Lp-Norm of Vector

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Lp Norm

1) Given a vector $X = (x_1, x_2 \cdots x_n)$
L_0 norm of x is defined by the number of non-zero values in X.

$$\|x\|_0 = L_0(x) = \sum_{i=0}^{n} \Delta(x_i = 0) \qquad \begin{cases} \Delta(\text{True}) = 1 \\ \Delta(\text{False}) = 0 \end{cases}$$

2) The L_1 norm of x is

$$\|x\|_1 = L_1(x) = \sum_{i=1}^{n} |x_i|$$

3) The L_2 norm, which is the most common one:

$$\|x\|_2 = L_2(x) = \sqrt[2]{x_1^2 + x_2^2 + \cdots x_n^2} = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}$$

4) Generally, Lp norm is defined by:

$$\|x\|_p = L_p(x) = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}$$

5) For the special case of $p = \infty$

$$\|x\|_\infty = L_\infty(x) = \left(\sum_{i=1}^{n} |x_i|^\infty\right)^{1/\infty}$$

If $x_j = \max_{1 \le i \le n}(|x_i|)$, we can have:

$$|x_j|^\infty \gg |x_i|^\infty \quad \text{for } i \neq j$$

For example: $(3^{100} \gg 2.99^{100})$
Therefore:

$$\|x\|_\infty = \left(\sum_{i=1}^{n} |x_i|^\infty\right)^{1/\infty} = \left(|x_j|^\infty\right)^{1/\infty} = |x_j|$$

Finally: $\|x\|_\infty = \max_{0 \le i \le n} |x_i|$