Ex. BrokenRank as a random walk

since

$$p_{ij} = \frac{w_{ij}}{d_i} \ , \ d_i = \sum_{k=1}^{N} w_{ik}$$

we have that

$$\begin{aligned}
\sum_{j=1}^{N} p_{ij} &= \sum_{j=1}^{N} \frac{w_{ij}}{d_i} \\
&= \frac{1}{d_i} \cdot \sum_{j=1}^{N} w_{ij} \\
&= \frac{1}{\sum_{k=1}^{N} w_{ik}} \cdot \sum_{j=1}^{N} w_{ij} \\
&= \frac{\sum_{k=1}^{N} w_{ik}}{\sum_{k=1}^{N} w_{ik}} \\
&= 1
\end{aligned}$$

Thus this complete the proof that

$$\sum_{j=1}^{N} p_{ij} = 1$$