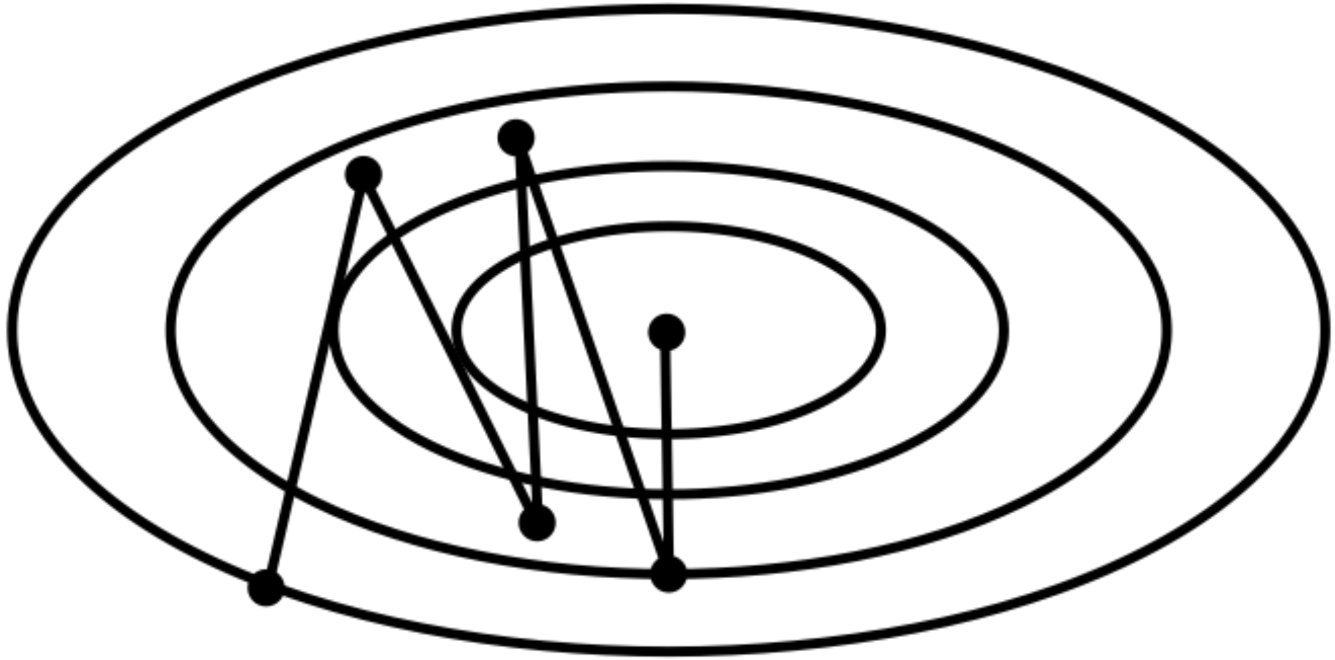
Gradient descent extensions

Difficult function



Mini-batch gradient descent

w^0 — initialization

while True:

$i_1, \dots, i_m$ = random indices between 1 and ℓ

$g_t = \frac{1}{m}\sum_{j=1}^{m} \nabla L\left(w^{t-1}; x_{i_j}; y_{i_j}\right)$

$\boldsymbol{w^t = w^{t-1} - \eta_t g_t}$

if $\|w^t - w^{t-1}\| < \epsilon$ then break

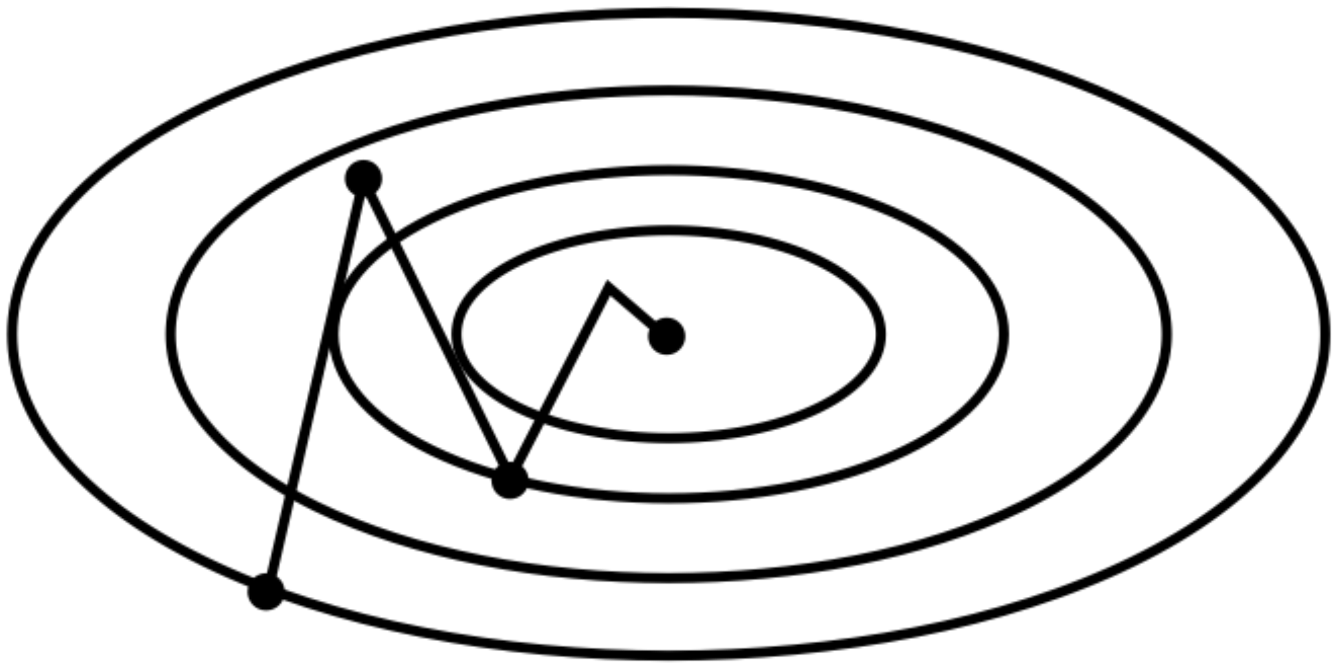
Momentum

$$h_t = \alpha h_{t-1} + \eta_t g_t$$

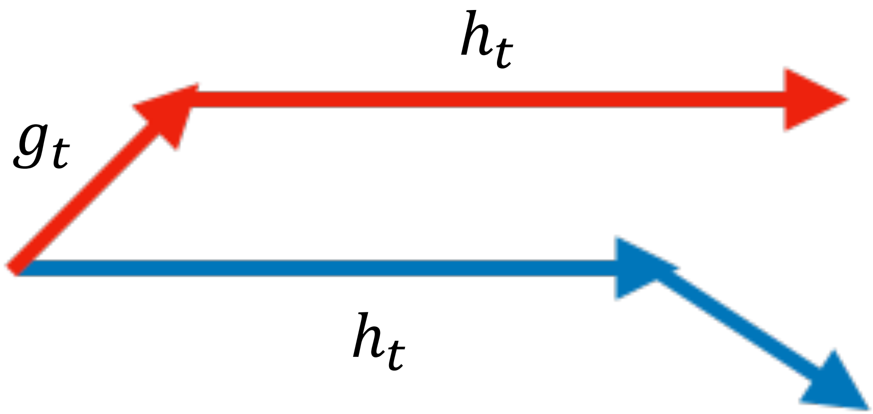
$$w^t = w^{t-1} - h_t$$

- Tends to move in the same direction as on previous steps
- h_t accumulates values along dimensions where gradients have the same sign
- Usually: $\alpha = 0.9$

Momentum



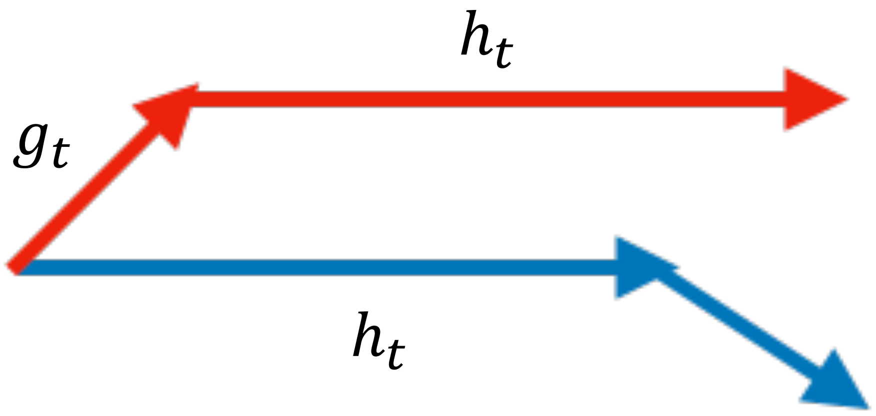
Nesterov momentum



Nesterov momentum

$$h_t = \alpha h_{t-1} + \eta_t \nabla L(w^{t-1} - \alpha h_{t-1})$$

$$w^t = w^{t-1} - h_t$$



AdaGrad

$$G_j^t = G_j^{t-1} + g_{tj}^2$$

$$w_j^t = w_j^{t-1} - \frac{\eta_t}{\sqrt{G_j^t + \epsilon}} g_{tj}$$

- g_{tj} — gradient with respect to j-th parameter
- Separate learning rates for each dimension
- Suits for sparse data
- Learning rate can be fixed: $\eta_t = 0.01$
- G_j^t always increases, leads to early stops

RMSprop

$$G_j^t = \alpha G_j^{t-1} + (1-\alpha) g_{tj}^2$$

$$w_j^t = w_j^{t-1} - \frac{\eta_t}{\sqrt{G_j^t + \epsilon}} g_{tj}$$

- α is about 0.9
- Learning rate adapts to latest gradient steps

Adam

$$v_j^t = \frac{\beta_2 v_j^{t-1} + (1-\beta_2) g_{tj}^2}{1-\beta_2^t}$$

$$w_j^t = w_j^{t-1} - \frac{\eta_t}{\sqrt{v_j^t} + \epsilon}\, g_{tj}$$

Adam

$$m_j^t = \frac{\beta_1 m_j^{t-1} + (1-\beta_1) g_{tj}}{1-\beta_1^t}$$

$$v_j^t = \frac{\beta_2 v_j^{t-1} + (1-\beta_2) g_{tj}^2}{1-\beta_2^t}$$

$$w_j^t = w_j^{t-1} - \frac{\eta_t}{\sqrt{v_j^t} + \epsilon} m_j^t$$

Combines momentum and individual learning rates

Summary

- Momentum methods smooth gradients and speed up convergence
- Adaptive methods eliminate sensitive learning rate
- Adam combines both approaches