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关注 小象学院

Machine Learning

Part 5: Statistical Learning

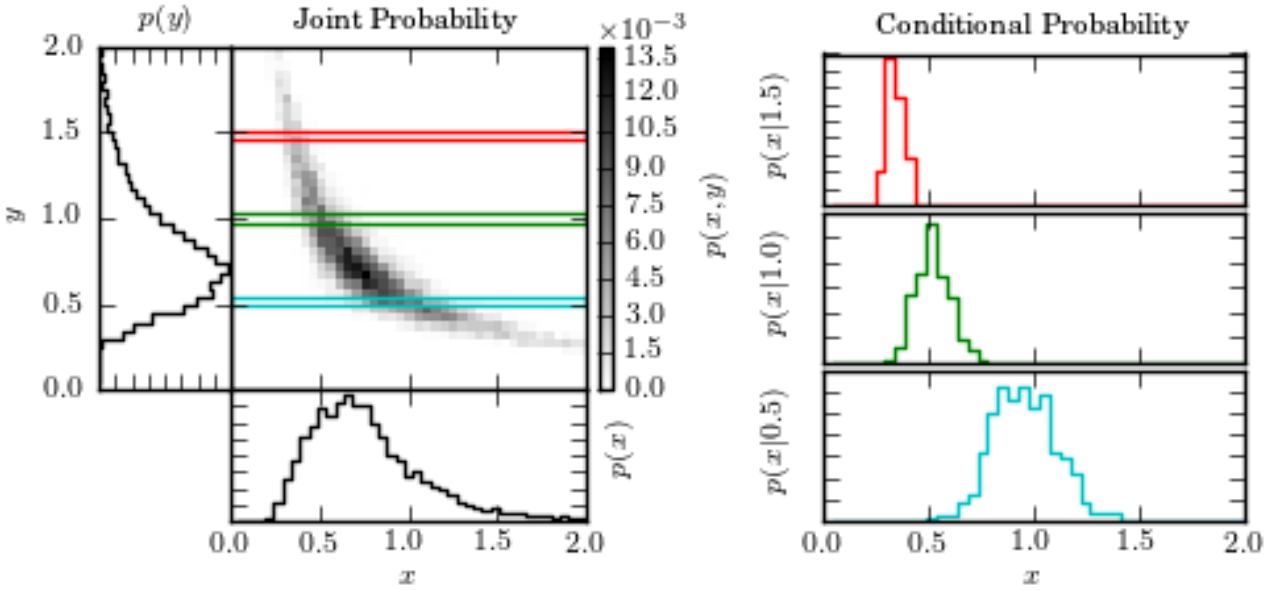
Generative Learning Models

Discriminative and Generative Learning

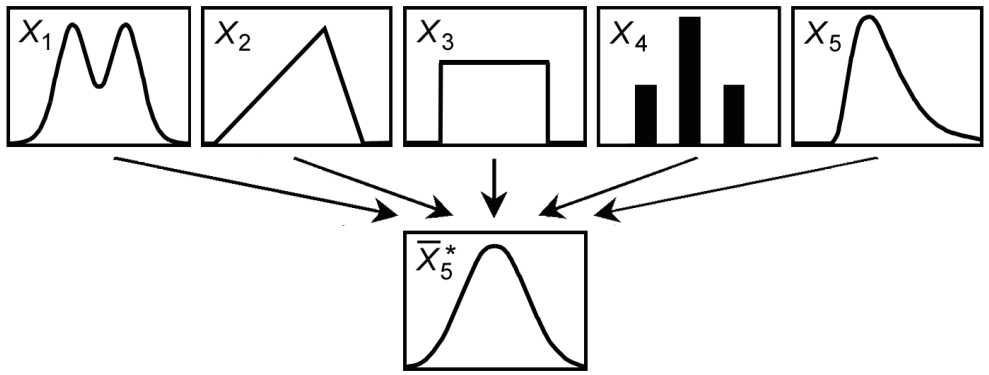
1. Algorithms that try to learn $p(x|y)$ directly from the space of inputs X to the labels {0,1}, are called discriminative learning algorithms. Aim to minimize the cost function.

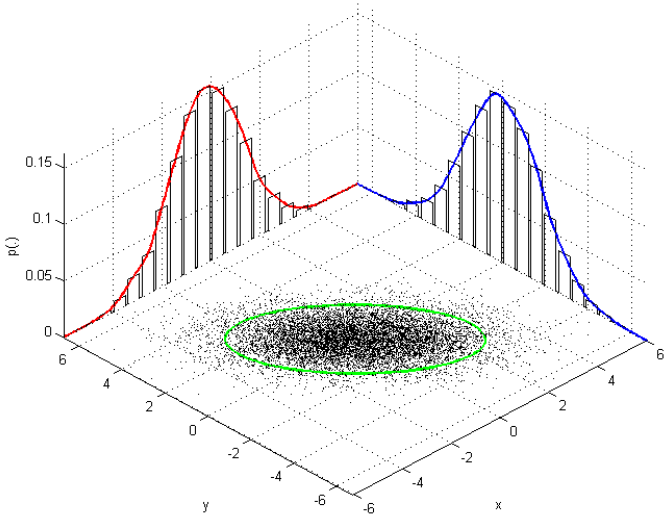
2. For generative learning algorithms, we usually learn $p(x, y)$, if y indicates whether an example is a dog (0) or an elephant (1), then $p(x|y = 0)$ models the distribution of dogs' features. Aim to recreate how the observed data are generated following some hidden patterns (or rules).

3. Based on my generation assumptions, which category is most likely to generate this signal? A discriminative algorithm does not care about how the data was generated.

