Distributions: Beta

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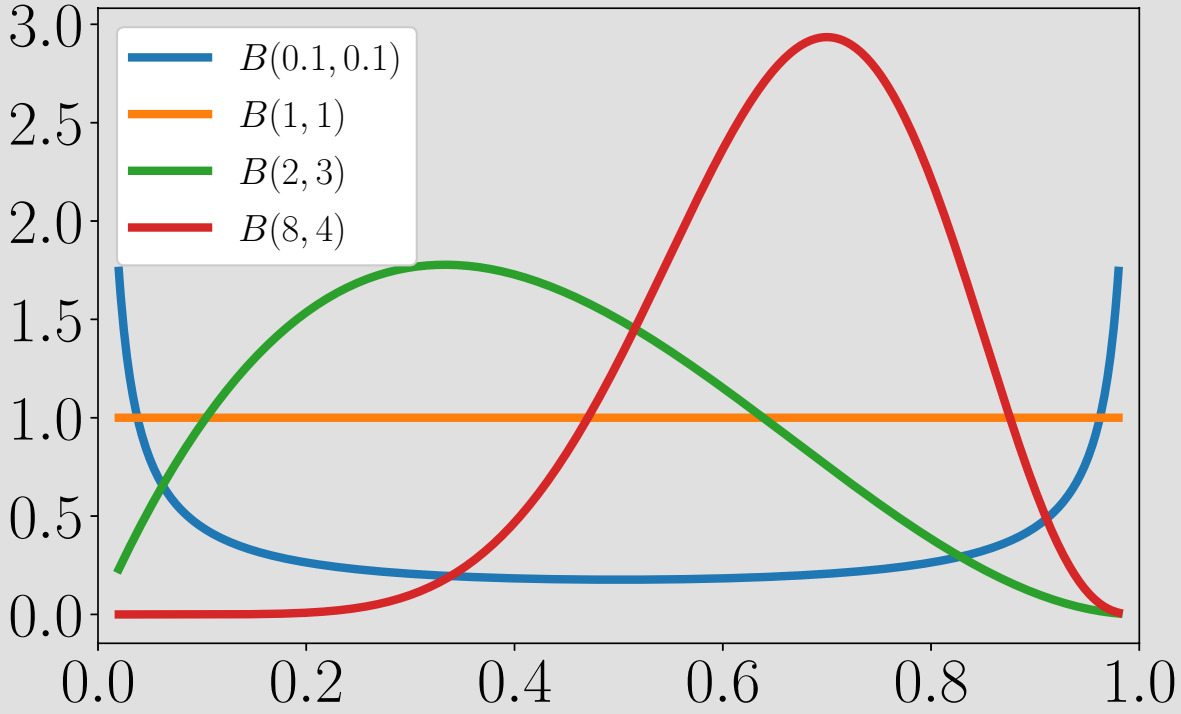

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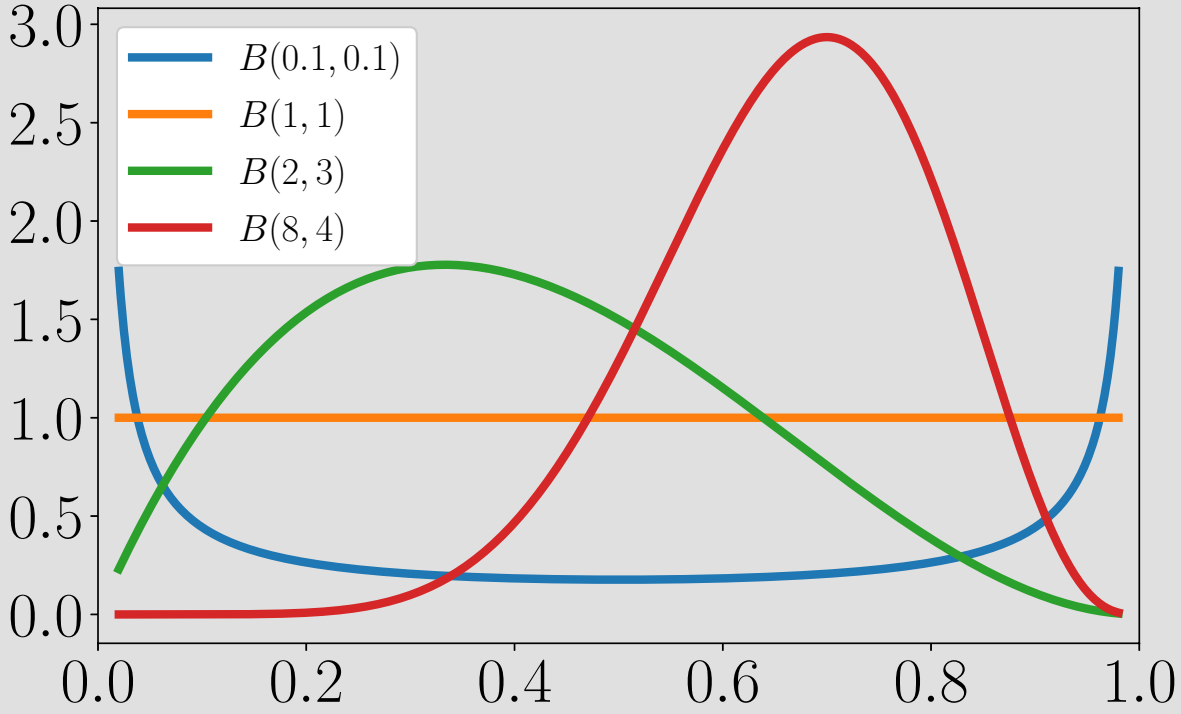

