Geometrical Interpretation of OLS

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GEOMETRICAL INTERPRETATION

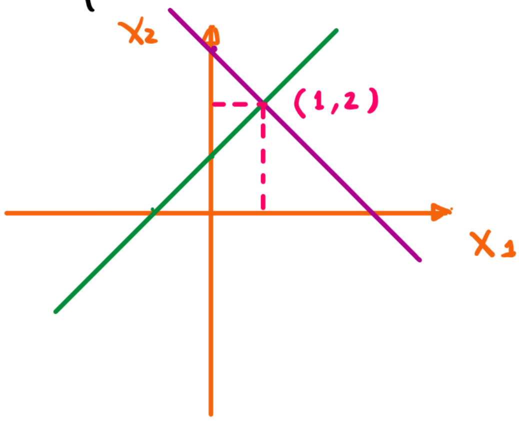
1) Given two equations:

$$\begin{cases} x_1 + x_2 = 3 \\ -x_1 + x_2 = 1 \end{cases}$$

we can easily solve the equations and have:

$$\begin{cases} x_1 = 1 \\ x_2 = 2 \end{cases}$$

That is where these two lines meet.



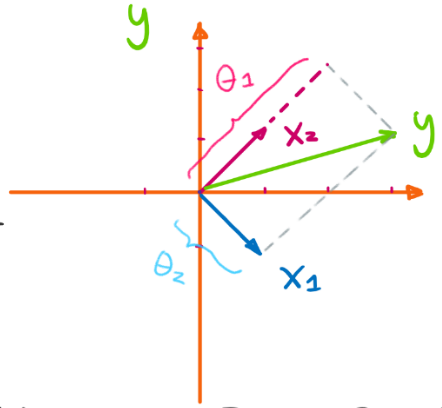
2) We can rewrite the equations by matrices using θ:

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad \boxed{X\theta = y}$$

or: $\underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{x_1} \theta_1 + \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{x_2} \theta_2 = \underbrace{\begin{bmatrix} 3 \\ 1 \end{bmatrix}}_{y}$

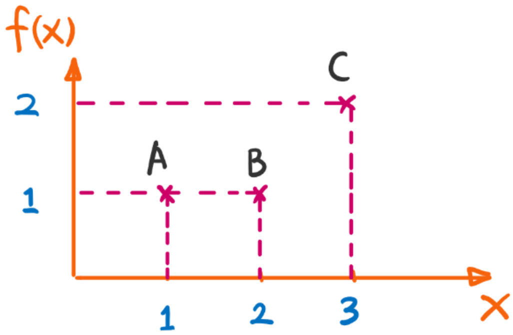
$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ lies in the plane of x_1 and x_2. Therefore we can solve θ:

$\theta_1 = 2, \quad \theta_2 = 1$



It is a perfect regression with zero error.

3) Consider the following case of linear regression



$h_\theta(x) = \theta_0 + \theta_1 x$

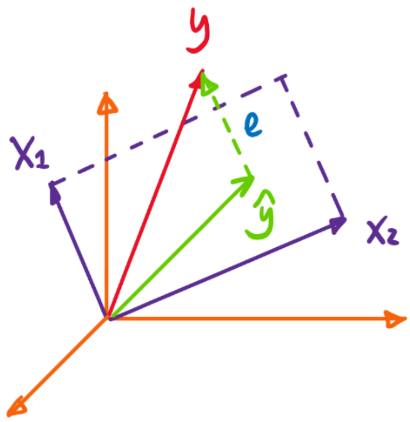
A: $\theta_1 \times 1 + \theta_0 = 1$

B: $\theta_1 \times 2 + \theta_0 = 1$

C: $\theta_1 \times 3 + \theta_0 = 2$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \theta_0 + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \theta_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$



$y - \hat{y} = e$ where $\hat{y}$ is the shadow of y and the plane of $L(X_1, X_2)$

$$\hat{y} = X\theta \quad (\hat{y} = \theta_0 + x_1\theta_1)$$

$$\begin{cases} X_1^T e = 0 \\ X_2^T e = 0 \end{cases} \Rightarrow X^T e = 0$$

$$X^T e = X^T(y - \hat{y}) = X^T(y - X\theta) = 0$$

$$\Rightarrow X^T y = X^T X \theta$$

$$\Rightarrow \theta = (X^T X)^{-1} X^T y$$

4) The geometrical interpretation of OLS can be generalized to high-dimensional space.