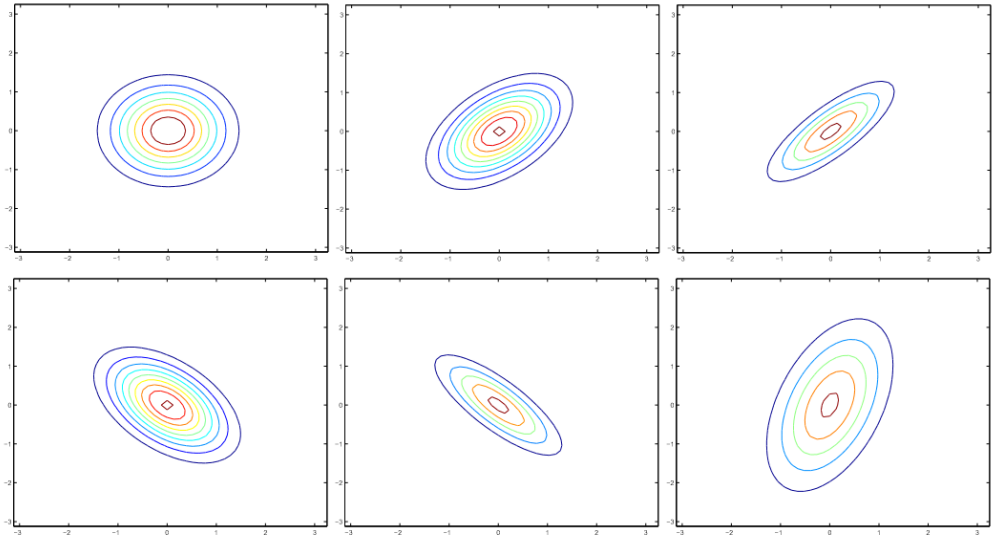


Gaussian (Central Limit Theorem)





$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad \Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}; \quad .\Sigma = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

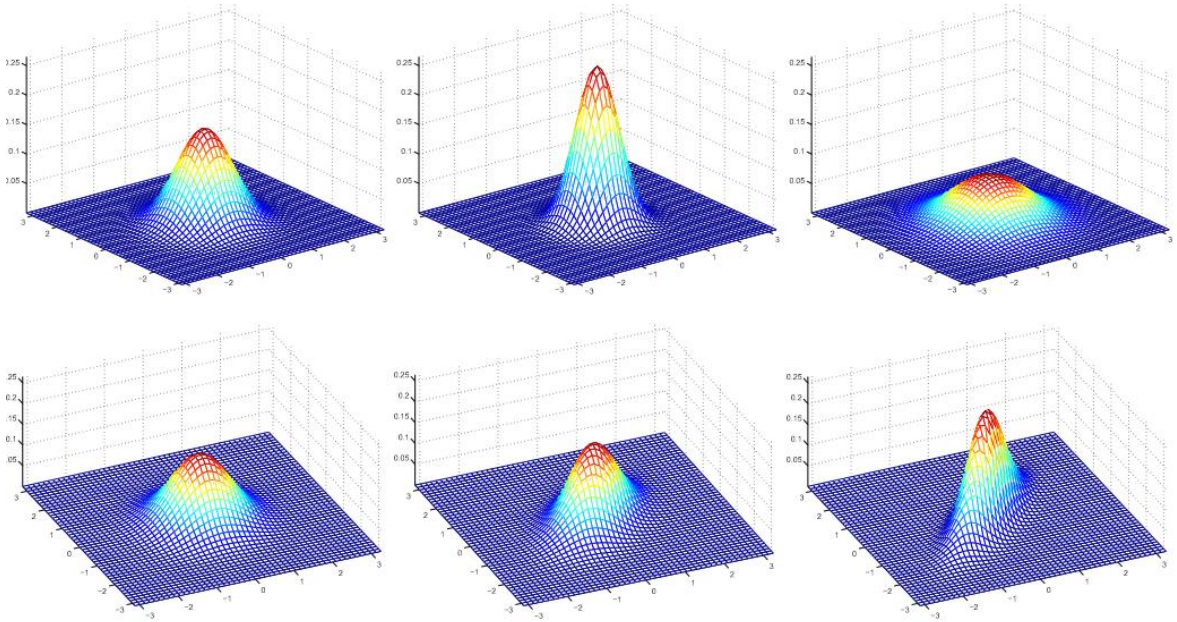


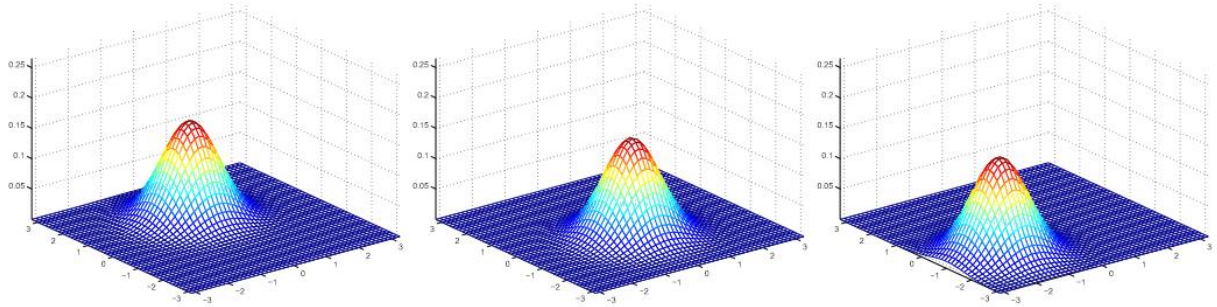
$$\Sigma = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}; \quad \Sigma = \begin{bmatrix} 1 & -0.8 \\ -0.8 & 1 \end{bmatrix}; \quad .\Sigma = \begin{bmatrix} 3 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