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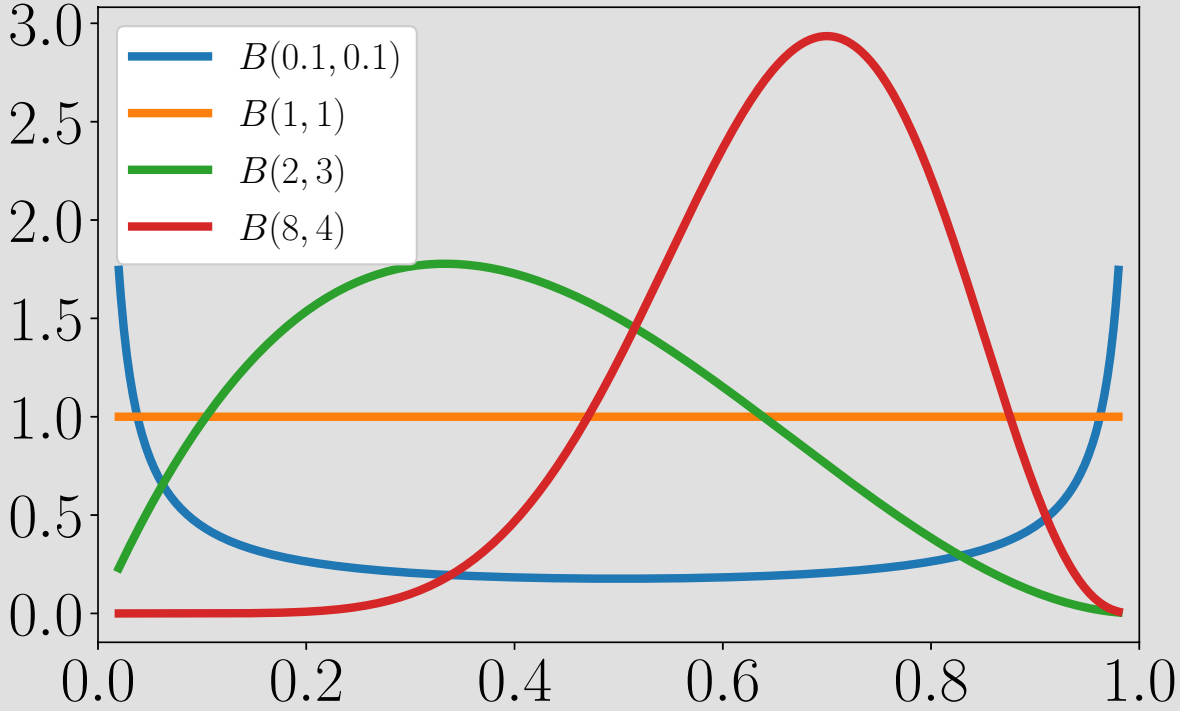
$$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$$



Statistics

$$B(x|a,b) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}$$

$$\mathbb{E}x = \frac{a}{a+b}$$

$$\text{Mode}[x] = \frac{a-1}{a+b-2}$$

$$\text{Var}[x] = \frac{ab}{(a+b)^2(a+b-1)}$$

Example

Movie rank is 0.8 ± 0.1

Example ТЕХНИЧЕСКИЙ СЛАЙД

Movie rank is 0.8 ± 0.1

1 — best movie

0 — Batman & Robin

Example

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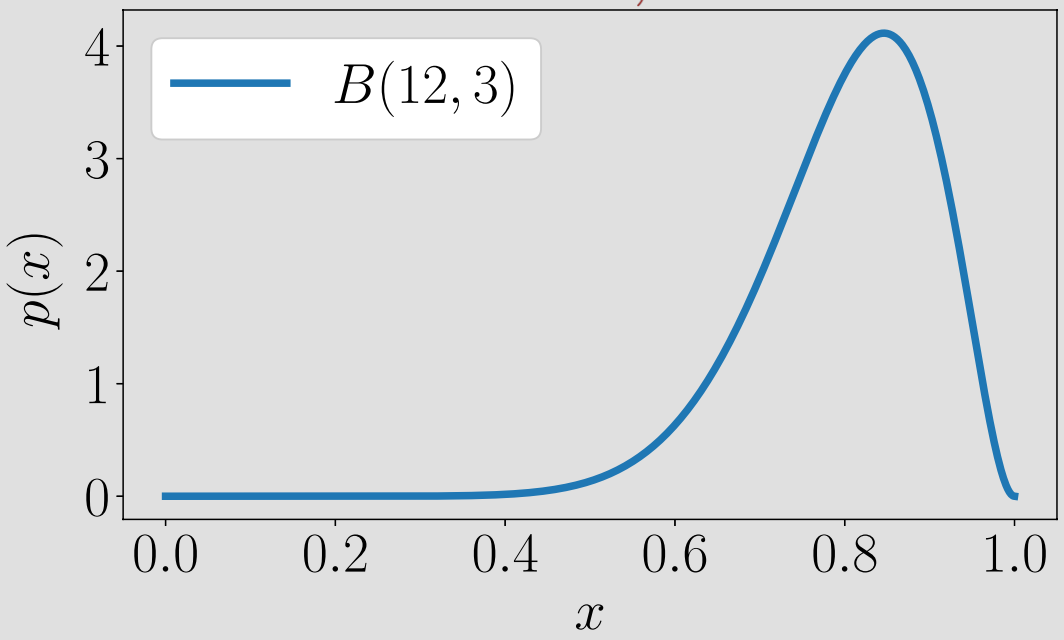

Example: Bernoulli

Beta prior

$$p(X|\theta) = \theta^{N_1}(1-\theta)^{N_0}$$

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$$p(\theta|X) = B(N_1+a, N_0+b)$$

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E.g. $p(\theta|X) = B(N_1 + a, N_0 + b)$

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Cons:

- Conjugate prior may be inadequate