Naïve Bayes Notes

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Naive Bayes

Given Bayes Theorem $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

In a classification problem, we hope to learn $X \rightarrow C$, where $C \in \{C_1, C_2, \cdots C_m\}$ is a set of class labels. Therefore:

$$P(C|X) = \frac{P(X|C)P(C)}{P(X)}$$

Since we only need to evaluate relative values of $P(C_j|X)$ for $j = 1, 2 \cdots m$. We can have:

$$P(C_j|X) = \frac{P(X|C_j)P(C_j)}{\sum_k P(X|C_k)P(C_k)}$$

Since $\sum_j P(C_j|X) = 1$, so the above equation can be explained by:

$$P(C_1|X) + P(C_2|X) + \cdots P(C_m|X) = 1$$

which implies:

$$\frac{P(X|C_1)P(C_1)}{D} + \frac{P(X|C_2)P(C_2)}{D} + \cdots \frac{P(X|C_m)P(C_m)}{D} = 1$$

which yields:

$$D = \sum_k P(X|C_k)P(C_k)$$

$P(C_j)$ is the prior probability.

If we assume all features are independent

The likelihood becomes:

$$P(x|c) = P(x_1|c)\,P(x_2|c) \cdots P(x_n|c)$$
$$= \prod_{i=1}^{N} P(x_i|c)$$

Finally, the posterior probability becomes:

$$P(c|x) = \frac{\prod_{i=1}^{n} P(x_i|c)\,P(c)}{D} \propto \prod_{i=1}^{n} P(x_i|c)\,P(c)$$