Multivariate Gaussian

The multivariate Gaussian distribution, is parameterized by a mean vector $\mu \in R^n$ and a covariance matrix $\Sigma \in R^{n \times n}$ where $\Sigma \geq 0$ is symmetric and positive semi-definite. $|\Sigma|$ denotes the determinant of Σ:

$$p(x; \mu, \Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right)$$



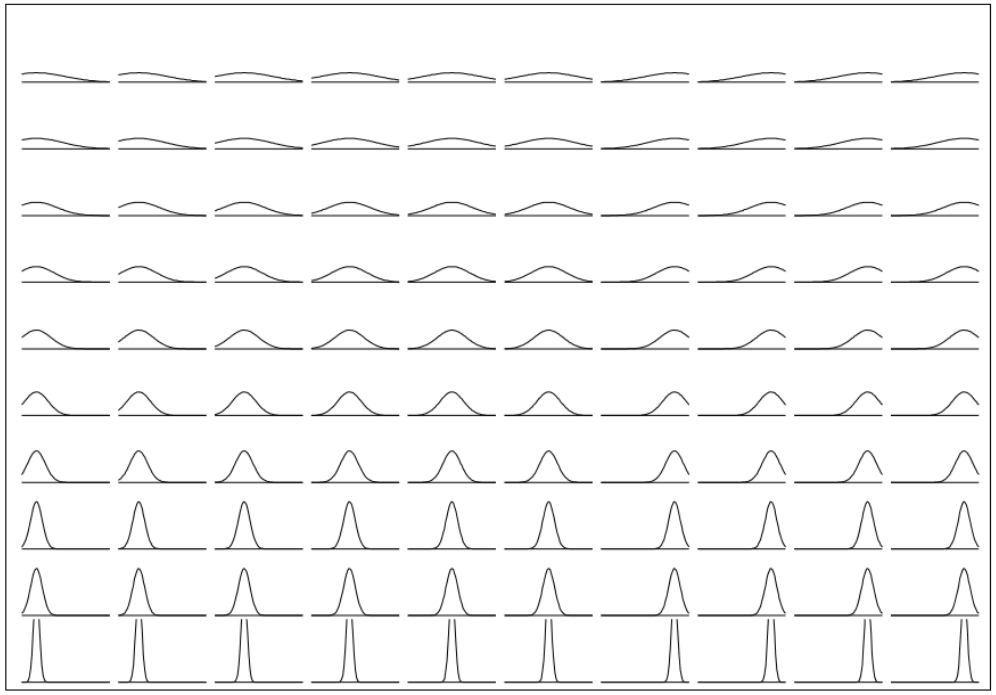


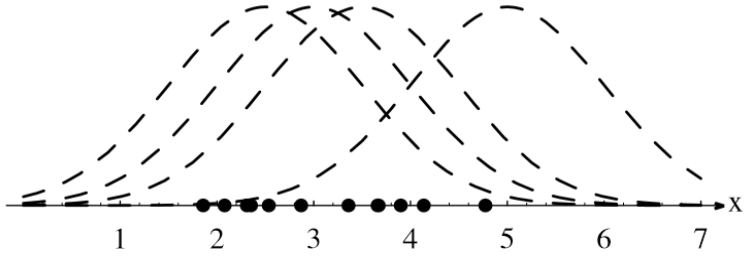
$$\mu = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad \mu = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}; \quad \mu = \begin{bmatrix} -1 \\ -1.5 \end{bmatrix}$$

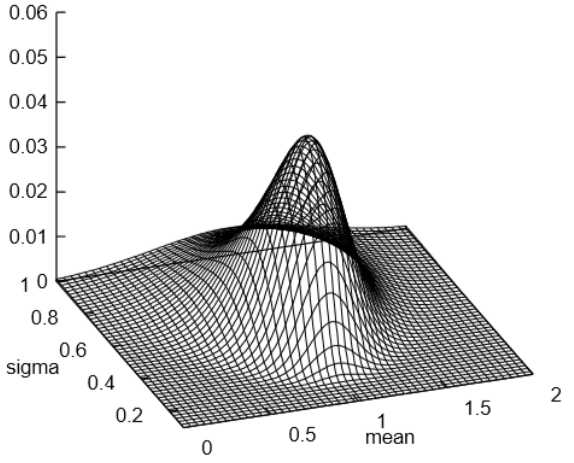
Maximum Likelihood

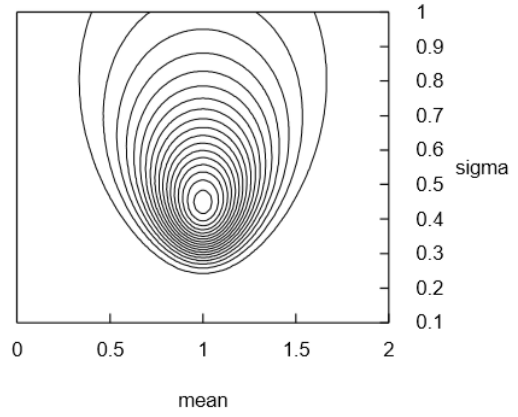
We change the mean and std in order to obtain the maximum likelihood of $\{x_1, x_2 \dots x_N\}$ (e.g. N=5):

Q5: How to calculate the likelihood?









