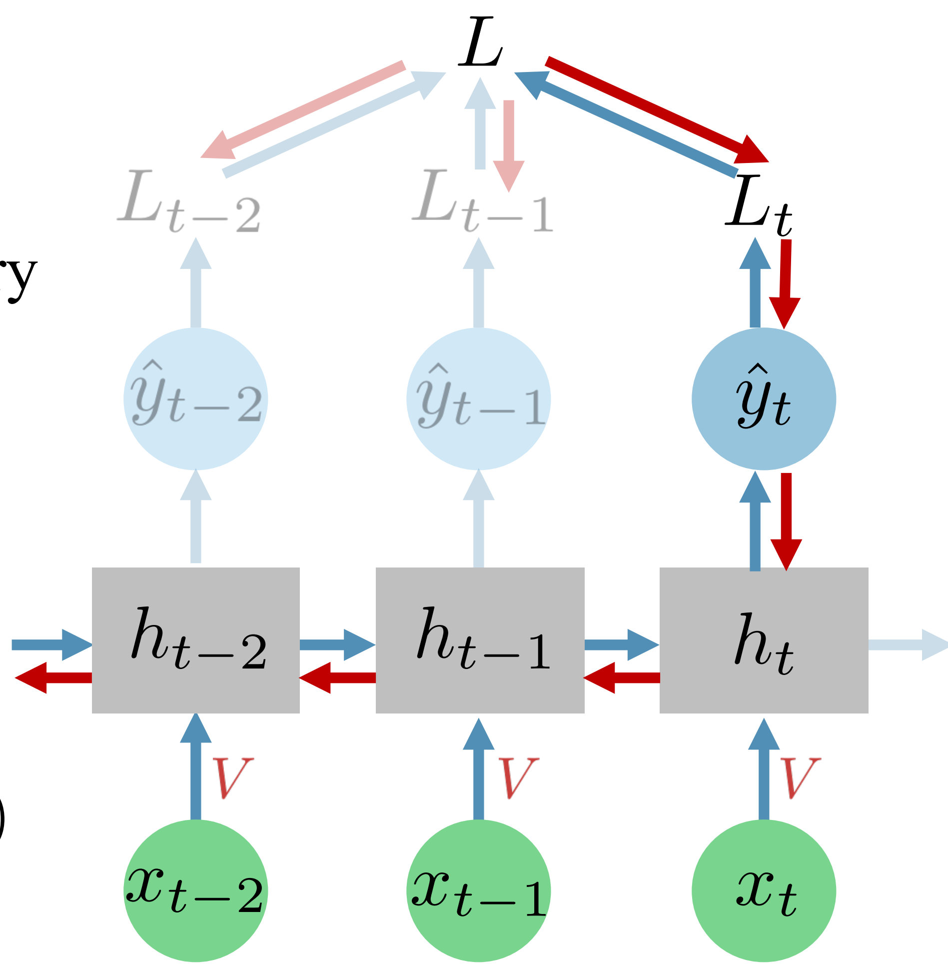
Backpropagation Through Time (BPTT)

And what about the last weight matrix V? Is it necessary to go backwards in time to calculate $\dfrac{\partial L}{\partial V}$?

Yes! Here we have the same situation as with W:

$$h_t = f_h(Vx_t + W\boxed{h_{t-1}} + b_h)$$

Depends on W too!



Summary

- We have learned what is a simple RNN.
- RNNs are trained using simple Backpropagation. BPTT is just a fancy name for it.

In the next video:

Is it really that simple to train an RNN?