第六章作业

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P188. 3

$f(x)=e^x$

x	0.82	0.83	0.84
$f(x)$	2.270500	2.293319	2.316367

① 对于节点 x_0, x_1 的一次插值基函数

$$l_0(x)=\frac{x-0.83}{0.82-0.83},\quad l_1(x)=\frac{x-0.82}{0.83-0.82}$$

函数值为 $f(x_0)=f(0.82)=2.270500$

$f(x_1)=f(0.83)=2.293319$

插值结果为

$$L_1(x)=f(x_0)\cdot l_0(x)+f(x_1)\cdot l_1(x)$$
$$=2.270500\cdot\frac{x-0.83}{0.82-0.83}+2.293319\cdot\frac{x-0.82}{0.83-0.82}$$
$$=-227.0500\cdot(x-0.83)+229.3319\cdot(x-0.82)$$

$\therefore L_1(0.826)\approx 2.284191$

$f(0.826)=e^{0.826}\approx 2.284164$

$\therefore$ 实际误差 $e=|f(0.826)-L_1(0.826)|\approx 2.7\times10^{-5}$

估计误差界：

$$|R_1(x)|=\left|\frac{1}{(n+1)!}f^{(n+1)}(\xi)w_{n+1}(x)\right|$$
$$=\left|\frac{1}{2!}f^{(2)}(\xi)w_2(x)\right|=\frac{1}{2}|(x-x_0)(x-x_1)e^{\xi}|\le\frac{h^{n+1}}{4(n+1)}\max_{a\le x\le b}|f^{(n+1)}(x)|$$
$$\le\frac{1}{2}\cdot\frac{(x_1-x_0)^2}{4}e^{0.83}=\frac{1}{2}\cdot\frac{(0.83-0.82)^2}{4}\cdot e^{0.83}=2.87\times10^{-5}$$

② 对于节点 x_0, x_1, x_2 的二次 Lagrange 插值基函数：

$f(x)=e^x$

x	0.82	0.83	0.84
$f(x)$	2.270500	2.293319	2.316367

$$l_0(x)=\frac{(x-0.83)(x-0.84)}{(0.82-0.83)(0.82-0.84)}$$
$$l_1(x)=\frac{(x-0.82)(x-0.84)}{(0.83-0.82)(0.83-0.84)}$$
$$l_2(x)=\frac{(x-0.82)(x-0.83)}{(0.84-0.82)(0.84-0.83)}$$

对应函数值为 $f(x_0)=2.270500$, $f(x_1)=2.293319$, $f(x_2)=2.316367$

$\therefore L_2(x) = f(x_0)\cdot l_0(x) + f(x_1)\cdot l_1(x) + f(x_2)\cdot l_2(x)$

$= 2.270500\cdot\frac{(x-0.83)(x-0.84)}{(0.82-0.83)(0.82-0.84)} + 2.293319\cdot\frac{(x-0.82)(x-0.84)}{(0.83-0.82)(0.83-0.84)} + 2.316367\cdot\frac{(x-0.82)(x-0.83)}{(0.84-0.82)\cdot(0.84-0.83)}$

$\therefore L_2(0.826) = 2.28416392$

$f(0.826) = e^{0.826} \approx 2.28416379$

$\therefore$ 实际误差 $e = |f(0.826) - L_2(0.826)| \approx 1.3\times 10^{-7}$

$|R_2(x)| = \frac{1}{3\times 2}|(x-x_0)(x-x_1)(x-x_2)|e^{0.84}$

$\leq \frac{\sqrt{3}}{27}M_3h^3$

$= \frac{\sqrt{3}}{27}\cdot e^{0.84}\cdot h^3 = \frac{\sqrt{3}}{27}e^{0.84}\cdot(0.01)^3 = 1.49\times 10^{-7}$

P189.9

用 $H_3(x)$ 近似计算 $f(1.25)$

$f(x) = e^{0.1x^2}$, $f(1.25)$? $(H_3(x))$

x	$f(x) = e^{0.1x^2}$	$f'(x) = 0.2xe^{0.1x^2}$
1	1.105170918	0.2210341836
1.5	1.252322716	0.375696148

	x_i	$f(x_i)$	一阶差商	二阶差商	三阶差商
x_0	1	1.105171			
x_0	1	1.105171	0.221034		
x_1	1.5	1.252323	0.294304	0.146540	
x_1	1.5	1.252323	0.375697	0.162786	0.032492