Gaussian Discriminant Analysis

Given y is a binary random variable follows Bernoulli distribution parameterized with ∅:

$$\begin{aligned} y &\sim \text{Bernoulli}(\phi) \\ x|y=0 &\sim \mathcal{N}(\mu_0, \Sigma) \\ x|y=1 &\sim \mathcal{N}(\mu_1, \Sigma) \end{aligned}$$

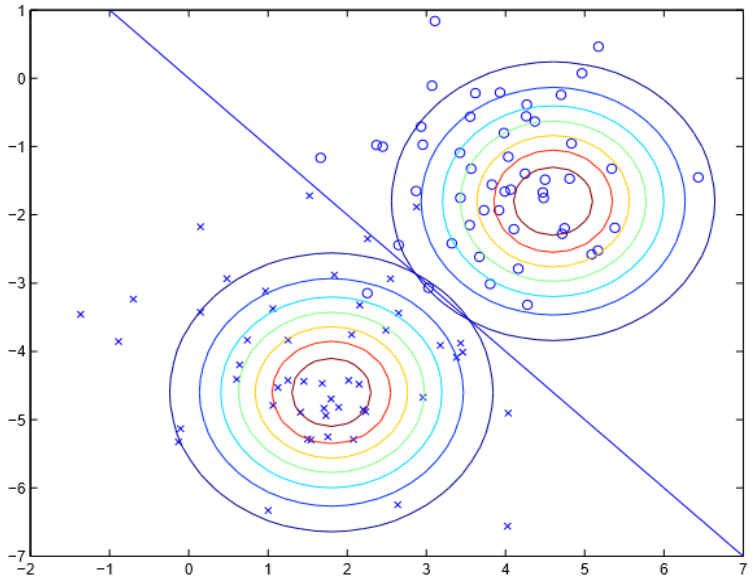
Specifically:

$$\begin{aligned} p(y) &= \phi^y(1-\phi)^{1-y} \\ p(x|y=0) &= \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu_0)^T\Sigma^{-1}(x-\mu_0)\right) \\ p(x|y=1) &= \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right) \end{aligned}$$

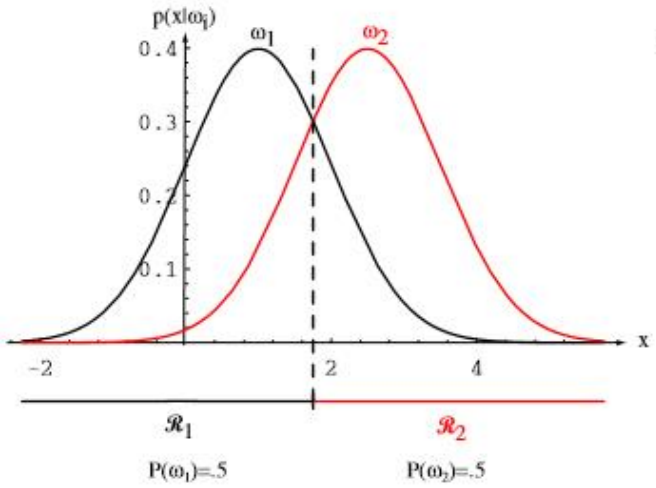
Q5: Generative or discriminative ?

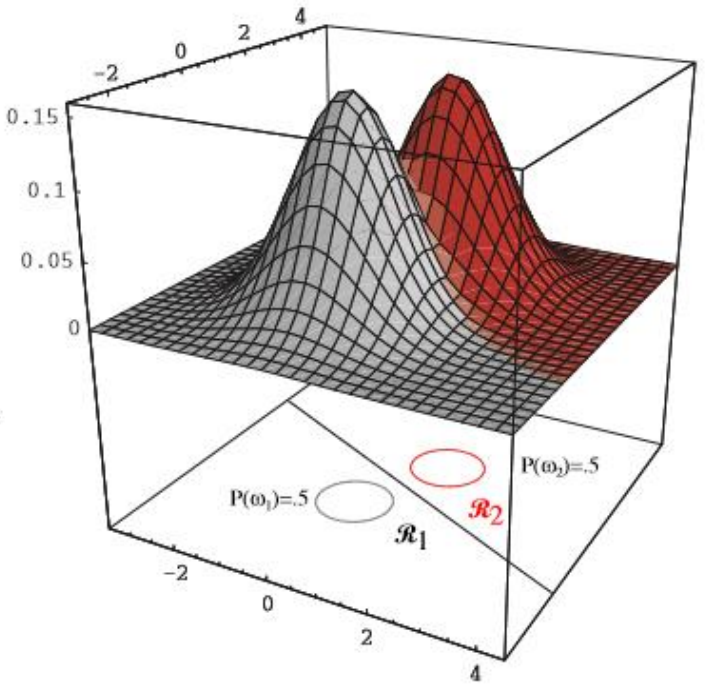
For generative learning, the log likelihood is:

