2.5.1

假设u,v是相同维度向量, 请证明下面等式： $u^Tv = tr(vu^T)$

solution:

$u = (x_1, x_2, ..., x_n)^T$

$v = (y_1, y_2, ..., y_n)^T$

$u^Tv = x_1y_1 + x_2y_2 + ... + x_ny_n = \sum_{i=1}^n x_iy_i$

$$uv^T = \begin{bmatrix} x_1y_1 & \cdots & \cdots & \cdots \\ \cdots & x_2y_2 & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \cdots & \cdots & \cdots & x_ny_n \end{bmatrix}$$

$tr(uv^T) = \sum_{i=1}^n x_iy_i = u^Tv$

2.5.2

如果有两个相互独立的随机变量x,y,它们的联合分布为$p(x, y)$,请证明它们概率的香浓信息等于各自独立香浓信息的和:

$H(x, y) = H(x) + H(y)$

solution:

$$\begin{aligned} &H(x, y) \\ &= -E_{(x,y)}(ln(f(x, y))) \\ &= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)ln(f(x, y))dxdy \end{aligned}$$

因为x,y独立

$$\begin{aligned} &H(x, y) \\ &= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)f(y)[ln(f(x)) + ln(f(y))]dxdy \\ &= -[\int_{-\infty}^{\infty}(f(x)ln(f(x))dx] * \int_{-\infty}^{\infty} f(y)dy - [\int_{-\infty}^{\infty} f(y)ln(f(y))dy] * \int_{-\infty}^{\infty} f(x)dx \\ &= -\int_{-\infty}^{\infty}(f(x)ln(f(x))dx - \int_{-\infty}^{\infty} f(y)ln(f(y))dy \\ &= H(x) + H(y) \end{aligned}$$