$H_3(x) = f(x_0) + f[x_0,x_0](x-x_0) + f[x_0,x_0,x_1](x-x_0)^2$
$+ f[x_0,x_0,x_1,x_1](x-x_0)^2(x-x_1)$

$= 1.105171 + 0.221034(x-1) + 0.146540(x-1)^2 + 0.032492(x-1)^2(x-1.5)$

$\therefore H_3(1.25) = 1.169081$

$f(1.25) = 1.169118$

$|f(1.25) - H_3(1.25)| \approx 3.7\times 10^{-5}$

P189.14

$f(x)$ $[-3,4]$ $x_0=-3$, $x_1=-2$, $x_2=1$, $x_3=4$.

$f(x_0)=2$, $f(x_1)=0$, $f(x_2)=3$, $f(x_3)=1$

$f''(x_0)=0$, $f''(x_3)=0$

解: 由部分节点知, $h_0=1$, $h_1=3$, $h_2=3$

$M_0=0$, $M_3=0$

$$\mu_1 = \frac{h_0}{h_0+h_1} = \frac{1}{1+3} = \frac{1}{4}, \quad \mu_2 = \frac{h_1}{h_1+h_2} = \frac{3}{3+3} = \frac{1}{2}$$

$$\lambda_1 = 1-\mu_1 = \frac{3}{4}, \quad \lambda_2 = 1-\mu_2 = \frac{1}{2}$$

x_i	$f(x_i)$	一阶	二阶
$x_0=-3$	2		
$x_1=-2$	0	-2	
$x_2=1$	3	1	$\frac{3}{4}$
$x_3=4$	1	$-\frac{2}{3}$	$-\frac{5}{18}$

$$d_1 = 6f[x_0, x_1, x_2] = 6\times\frac{3}{4} = \frac{9}{2}$$

$$d_2 = 6f[x_1, x_2, x_3] = 6\times -\frac{5}{18} = -\frac{5}{3}$$

由 $$\begin{bmatrix} 2 & \lambda_1 \\ \mu_2 & 2 \end{bmatrix}\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} d_1 - \mu_1 M_0 \\ d_2 - \lambda_2 M_3 \end{bmatrix}$$

得 $$\begin{bmatrix} 2 & \frac{3}{4} \\ \frac{1}{2} & 2 \end{bmatrix}\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} \frac{9}{2} - \frac{1}{4}\times 0 \\ -\frac{5}{3} - \frac{1}{2}\times 0 \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ -\frac{5}{3} \end{bmatrix}$$

$f(x_0)=2,\ f(x_1)=0,\ f(x_2)=3,\ f(x_3)=1$

$x_0=-3,\ x_1=-2,\ x_2=1,\ x_3=4.$

$h_0=1,\ h_1=3,\ h_2=3$

$M_0=0,\ M_3=0$

解得 $M_1 = \frac{82}{29}$

$M_2 = -\frac{134}{87}$

代入 $S(x)$ 表达式：

$$S(x) = \begin{cases} \frac{82}{29}\cdot\frac{(x+3)^3}{6} + 2\cdot(-2-x) + \left(-\frac{82}{29\times 6}\right)\cdot(x+3), & x\in[-3,-2] \\ \frac{82}{29}\cdot\frac{(1-x)^3}{18} + \left(-\frac{134}{87}\right)\cdot\frac{(x+2)^3}{18} + \left(\frac{82\times 9}{29\times 6}\right)\cdot\frac{(1-x)}{3} + \left(3+\frac{134\times 9}{87\times 6}\right)\cdot\frac{(x+2)}{3}, & x\in[-2,1] \\ -\frac{134}{87}\cdot\frac{(4-x)^3}{18} + \left(3+\frac{134\times 9}{87\times 6}\right)\cdot\frac{(4-x)}{3} + \frac{x-4}{3}, & x\in[1,4] \end{cases}$$