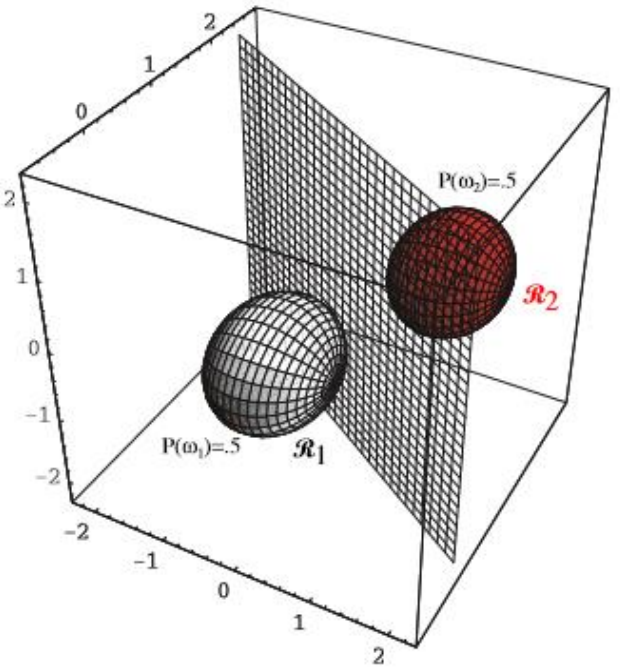
$$\begin{aligned} \ell(\phi, \mu_0, \mu_1, \Sigma) &= \log \prod_{i=1}^{m} p(x^{(i)}, y^{(i)}; \phi, \mu_0, \mu_1, \Sigma) \\ &= \log \prod_{i=1}^{m} p(x^{(i)}|y^{(i)}; \mu_0, \mu_1, \Sigma) p(y^{(i)}; \phi) \end{aligned}$$



Higher Dimensional Gaussian







Q6: What the boundary looks like if we have different covariance matrix?

Topic Model

Probabilistic Latent Semantic Analysis

Probabilistic latent semantic analysis (PLSA), is a statistical technique for the analysis of two-mode and co-occurrence data. In effect, one can derive a low-dimensional representation of the observed variables in terms of their affinity to certain hidden variables, just as in latent semantic analysis, from which PLSA evolved.

Considering observations in the form of co-occurrences (w,d) of words and documents, PLSA models the probability of each co-occurrence as a mixture of conditionally independent multinomial distributions:

d is the document index variable, c is a word's topic drawn from the document's topic distribution, $P(c|d)$, and w is a word drawn from the word distribution of this word's topic, $P(w|c)$. The d and w are observable variables, the topic c is a latent variable.

$$P(w,d) = \sum_c P(c)P(d|c)P(w|c) = P(d)\sum_c P(c|d)P(w|c)$$

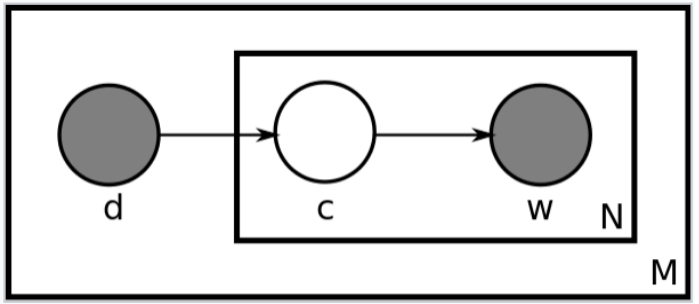
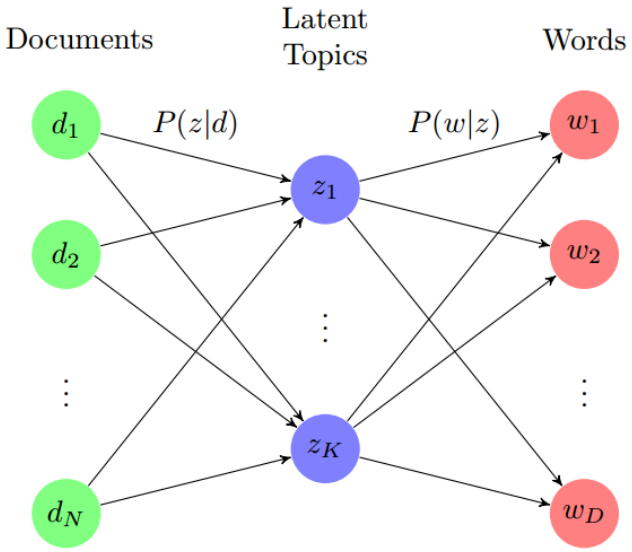
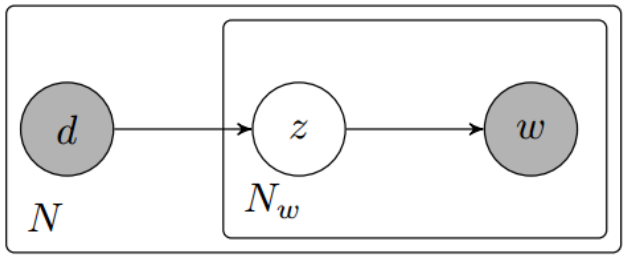


Plate Representation of P-LSA





The joint probability is represented by:

$$P(d,w) = P(d)P(w|d)$$

$$P(w|d) = \sum_{z\in\mathcal{Z}} P(w,z|d)$$

$$= \sum_{z\in\mathcal{Z}} P(w|d,z)P(z|d)$$

$$P(w|d) = \sum_{z\in\mathcal{Z}} P(w|z)P(z|d)$$

$$P(w,d) = \sum_{z\in\mathcal{Z}} P(z)P(d|z)P(w|z)$$

The likelihood is written as:

$$L = \prod_{(d,w)} P(w|d) = \prod_{d\in\mathcal{D}} \prod_{w\in\mathcal{W}} P(w|d)^{n(d,w)}$$

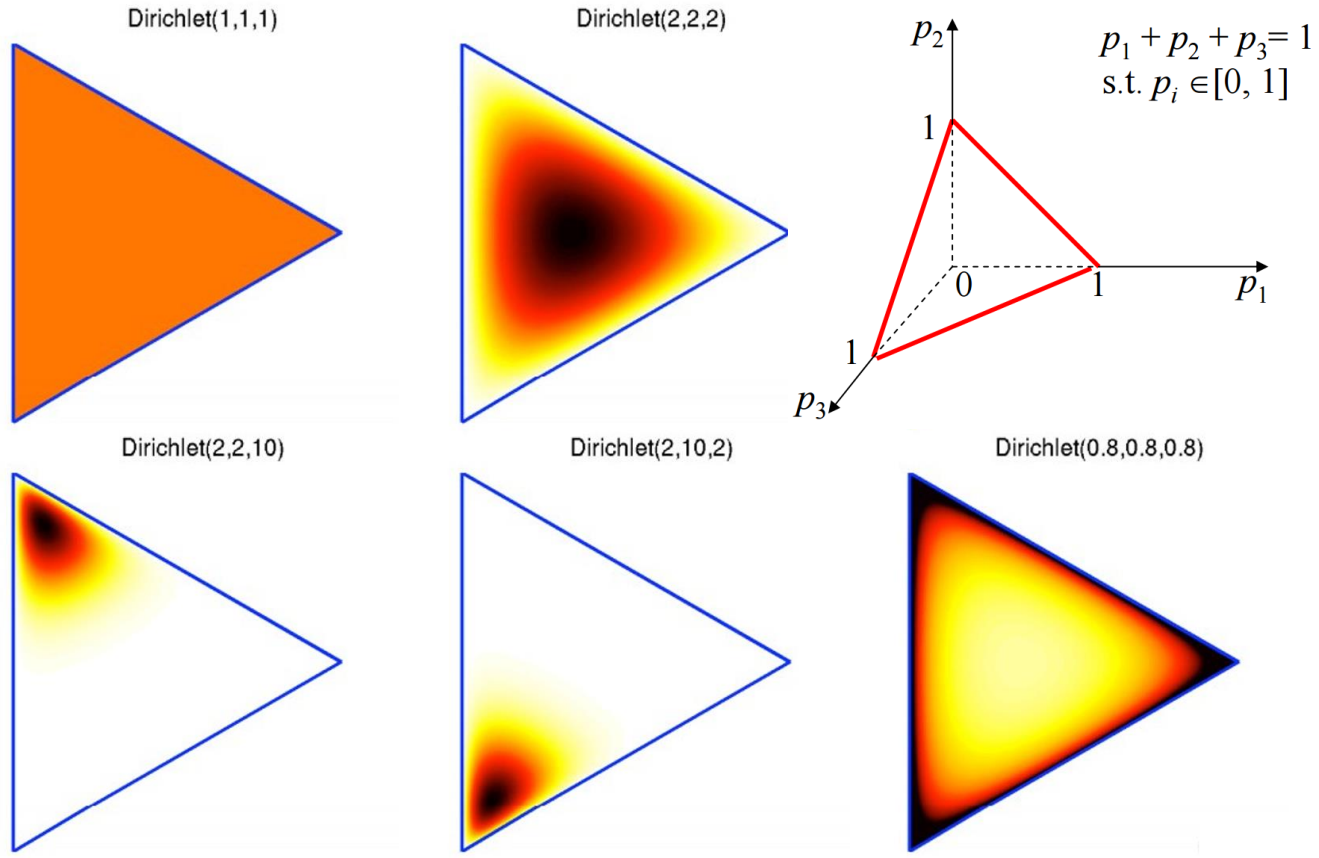
$$\mathcal{L} = \log L = \sum_{d\in\mathcal{D}} \sum_{w\in\mathcal{W}} n(d,w) \cdot \log \sum_{z\in\mathcal{Z}} P(w|z)P(z|d)$$

Dirichlet Distribution

The Dirichlet distribution is the multivariate generalization of the beta distribution. A Dirichlet distribution can be conceptualized as a probability distribution of distributions. Mathematically, it is represented by:

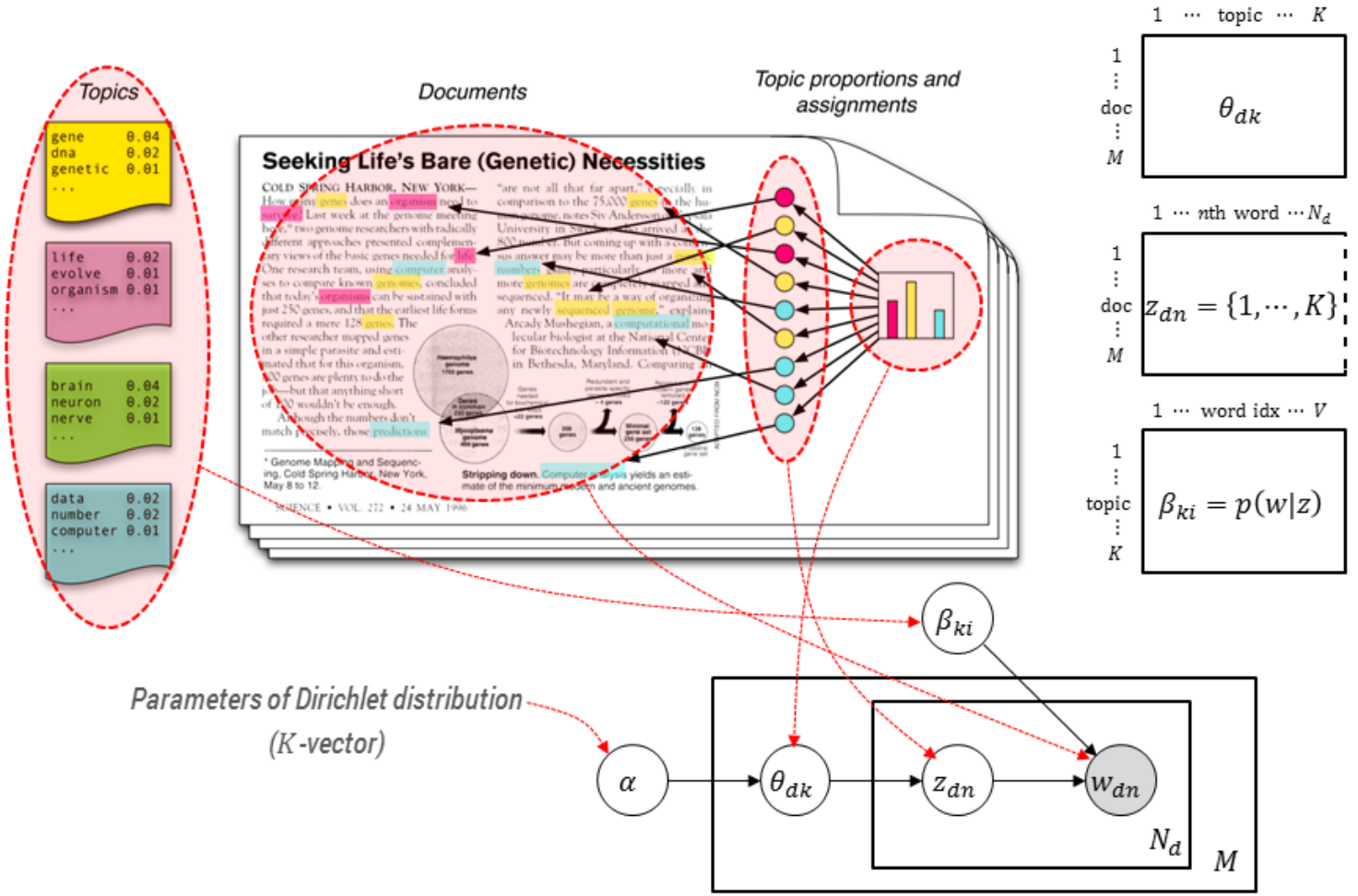
$$f(x_1, x_2, ..., x_K; \alpha_1, \alpha_2, ..., \alpha_K) = \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$

Where $0 < x_i < 1$ and $\sum_{i=1}^{K} x_i = 1$



Latent Dirichlet Allocation

Latent Dirichlet allocation (**LDA**) is a hierarchical generative statistical model. For example, if observations are words collected into documents, it posits that each document is a mixture of a small number of topics and that each word's creation is attributable to one of the document's topics.



An Example of Topics

Topic samples from training (Top words of the list).

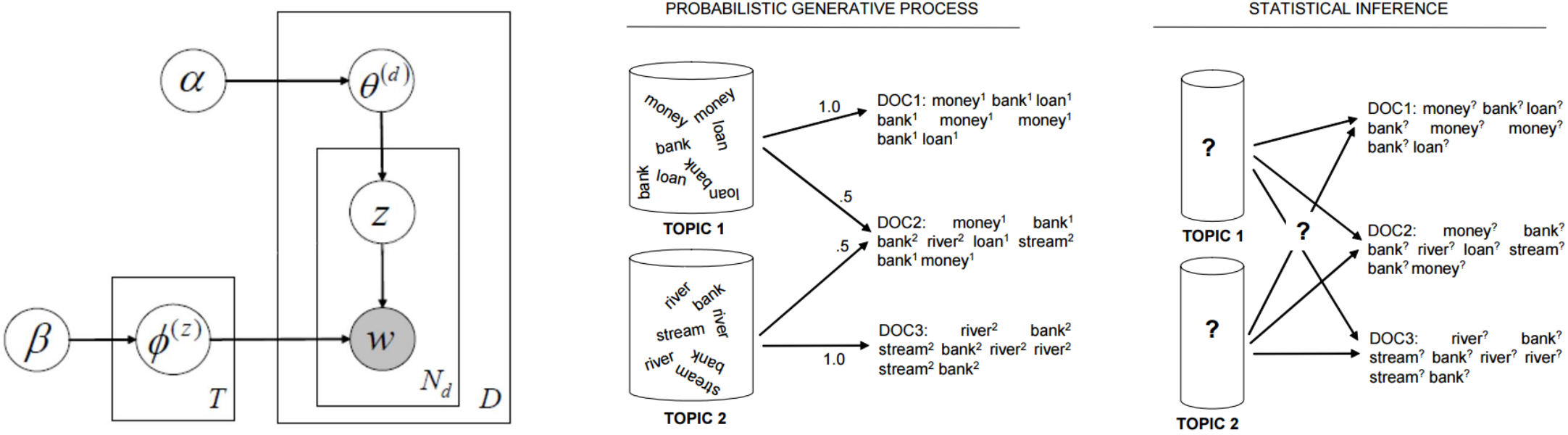
"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
THEATER	PROGRAMS	PERCENT	PRESIDENT
ACTRESS	GOVERNMENT	CARE	ELEMENTARY
LOVE	CONGRESS	LIFE	HAITI

Each topic is a distribution of words and a document can be represented as a mixture of topics. The following paragraph is a mixture of Arts, Budges, Children and Education.

The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.

Topic Models

Illustration of the generative process and the problem of statistical inference underlying topic models



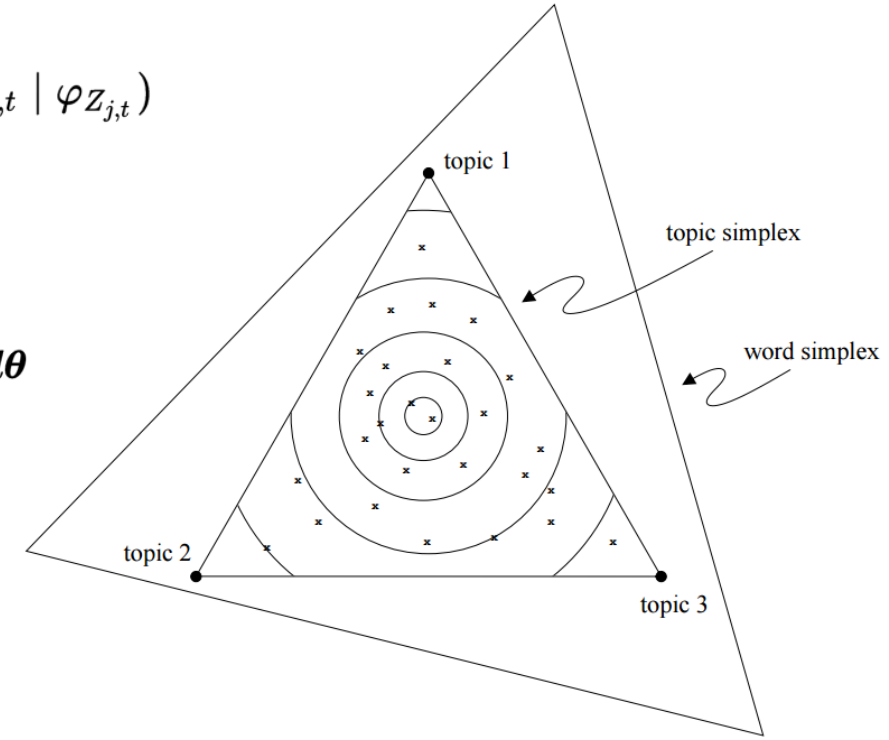
Likelihood

Illustration of the generative process and the problem of statistical inference underlying topic models

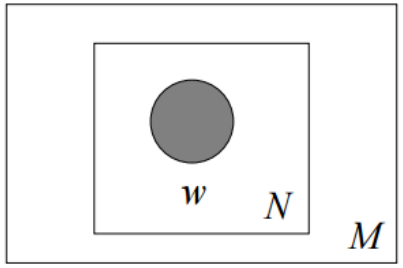
$$P(\boldsymbol{W}, \boldsymbol{Z}, \boldsymbol{\theta}, \boldsymbol{\varphi}; \alpha, \beta) = \prod_{i=1}^{K} P(\varphi_i; \beta) \prod_{j=1}^{M} P(\theta_j; \alpha) \prod_{t=1}^{N} P(Z_{j,t} \mid \theta_j) P(W_{j,t} \mid \varphi_{Z_{j,t}})$$

$$P(\boldsymbol{Z}, \boldsymbol{W}; \alpha, \beta) = \int_{\boldsymbol{\theta}} \int_{\boldsymbol{\varphi}} P(\boldsymbol{W}, \boldsymbol{Z}, \boldsymbol{\theta}, \boldsymbol{\varphi}; \alpha, \beta)\, d\boldsymbol{\varphi}\, d\boldsymbol{\theta}$$

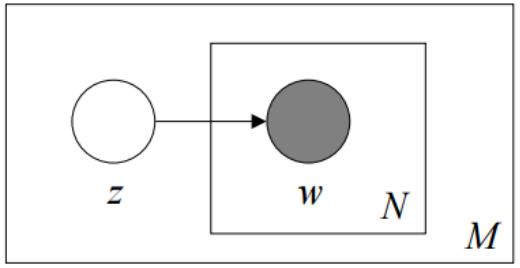
$$= \int_{\boldsymbol{\varphi}} \prod_{i=1}^{K} P(\varphi_i; \beta) \prod_{j=1}^{M} \prod_{t=1}^{N} P(W_{j,t} \mid \varphi_{Z_{j,t}})\, d\boldsymbol{\varphi} \int_{\boldsymbol{\theta}} \prod_{j=1}^{M} P(\theta_j; \alpha) \prod_{t=1}^{N} P(Z_{j,t} \mid \theta_j)\, d\boldsymbol{\theta}$$



Model Comparisons



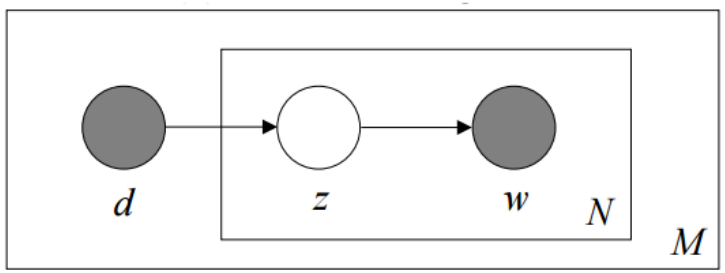
(a) unigram



(b) mixture of unigrams

$$p(\mathbf{w}) = \prod_{n=1}^{N} p(w_n)$$

$$p(\mathbf{w}) = \sum_{z} p(z) \prod_{n=1}^{N} p(w_n|z)$$



(c) pLSI/aspect model

$$p(d, w_n) = p(d) \sum_{z} p(w_n|z) p(z|d)$$

We compare LDA to simpler latent variable models for text—the unigram model, a mixture of unigrams, and the pLSI model. The pLSI model attempts to relax the simplifying assumption made in the mixture of unigrams model that each document is generated from only one topic.

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